Calculus I Exam 2 Review

2.5: 26 . Find the derivative

$$s(t) = \sqrt{\frac{1+\sin t}{1+\cos t}}$$

$$s'(t) = \frac{1}{2}\left(\frac{1+\sin t}{1+\cos t}\right)^{-\frac{1}{2}} \frac{(1+\cos t)\cos t - (1+\sin t)(-\sin t)}{(1+\cos t)^2}$$

$$= \frac{\cos t\,(1+\cos t) + \sin t\,((-\sin t)}{2(1+\cos t)^{3/2}(1+\sin t)^{1/2}}$$

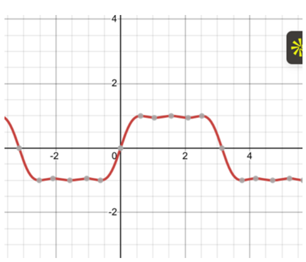
2.5: 40. Find the derivative

$$g(u) = \left[\left(u^2 - 1\right)^6 - 3u\right]^4$$

$$g'(u) = 4[(u^2-1)^6 - 3u]^3[12u(u^2-1)^5 - 3]$$

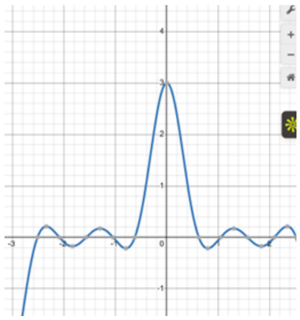
2.5: 60. The function $f(x) = \sin(x + \sin 2x)$, $0 \le x \le \pi$, arises in application to frequency modulation (FM) synthesis.

a. Use a graph of f produced by a calculator or computer to make a rough sketch of the graph of f'.



b. Calculate $f'(x)$ and use this expression, with a calculator or computer, to graph f'. Compare with your sketch in part a.

$f'(x) = \cos(x + \sin 2x)\,(1 + 2\cos 2x)$



2.5: 62. At what point on the curve $y = \sqrt{1 + 2x}$ is the tangent line perpendicular to the line $6x + 2y = 1$

$2y = -6x + 1 \implies y = -3x + \frac{1}{2} \implies m_1 = -3$

To make two lines perpendicular: $m_1 m_2 = -1 \implies m_2 = \frac{1}{3}$

$y' = \frac{1}{2}(1 + 2x)^{-1/2}\,2 = \frac{1}{\sqrt{1+2x}} = \frac{1}{3}$

$\sqrt{1 + 2x} = 3 \implies 1 + 2x = 9 \implies x = 4$

$y = \sqrt{1 + 2(4)} = 3$

Thus, the point is $(4, 3)$

2.6: 4 $\quad \frac{2}{x} - \frac{1}{y} = 4$

a) Find y' by implicit differentiation

$-\frac{2}{x^2} + \frac{1}{y^2}y' = 0 \Rightarrow y' = \frac{2y^2}{x^2}$

b) Solve the equation explicitly for y and differentiate to get y' in terms of x.

$\frac{1}{y} = \frac{2}{x} - 4 \Rightarrow y = \frac{1}{\frac{2}{x} - 4} = \frac{x}{2-4x}$

c) Check that your solutions are consistent by substituting the expression for y into your solution for part a

$y' = \frac{2\left(\frac{x}{2-4x}\right)^2}{x^2} = \frac{2x^2}{(2-4x)^2 x^2} = \frac{2}{(2-4x)^2}$ (consistent)

2.6: 24: Regard y as the independent variable and x as the dependent variable and use implicit differentiation to find dx/dy

$$y\sec x = x\tan y$$

$\frac{d}{dy}(y\sec x) = \frac{d}{dy}(x\tan y) \Rightarrow \sec x + y\sec x\tan x\frac{dx}{dy} = \frac{dx}{dy}\tan y + x\sec^2 y$

$\Rightarrow y\sec x\tan x\frac{dx}{dy} - \tan y\frac{dx}{dy} = x\sec^2 - \sec x \Rightarrow \frac{dx}{dy}(y\sec x\tan x - \tan y) = x\sec^2 - \sec x$

$\frac{dx}{dy} = \frac{x\sec^2 - \sec x}{y\sec x\tan x - \tan y}$

2.6: 30. Use implicit differentiation to find an equation of the tangent line to the curve at the given point.

a) $x^2 + 2xy + 4y^2 = 12,\ (2, 1)$ (ellipse)

$\frac{d}{dx}(x^2 + 2xy + 4y^2) = 2x + 2y + 2x\frac{dy}{dx} + 8y\frac{dy}{dx} = 0$

$(2x + 8y)\frac{dy}{dx} =- (2x + 2y) \Rightarrow \frac{dy}{dx} = \frac{-(2x+2y)}{2x+8y} = \frac{-(x+y)}{x+4y} = \frac{-(2+1)}{2+4(1)} = \frac{-3}{6} =- \frac{1}{2}$

$\Rightarrow$ Slope $m = \frac{-1}{2}$

$y - y_1 = m(x - x_1) \Rightarrow y - 1 = \frac{-1}{2}(x - 2) \Rightarrow y =- \frac{1}{2}x + 2$

b) $x^2 + 3xy + 9y^2 = 19,\ (2, 1)$ (ellipse)

$\frac{d}{dx}(x^2 + 3xy + 9y^2) = 2x + 3y + 3x\frac{dy}{dx} + 18y\frac{dy}{dx} = 0$

$(3x + 18y)\frac{dy}{dx} =- (2x + 3y) \Rightarrow \frac{dy}{dx} = \frac{-(2x+3y)}{3x+18y} = \frac{-(4+3)}{6+18} = \frac{-7}{24}$

$\Rightarrow$ Slope $m = \frac{-7}{24}$

$y - y_1 = m(x - x_1) \Rightarrow y - 1 = \frac{-7}{24}(x - 2) \Rightarrow y =- \frac{7}{24}x + \frac{19}{12}$

2.6: 36.

a) The curve with equation $y^2 = x^3 + 3x^2$ is called the Tschirnhausen cubic. Find an equation of the tangent line to this curve at the point $(1, \ -2)$.

$\frac{d}{dx}(y^2) = \frac{d}{dx}(x^3 + 3x^2) \Rightarrow 2y\frac{dy}{dx} = 3x^2 + 6x \Rightarrow \frac{dy}{dx} = \frac{3x^2+6x}{2y} = \frac{3(1)^2+6(1)}{2(-2)} = -\frac{9}{4}$

$m = -\frac{9}{4}$

$y - y_1 = m(x - x_1) \Rightarrow y - (-2) = -\frac{9}{4}(x - 1) \ => \ y = -\frac{9}{4}x + \frac{1}{4}$

b) At what points does this curve have horizontal tangents?

For a horizontal tangent, slope = 0

$\frac{dy}{dx} = \frac{3x^2+6x}{2y} = 0 \Rightarrow 3x^2 + 6x = 0 \Rightarrow 3x(x + 2) \Rightarrow x = 0 \ or \ x = -2$

For $x = 0$:

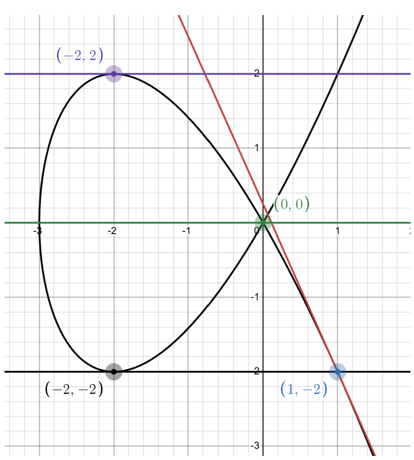
$$y^2 = (0)^3 + 3(0)^2 = 0 \Rightarrow y = 0$$

For $x = -2$:

$$y^2 = (-2)^3 + 3(-2)^2 = -8 + 12 = 4 \Rightarrow y = \pm 2$$

Horizontal tangent points: $(0, 0)$, $(-2, 2)$, $(-2, \ -2)$

c) Illustrate parts (a) and (b) by graphing the curve and the tangent lines on a common screen.



2.7: 18

a. The volume of a growing spherical cell is $V = \frac{4}{3}\pi r^3$, where the radius r is measured in micrometers ($1\ \mu m = 10^{-6} m$). Find the average rate of change of V with respect to r when r changes from

i. $5\ to\ 8\ \mu m$

$\frac{\Delta V}{\Delta r} = \frac{\frac{4}{3}(8^3 - 5^3)}{8-5}\pi = 172\pi\ \mu m^2$

ii. $5\ to\ 6\ \mu m$

$\frac{\Delta V}{\Delta r} = \frac{\frac{4}{3}(6^3 - 5^3)}{6-5}\pi = \frac{364}{3}\pi\ \mu m^2$

iii. $5\ to\ 5.1\ \pi m$

$\frac{\Delta V}{\Delta r} = \frac{\frac{4}{3}(5^3 - 5.1^3)}{5-5.1}\pi = 102\pi\ \mu m^2$

b. Find the instantaneous rate of change of V with respect to r when $r = 5\ \mu m$.

$\frac{dV}{dr} = 4\pi r^2 = 4\pi(5^2) = 100\pi\ \mu m$

c. Show that the rate of change of the volume of a sphere with respect to its radius is equal to its surface area. Explain geometrically why this result is true.

When you add a thin layer around the sphere, the new volume you get is like covering the whole surface with that thin layer's thickness.

2.7: 30 The frequency of vibrations of a vibrating violin string is given by

$$f = \frac{1}{2L}\sqrt{\frac{T}{\rho}}$$

where L is the length of the string, T is its tension, and ρ is its linear density.

a) Find the rate of change of the frequency with respect to

i. the length (when T and ρ are constant),

$f = (\frac{1}{2}\sqrt{\frac{T}{\rho}})L^{-1} \Rightarrow \frac{df}{dL} =- (\frac{1}{2}\sqrt{\frac{T}{\rho}})L^{-2} =- \frac{1}{2L^2}\sqrt{\frac{T}{\rho}}$

ii. the tension (when L and ρ are constant), and

$f = (\frac{1}{2L\sqrt{\rho}})T^{1/2} \Rightarrow \frac{df}{dT} = \frac{1}{4L\sqrt{\rho}}T^{-1/2}$

iii. the linear density (when L and T are constant).

$f = \frac{\sqrt{T}}{2L}\rho^{-1/2} \Rightarrow \frac{df}{d\rho} =- \frac{\sqrt{T}}{4L\rho^{3/2}}$

b) The pitch of a note (how high or low the note sounds) is determined by the frequency f. (The higher the frequency, the higher the pitch.) Use the signs of the derivatives in part a to determine what happens to the pitch of a note

i. when the effective length of a string is decreased by placing a finger on the string so a shorter portion of the string vibrates, the note becomes higher because $\frac{df}{dL}$ is negative.

ii. when the tension is increased by turning a tuning peg, the note becomes higher because $\frac{df}{dT} > 0$

iii. when the linear density is increased by switching to another string, produce a lower note because $\frac{df}{d\rho} < 0$

2.8: 6. The length of a rectangle is increasing at a rate of 8 cm/s and its width is increasing at a rate of 3 cm/s. When the length is 20 cm and the width is 10 cm, how fast is the area of the rectangle increasing (in square cm/s)?

Let L be the length and W be the width of the rectangle.

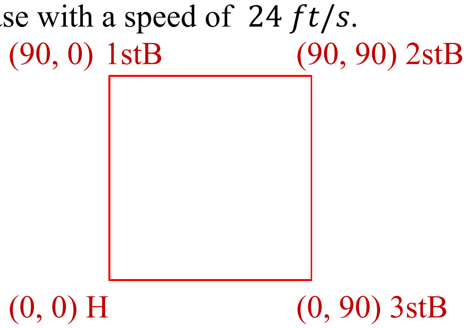
$\frac{dL}{dt} = 8, \frac{dW}{dt} = 3$

The area: $A = LW$

Then, $\frac{dA}{dt} = \frac{dL}{dt}W + L\frac{dW}{dt} = (8)(20) + (3)(10) = 160 + 30 = 190\, cm^2/s$

So, the area is increasing at the rate of 190 square cm/s.

2.8: 20. A baseball diamond is a square with side $90\ ft$. A batter hits the ball and runs toward first base with a speed of $24\ ft/s$.



a) At what rate is his distance from second base decreasing when he is halfway to first base?

Let x be the distance from the batter to H.

The distance from the batter to 2nd base: $D_2 = \sqrt{(90 - x)^2 + 90^2}$

$$\frac{dD_2}{dt} = \frac{1}{2\sqrt{(90-x)^2+90^2}} 2(90 - x)(-1)\frac{dx}{dt} = \frac{-90+x}{\sqrt{(90-x)^2+90^2}}\frac{dx}{dt} = \frac{90+45}{\sqrt{(90-45)^2+90^2}}24 =- 10.7 ft/s$$

b) At what rate is his distance from third base increasing at the same moment?

The distance from the batter to 3rd base: $D_3 = \sqrt{x^2 - 90^2}$

$$\frac{dD_3}{dt} = \frac{1}{2\sqrt{x^2-90^2}}2x\frac{dx}{dt} = \frac{x}{\sqrt{x^2-90^2}}\frac{dx}{dt} = \frac{45}{\sqrt{45^2-90^2}}24 = 10.7 ft/s$$

2.8: 28. A trough is $10\ ft$ long and its ends have the shape of isosceles triangles that are $3\ ft$ across at the top and have a height of $1\ ft$. If the trough is being filled with water at a rate of $12\ ft^3/min$, how fast is the water level rising when the water is $6\ inches$ deep?

$$A = \frac{1}{2}bh = \frac{1}{2}(3h)(h) = \frac{3}{2}h^2$$

$$V = AL = \frac{3}{2}h^2(10) = 15h^2$$

$$\frac{dV}{dt} = 30h\frac{dh}{dt} \Rightarrow 12 = 30(0.5)\frac{dh}{dt} \Rightarrow \frac{dh}{dt} = 0.8\ ft/min$$

2.9: 2. Find the linearization $L(x)$ of the function at a

$f(x) = \cos 2x\,, a = \pi/6$

$$f(\frac{\pi}{6}) = cos(2\frac{\pi}{6}) = cos(\frac{\pi}{3}) = \frac{1}{2}$$

$$f'(\frac{\pi}{6}) =- 2sin(2a) =- 2sin(2\frac{\pi}{6}) =- 2sin(\frac{\pi}{2}) =- 2\frac{\sqrt{3}}{2} =- \sqrt{3}$$

$$L(x) = f(\frac{\pi}{6}) + f'(\frac{\pi}{6})(x - \frac{\pi}{6}) = \frac{1}{2} - \sqrt{3}(x - \frac{\pi}{6}) =$$

2.9: 4. Find the linearization $L(x)$ of the function at a

$$f(x) = 2/\sqrt{x^2 - 5},\ a = 3$$

$$f(3) = \frac{2}{\sqrt{3^2-5}} = 1$$

$$f'(3) = 2\frac{-1}{2}(x^2 - 5)^{-3/2}(2x) = \frac{-2x}{(x^2-5)^{3/2}} = \frac{-2(3)}{(3^2-5)^{3/2}} =- \frac{3}{4}$$

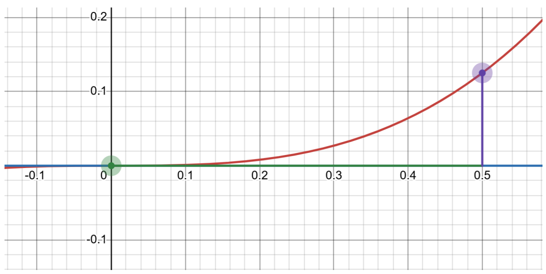
$$L(x) = f(3) + f'(3)(x - 3) = 1 - \frac{3}{4}(x - 3)$$

2.9: 26. Compute Δy and dy for the given values of x and $dx = \Delta x$. Then sketch a diagram showing the line segments with lengths dx, dy, and Δy.

$y = x^3$, $x = 0$, $\Delta x = 0.5$

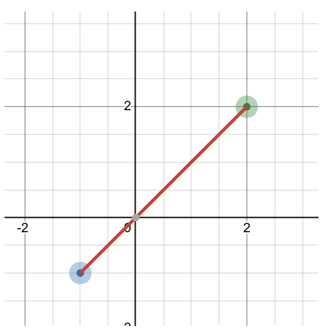
$\Delta y = f(x + \Delta x) - f(x) = (x + \Delta x)^3 - x^3 = (0 + 0.5)^3 - 0^3 = 0.125$

$dy = f'(0)dx = 3x^2 dx = 3(0^2)(0.5) = 0$



3.1: 12

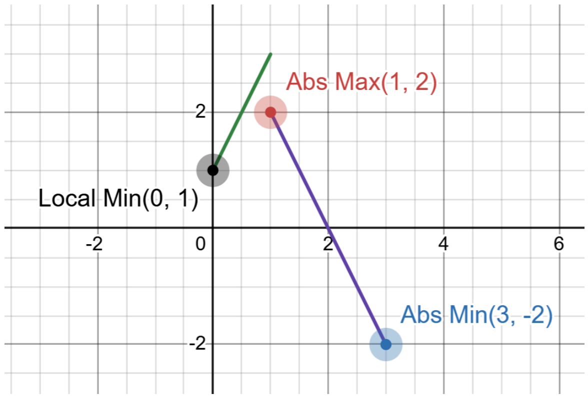
a) Sketch the graph of a function on $[-1, 2]$ that has an absolute maximum but no local maximum.



b) Sketch the graph of a function on $[-1, 2]$ that has a local maximum but no absolute maximum.

3.1: 28. Sketch the graph of f by hand and use your sketch to find the absolute and local maximum and minimum values of f.

$f(x) = \{ \ 2x + 1 \quad if\ 0 \le x < 1 \quad and\ 4 - 2x \quad if\ 1 \le x \le 3\}$



3.1: 42. Find the critical numbers of the function.

$h(x) = x^{-1/3}(x - 2)$

Firstly, we can simplify the function: $h(x) = x^{2/3} - 2x^{-1/3}$

$h'(x) = \frac{2}{3}x^{-1/3} + \frac{2}{3}x^{-4/3} = \frac{2(x+1)}{3x^{4/3}}$

$h'(x)$ = 0 when x = -1.

$h'(x)$ is undefined when x = 0.

So, the critical numbers of the function are: x=0 and x=-1.

3.1: 46. Find the critical numbers of the function.

$g(x) = \sqrt{1 - x^2}$

$1 - x^2 >= 0 \Rightarrow x^2 <= 1 \Rightarrow$ The Domain is $[-1, 1]$

$g'(x) = \frac{1}{2}(1 - x^2)^{-1/2}(-2x) = \frac{-x}{\sqrt{1-x^2}}$

$\frac{-x}{\sqrt{1-x^2}} = 0 \Rightarrow x = 0 \Rightarrow x = 0$ is one critical number

$\sqrt{1 - x^2} = 0 \Rightarrow x = \pm 1$. Since they are in the domain, so they are also critical number

Critical number of $g(x)$: $x = -1, 0, 1$

3.1: 54. Find the absolute maximum and absolute minimum values of f on the given interval.

$f(x) = (t^2 - 4)^3,\ [-2,\ 3]$

$f'(t) = 3(t^2 - 4)^2(2t) = 6t(t^2 - 4)^2$

Set $f'(t) = 0$: $6t(t^2 - 4)^2 = 0 \Rightarrow t = 0,\ t = -2,\ t = 2$

$f(-2) = ((-2)^2 - 4)^3 = 0$

$f(0) = (0^2 - 4^3)^3 = -64$ (Absolute minimum)

$f(2) = (2^2 - 4)^3 = 0$

$f(3) = (3^3 - 4)^3 = 125$ (Absolute Maximum)

3.2: 16. Verify that the function satisfies the hypotheses of the Mean Value Theorem on the given interval. Then find all numbers that satisfy the conclusion of the Mean Value Theorem.

$f(x) = x^3 - 3x + 2,\ [-2,\ 2]$

Because $f(x)$ is a polynomial, it is continuous and differentiable everywhere

$f(-2) = (-2)^3 - 3(-2) + 2 = 0$

$f(2) = 2^3 - 3(2) + 2 = 4$

$f'(c) = \frac{f(b)-f(a)}{b-a} = \frac{4-0}{2-(-2)} = 1$

$f'(x) = 3x^2 - 3 = 1 \Rightarrow 3x^2 = 4 => x = \pm\frac{2}{\sqrt{3}}$, and both are in $[-2,\ 2]$

3.2: 20. Find the number c that satisfies the conclusion of the Mean Value Theorem on the given interval. Graph the function, the secant line through the endpoints, and the tangent line at $(c,\ f(c))$. Are the secant line and the tangent line parallel?

$f(x) = x^3 - 2x,\ [2,\ -2]$

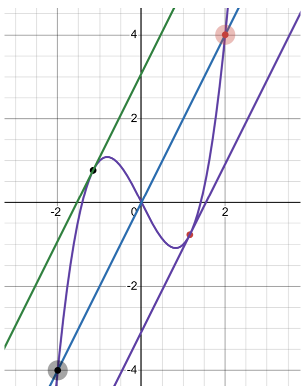
Since $f(x)$ is a polynomial, it's continuous and differentiable everywhere.

$f(2) = 2^3 - 2(2) = 4$

$f(-2) = (-2)^3 - 2(-2) = -4$

$f'(c) = \frac{f(2)-f(-2)}{2-(-2)} = \frac{4-(-4)}{4} = 2$

$f'(c) = 3c^2 - 2 = 2 \Rightarrow 3c^2 = 4 \Rightarrow c = \pm \frac{2}{\sqrt{3}}$, and both are in $(-2, 2)$



3.2: 23. Show that the equation has exactly one real solution.

$2x + \cos x = 0$

$f(x)$ is a sum of continuous functions ($2x$ and $\cos x$), so $f(x)$ is continuous everywhere.

Since $f(-10) < 0 < f(10)$, and $f(x)$ is continuous on $[-10, 10]$, by Intermediate Value Theorem, there exists some x between -10 and 10 that $f(x) = 0$. So, the function must have at least one solution.

$f'(x) = 2 - \sin x$

$-1 \leq \sin x \leq 1 \Rightarrow f'(x) = 2 - \sin x \geq 2 - 1 = 1 \Rightarrow f'(x)$ is always positive

$\Rightarrow f(x)$ is strictly increasing $\Rightarrow f(x)$ has at most one solution.

Thus, the equation has exactly one real solution.

3.3: 16. Find the intervals on which f is concave upward or concave downward, and find the infection points of f.

$f(x) = 2x^3 - 9x^2 + 12x - 3$

$f'(x) = 6x^2 - 18x + 12$

$f''(x) = 12x - 18$

$f''(x) = 0\, when\, 12x - 18 = 0 \Rightarrow 12x = 18 \Rightarrow x = \frac{3}{2}$

When $x < \frac{3}{2}$: $Pick\ x = 0 \Rightarrow f''(0) = 12(0) - 18 =- 18 < 0$

Concave downward on $(- \infty, \frac{3}{2})$

When $x < \frac{3}{2}$: $Pick\ x = 2 \Rightarrow f''(2) = 12(0) - 18 = 6 > 0$

Concave upward on $(\frac{3}{2}, \infty)$

$f(\frac{3}{2}) = 2(\frac{3}{2})^3 - 9(\frac{3}{2})^2 + 12(\frac{3}{2}) - 3 = \frac{3}{2}$

Inflection Point: $(\frac{3}{2}, \frac{3}{2})$

3.3: 22. Find the intervals on which f is concave upward or concave downward, and find the infection points of f.

$f(x) = cos^2 x - 2sin\, x,\ 0 \leq x \leq 2\pi$

$f(x) = cos^2 x - 2sin\, x = 1 - sin^2 x - 2sin\, x$

$f'(x) =- 2sin\, xcos\, x - 2cos\, x =- 2cos\, x(sin\, x + 1)$

$f''(x) =- 2[(- sin\, x)(sin\, x + 1) + cos\, xcos\, x] =- 2[- sin^2 x - sin\, x + cos^2 x]$

$= 2[sin^2 x + sin\, x - cos^2 x] = 2[sin^2 x + sin\, x - 1 + sin^2 x] = 2[2sin^2 x + sin\, x - 1]$

$= 4sin^2 x + 2sin\, x - 2$

When $f''(x) = 0,\ 4sin^2 x + 2sin\, x - 2 = 0 \Rightarrow 2sin^2 x + sin\, x - 1 = 0$

$\Rightarrow 2sin^2 x + 2sin\, x - sin\, x - 1 = 0 \Rightarrow (2sin^2 x + 2sin\, x) - (sin\, x + 1) = 0$

$\Rightarrow 2sin\, x(sin\, x + 1) - (sin\, x + 1) = 0 \Rightarrow (2sin\, x - 1)(sin\, x + 1) = 0$

$\Rightarrow sin\, x = \frac{1}{2}$ $or\ sin\, x =- 1 \Rightarrow x = \frac{\pi}{6}, \frac{5\pi}{6}$ $or\ x = \frac{3\pi}{2}$

At $(0, \frac{\pi}{6})$, $f''(x) =- 2 \Rightarrow$ Concave Down

At $(\frac{\pi}{6}, \frac{5\pi}{6})$, $f''(x) = 4 \Rightarrow$ Concave Up

At $(\frac{5\pi}{6}, \frac{3\pi}{2})$, $f''(x) =- 2 \Rightarrow$ Concave Down

At $(\frac{3\pi}{2}, 2\pi)$, $f''(x) =- 2 \Rightarrow$ Concave Down

3.3: 24. Find the local maximum and minimum values of using both the First and Second Derivative Tests. Which method do you prefer?

$f(x) = \frac{x^2}{x-1}$

Denominator: $x - 1 = 0 \Rightarrow x =- 1$

So the domain is: $x \neq- 1$

$f'(x) = \frac{(2x)(x-1)-x^2(1)}{(x-1)^2} = \frac{2x^2-2x-x^2}{(x-1)^2} = \frac{x^2-2x}{(x-1)^2}$

$f'(x) = 0 \Rightarrow \frac{x^2-2x}{(x-1)^2} = 0 \Rightarrow x^2 - 2x = 0 \Rightarrow x(x - 2) = 0 \Rightarrow x = 0\ or\ x = 2$

$f'(x)$ is undefined at $x = 1$, which is also not a part of the domain.

First Derivative Test:

$(-\infty, 0)$: $x =- 1 \Rightarrow x(x - 2) = 3 \Rightarrow$ Increasing

$(0, 1)$: $x = 0.5 \Rightarrow x(x - 2) =- 0.75 \Rightarrow$ Decreasing

$(1, 2)$: $x = 1.5 \Rightarrow x(x - 2) =- 0.75 \Rightarrow$ Decreasing

$(2, \infty)$: $x = 3 \Rightarrow x(x - 2) = 3 \Rightarrow$ Increasing

$x = 0$: $f(x)$ change from positive to negative $\Rightarrow$ Local maximum: $(0, 0)$

$x = 2$: $f(x)$ change from negative to positive $\Rightarrow$ Local minimum: $(2, 4)$

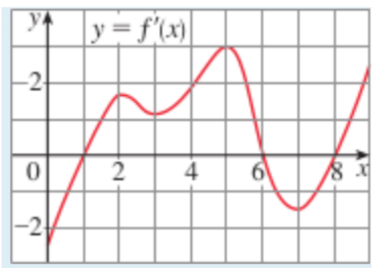
Second Derivative Test

$$f''(x) = \frac{2(x-1)^2 - x(x-2)2(x-1)}{(x-1)^4} = \frac{2(x-1)[(x-1)-(x^2-2x)]}{(x-1)^4} = \frac{2(x^2-2x+2)}{(x-1)^3}$$

$$f''(0) = \frac{2(0^2-2(0)+2)}{(0-1)^3} =- 4 > 0 \Rightarrow \text{ Concave down} \Rightarrow \text{Local max}$$

$$f''(2) = \frac{2(2^2-2(2)+2)}{(2-1)^3} = 4 > 0 \Rightarrow \text{Concave up} \Rightarrow \text{Local min}$$

3.3: 38. The graph of the derivative f' of a continuous function f is shown.



a) On what intervals is f increasing? Decreasing?

$(0, 1)$: $f'(x) < 0$

$(1, 6)$: $f'(x) > 0$

$(6, 8)$: $f'(x) < 0$

After 8: $f'(x) > 0$

Therefore, increasing on $(1, 6)$ and $(x > 8)$ and decreasing on $(0, 1)$ and $(6, 8)$

b) At what values of x does f have a local maximum? Local minimum?

Local minimum: $x = 1$ and $x = 8$ because $f'(x)$ change from negative to positive

Local maximum: $x = 6$ because $f'(x)$ change from positive to negative

c) On what intervals is f concave upward? Concave downward?

Concave up: $(0,\ 2),\ (3,\ 6),\ (7,\ 8)$

Concave down: $(2,\ 3),\ (6,\ 7)$

d) State the x-coordinate (s) of the point(s) of inflection. $x\ =\ 2,\ 3,\ 6,\ 7$

e) Assuming that $f(0)\ =\ 0$, sketch a graph of f.