LUMEN LEARNING

Calculus 1 Instructor Guide

Module 3: Understanding Limits

Table of Contents

Learning Outcomes

Detailed Course Learning Outcome Spreadsheet is linked here.

Topic	Student Learning Goals
A Preview of Calculus	• Identify the line that just touches a curve at one point by looking at how nearby lines approach it • Describe how integration can be used to calculate the area under a curve
Introduction to the limit of a function	• Understand how to write the limit of a function using the correct symbols and estimate limits by examining tables and graphs • Understand one-sided limits (approaching a point from only one direction) and how they relate to two-sided limits • Understand and use the proper notation for infinite limits and define vertical asymptotes

Summary of Module

Background You'll Need

The assumed prerequisite skills:

- Determine whether a linear function is increasing, decreasing, or constant.
- Identify the vertical and horizontal asymptotes of a function
- Define slope for a linear function and calculate slope given two points

A Preview of Calculus

In this section, students were introduced to fundamental concepts in calculus, focusing on the tangent problem and differential calculus. They explored how the tangent line to a curve at a point can be used to understand the concept of the derivative. Students practiced finding slopes of tangent lines and learned how these relate to instantaneous rates of change.

The section also provided a preview of calculus, highlighting the key ideas and concepts that form the foundation of the subject. Students examined the difference between average and instantaneous rates of change and were introduced to the notion of limits.

Additionally, students explored the area problem, which leads to the concept of integral calculus. They learned how to approximate the area under a curve using methods such as Riemann sums, and how these approximations improve as the intervals get smaller.

Introduction to the limit of a function

In this section, students focused on the concept of limits, a foundational idea in calculus. They began with the formal definition of a limit and explored how to evaluate limits of functions as they approach a particular point. Through examples and exercises, students practiced finding limits using various techniques, including direct substitution and algebraic manipulation.

Students also examined one-sided limits, learning how to determine the limit of a function as it approaches a specific value from either the left or the right. This included understanding and applying the notation for one-sided limits. The section further introduced infinite limits, where students explored situations in which functions grow without bound as they approach a certain point. They practiced identifying and working with vertical asymptotes and understanding the behavior of functions near these asymptotes.

Module Resources

Cheat Sheet

Understanding Limits: Cheat Sheet

Worksheets/Handouts

Limits Activity Tangent Lines: Station 1

Limits Activity Tangent Lines: Station 1 Answer Key

Limits Activity Area Under A Curve: Station 2

Limits Activity Area Under A Curve: Station 2 Answer Key

Limits Activity Estimating Limits: Station 3

Limits Activity Estimating Limits: Station 3 Answer Key

Limits Activity One-Sided and Infinite Limits: Station 4

Limits Activity One-Sided and Infinite Limits: Station 4 Answer Key

Limits and the Area of a Circle Discussion

The Tangent Problem in Real-World Applications Discussion

Limit Analysis in Rate of Change Problems Writing Task

Activity One: Limits Exploration Circuit: Navigating the World of Limits and Their Applications in Calculus

Evidence-Based Teaching Practices

Community-Building

Educators encourage students to build relationships with one another to establish peer networks of support by having groups work collaboratively at each station and participate in a cross-share session where they compare answers, clarify doubts, and assist each other with challenging concepts.

Collaborative

Educators design group projects that require students to work together in order to help them master both core content and critical skills related to collaboration and teamwork by splitting the class into groups to circulate through four stations, working collaboratively to complete each station's tasks.

Connections

Educators help students understand the relationships between the topics and ideas in the course by designing stations that connect different aspects of limits, from foundational concepts to more complex applications like tangent lines and integrals.

Background

Students have learned to recognize a tangent to a curve as the limit of secant lines and to explain integrals through the area problem. This activity builds on these concepts by having students circulate through four stations designed to enhance their understanding of limits and their applications in calculus. Each station is dedicated to a specific category: Station 1 focuses on Tangent Lines, Station 2 on Integrals and Area, Station 3 on Estimating Limits, and Station 4 on One-Sided and Infinite Limits.

Instructions

Time Estimate: 60-75 minutes

1. **Conversation starter**

   What is a limit in the mathematical sense? Can you think of real-life situations where we approach a value but never quite reach it?

2. **Review**

   You may wish to do a brief overview of the different types of limits mentioned in this module: one-sided limits, two-sided limits, and infinite limits. It may be helpful to provide a visual of each using graphs.

3. **Split class into groups of 3-5 students**

   Students will need to be able to write down their ideas either on paper or their own copy of the worksheets. Students will eventually go to all four stations and will work collaboratively to complete each station.

4. **Distribute Worksheet**

   A pdf of the station worksheets can be found in the Module Resources of this document. Students can either receive their own copy of each station or they can work on a separate sheet of paper. Use your own discretion, but students may need at least 12-15 minutes for each station.

5. **Group Cross Share**

   Allow time for groups to cross share their findings and discuss their answers collaboratively. Encourage students to compare their answers, clarify doubts, and assist each other in areas where they may be struggling.

Discussion Prompts

- **How does the concept of a limit help us find the tangent line to a curve at a specific point?**

  Misconception: Students often think that the tangent line touches the curve at exactly one point and doesn't cross it.
  Goal: Help students understand that the tangent line represents the instantaneous rate of change at a point, which is found by considering the limit of secant lines as they approach that point.

- **How does the concept of limits relate to continuity of a function?**

  Misconception: Students often believe that if a limit exists at a point, the function must be continuous at that point.
  Goal: Clarify that while the existence of a limit is necessary for continuity, it's not sufficient. The function must also be defined at the point, and its value must equal the limit.

- **What information can infinite limits provide about the behavior of a function?**

  Goal: This prompt aims to help students connect infinite limits to the concept of vertical asymptotes and understand how functions behave as they approach certain x-values. The discussion should lead students to recognize that infinite limits can indicate rapid growth or decay in a function, and often point to the presence of vertical asymptotes.

- **In what ways are limits used in fields outside of mathematics? Can you think of examples from physics, engineering, or other sciences where the concept of a limit is crucial?**

  Goal: Encourage students to see the broader applicability of limits beyond pure mathematics. This discussion should help students appreciate the practical importance of the concept they're learning. Possible examples to explore:

  - Physics: Approaching the speed of light, quantum mechanics
  - Engineering: Stress analysis, circuit behavior
  - Economics: Marginal cost and revenue
  - Biology: Population growth models

Reflection

After the activity, we recommend that students complete exit cards. Have each student write on a piece of paper one key concept they learned from the activity and one concept they have questions about. Below are some suggestions for students:

- Which type of limit (one-sided, two-sided, or infinite) do you feel most confident working with after today's activity, and why?
- How would you explain the relationship between limits and tangent lines to a peer who missed today's session?
- Describe a real-world scenario where understanding limits would be crucial.
- What strategies did you use to approach the station activities, and how effective were they?
- How did you adapt your approach when you encountered difficulties in understanding a particular limit concept?

Online Variation

Provide students with the four different worksheets digitally and have them complete on their own or with a group. To facilitate group discussions and cross-sharing, students can post their findings, answers, and insights in a discussion forum. Students can comment on each other's discussion boards allowing them to compare answers, ask questions, and assist each other in areas where they may be struggling.

For synchronous online sessions, utilize Zoom breakout rooms:

1. Create four breakout rooms, each corresponding to one of the stations.
2. Assign students to initial breakout rooms, ensuring a mix of students in each room.
3. After 12-15 minutes, rotate students to the next room, ensuring each group visits all four stations.
4. Use the main room for group cross-sharing and final discussions.
5. Utilize Zoom's whiteboard feature or a shared Google Doc for collaborative work within breakout rooms.

Assignments

The Tangent Problem in Real-World Applications Discussion

In this discussion, students will explore real-world applications of the tangent problem by finding examples where calculating instantaneous rates of change is important in fields such as physics, economics, medicine, or environmental science. Students will analyze their chosen application by identifying what the function represents, what the variables signify, and what the tangent line or instantaneous rate of change reveals about the situation. They'll create a post describing their real-world application, explaining why calculating instantaneous rates of change is essential in this context, and providing a specific example with realistic values to demonstrate the calculation process. After posting, students will engage with classmates by identifying connections between applications, asking questions about handling specific scenarios, and suggesting how the accuracy of rate of change calculations could be improved, helping to deepen everyone's understanding of how the tangent problem applies across various disciplines.

The Tangent Problem in Real-World Applications Discussion

We ask that you make your own copy to edit and adjust to fit the needs of your classroom

Limit Analysis in Rate of Change Problems Writing Task

In this writing task, students will analyze a pharmaceutical time-release medication model to understand instantaneous rates of change through limits. Students will first calculate average rates of change over progressively smaller time intervals to identify patterns and understand what these rates represent in the medication context. They'll then use the limit definition to find the instantaneous rate of change at a specific time, explaining what this value reveals about how the medication concentration is changing at that exact moment. Students will analyze when the medication reaches maximum concentration, determine when concentration drops below a threshold requiring another dose, and explain the connection between limits and instantaneous rates of change in medication dosing.

Limit Analysis in Rate of Change Problems Writing Task

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Limits and the Area of a Circle

In this discussion, students will explore the concept of limits by approximating the area of a circle using the method of exhaustion. They will work with the formula for the area of an n-sided regular polygon circumscribed around a circle of radius 1 to calculate and compare areas. Students will start with polygons of fewer sides and gradually increase the number of sides, observing how the estimated area approaches the actual area of the circle, π. They will calculate and record the areas for different values of n and determine how many sides are needed for the polygon's area to match π to five decimal places. The activity aims to deepen students' understanding of limits and historical mathematical methods while fostering collaborative learning and critical thinking.

Limits and the Area of a Circle Discussion

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