Riddles, Riddles In the Dark...

2-envelope problem

You have two indistinguishable envelopes that each contain money. One contains twice as much as the other. You may pick one envelope and keep the money it contains. You pick at random, but before you open the envelope, you are offered the chance to take the other envelope instead.

Soln: dunno, still a contested problem. See wikipedia

2-numbers problem

I draw two numbers “at random” off the reals (according to some unknown distribution) and hold one in each hand. You can pick a hand and I will show you the number in that hand, and then you will have a chance to stay with that number or switch to the other hand. You win if you wind up with the higher of the two numbers. What is a strategy that gives you better than 50% chance of winning?

Soln: You draw a number at random from any normalizable distribution with support on the reals. If the revealed number is higher than your randomly chosen number, stay with it, otherwise switch. This works because you are partitioning the sample space

into two pieces where your strategy has ½ chance of winning, and one tiny sliver where your strategy always wins. NOTE – in the problem you don’t have to say the numbers are randomly drawn – the solution works given no information about how the two numbers are chosen.

2 prisoners and 2 hats

Two prisoners are told that they will each be given a hat, each hat may be red or blue. They can see the other prisoners hat but not their own. But if just one of them can successfully right down the color of his own hat, they will both be released. Otherwise they will be executed! They can discuss a strategy beforehand, but not communicate after hats are handed out. What is a strategy that will work every time?

Soln: you must partition the sample space into two sub-spaces and have one prisoner cover each subspace with a strategy that always wins in that subspace. In this case one prisoner writes the color he sees on the other prisoners hat, the other person writes the color he doesn’t see.

7 prisoners and 7 hats

Exact same thing, 7 different possible colors, 7 different prisoners, only one needs to write down his own hat color correctly, and they will all be released.

Soln: TBD

Cookie problem (2 ppl)

You and your friend have a cookie to split – what is a strategy for splitting the cookie fairly where both people will be happy?

Soln: one person breaks the cookie, the other person gets to choose which piece he wants. In this way the first person will make a break where he thinks the exact center is (otherwise he will wind up with the small piece), and the other person will either agree that it is 50/50 split, or take what he thinks is the biggest piece.

Cookie problem (n ppl)

Same thing, but now there are n of you!

Soln: someone (or a clever contraption) moves a knife slowly across the cookie and whenever someone says stop the knife will cut and he will get that piece. No one will say stop before they believe an nth of the cookie has been covered, but whenever someone thinks an nth is covered they will say stop, otherwise someone else may say stop and get bigger than an nth, meaning they will get smaller than an nth.

Blue Eyes, Brown Eyes, Green Eyes

There is a remote island inhabited by 100 ppl with green eyes, 100 ppl with blue eyes and one Guru with green eyes. None of them can communicate with one another, and they can see everyone else's eye color but not their own. They don't know any prior information about the eye color distribution. Every night a ship comes to the island and anyone who can correctly state their own eye color can leave, but if they are incorrect they are drowned. One day the Guru stands up in front of all of them and says "I see someone with blue eyes." Who leaves the island and when? Ok, now explain why the guru needs to talk at all, when what he says is not new information for anyone on the island!

Soln: consider the case of n=1 (1 blue eyed, 1 brown eyed, 1 guru), then n=2. For n=2 nobody leaves the island on the first night, but observing that gives the two blue eyed people information they can act on the 2^{nd} night.

Bear Problem

A hunter leaves his cabin one day and travels due south, he turns and travels due east, when he encounters a bear. He screams like a little girl and runs due north right back to his cabin. What color is the bear?

Soln: white, he lives at the north pole.
Alternative Note: you can live an a series of concentric circles around the south pole which satisfy 2*pi*r*n = 1 mile for integer n, and the sequence 1 mile south, 1 mile east, 1 mile north will still put you back at your house.

2-Doors Problem

You die suddenly and find yourself in a white room with two identical doors in front of you, guarded by two identical guards (one in front of each door). A sign on the wall tells you that one door goes to heaven the other leads to hell. The door to heaven is guarded by an angel who always tells the truth, the door to hell is guarded by a demon who always lies. You are allowed to ask one guard one question to determine which door you want to go through – what do you do?

Soln: "If I asked the other guard which door leads to heaven, what door would he point to?"
Alternative Soln: "Do you always answer the same way?"

Monty Hall Problem

Classic

Polar Bear Dice Problem

Four dice are thrown and an algorithm determines how many polar bears are present from the four dice faces shown. As many example throws can be given as are desired. A strongly suggestive riddle can be provided as a hint: "The game is in the name of the game – polar bears *around an ice hole – invented in the days of Ghengis Khan. A clue for you to keep you true – like petals around a rose, you can count each bear's nose." What is the algorithm?*

Soln: The algorithm is to count the number of dots which are arranged around central dots on die faces (polar bear noses around ice holes).

Physician

I don't know where this one comes from, but it stumps many. In each blank in the following you have to place the same letters in the same order to make a sentence that is both grammatical and sensible

The _______ physician was _______ to operate because he had ______ .

Soln: "notable, not able, no table" – the key is realizing you can insert blanks.

Conway's Sequence

Consider the sequence:
1, 11, 21, 1211, 111221, 312211, 13112221,...
a) What is the next element in the sequence? (easier)
b) When will the digit 4 appear for the first time? (harder)

Soln: The next element in the sequence is generated by listing out the groupings of digits in the previous element e.g. "one, one one, two ones, one two and two ones…" The number four will never appear since a grouping of 4 of any number, x, would imply a description like "x x's and x x's" which should instead be described as "2x x's"
Note: This is known as the "look and say" sequence and has an interesting Wikipedia entry.

Counting

8809 = 6	7111 = 0	2172 = 0	6666 = 4	1111 = 0
3213 = 0	7662 = 2	9313 = 1	0000 = 4	2222 = 0
3333 = 0	5555 = 0	8913 = 3	8096 = 5	7777 = 0
9999 = 4	7756 = 1	6855 = 3	9881 = 5	5531 = 0

2581 = ????

Soln: Count the number of closed loops appearing in the digits of each number.

More Prisoners in Hats

Four prisoners are arrested for a crime, but the jail is full and the jailer has nowhere to put them. He eventually comes up with the solution of giving them a puzzle so if they succeed they can go free but if they fail they are executed.

The jailer puts three of the men sitting in a line. The fourth man is put behind a screen (or in a separate room). He gives all four men party hats (as in diagram). The jailer explains that there are two red and two blue hats; that each prisoner is wearing one of the hats; and that each of the prisoners only see the hats in front of them but not on themselves or behind. The fourth man behind the screen can't see or be seen by any other prisoner. No communication between the prisoners is allowed.

If any prisoner can figure out and say to the jailer what color hat he has on his head all four prisoners go free. If any prisoner suggests an incorrect answer, all four prisoners are executed. The puzzle is to find how the prisoners can escape, regardless of how the jailer distributes the hats.

Sol'n: B gives C a chance to speak – if he doesn't hear C speak he says the color that he doesn't observe on B. This works because B knows that if he and A are wearing the same color, then C can infer his own color. Since B can see A's color, and if C remains silent he knows he must have a different color from A, then B can know the color of his hat.

What's With All the Hats in Prison?

A group of prisoners are lined up all facing forward such that each prisoner can only see those who are in front of him (or her!) in line. Now a guard places a hat on each prisoner's head, which they know will be either black or white. The prisoners will be given a chance to each state the color of their own hat, starting with the person at the back of the line (who can see all the other prisoners) and moving forward. The prisoners can discuss ahead of time and agree on a strategy they will use. If all but one of them can correctly state their own hat color then they will all be free!

Sol'n: They agree that the first prisoner to go will say 'black' if he sees an odd number of hats and 'white' if he sees an even number. The second prisoner can see all but his own hats so if he sees an odd number in front of him AND the first prisoner says black then he knows his hat is white. The third prisoner now knows that the second prisoner is seeing an odd number of hats, so based on what he sees, he can deduce his own hat color etc.

It's Impolite to Talk About Money

Several co-workers would like to know their average salary. How can they calculate it, without disclosing their own salaries?

Sol'n: The first person adds a random number to their salary and writes the sum (salary + random) on a piece of paper and passes along to the second person. The second person adds their salary and writes the new total on a piece of paper and passes that on. The third person does likewise. After everyone has contributed the final sum is given to the first person who subtracts their random noise number to recover the true total.

Birthday Logic

3 friends A,B,C – A and B want to know C's birthday. C lists 10 possible dates:
May 15, 16, 19
June 17, 18
July 14, 16
August 14, 15, 17
C tells A the month and tells B the day
A then says "I don't know, but I know B doesn't know."
B then says "I didn't know, but now I do know."
A then says "Well then I know too."
What is C's birthday?

Sol'n: The first statement by A eliminates May and June. The second statement eliminates the 14^{th} from July or August. The third statement forces July 16^{th} b/c if A know the date that means it was a month that had only two options, one of which was eliminated by the second statement (i.e. 14^{th} of July being eliminated means only 16^{th} is left, whereas in August eliminate the 14^{th} still leaves 15^{th} and 17^{th} as possibility).

Saint Ives

As I was going to Saint Ives, I met a man with seven wives. Each wife had seven sacks, each sack had seven cats, each cat had seven kits. Kits, cats, sacks and wives, how many were going to Saint Ives?

Sol'n: Only one. Since he 'met' them they were going the other direction.

Odd Ball Out

You have 12 balls and a balance scale. One of the balls is slightly heavier or lighter (you don't know which) than the other 11. You need to identify the odd ball out in three moves (three uses of the balance).

Sol'n:
Starting with three groups of four, compare two of them. If they are the same you have two moves to choose among the other four which you didn't weigh - this is possible. On the other hand if the two groups you weigh are different then proceed as follows:

Remove three balls from the left side of the scale and replace them with three of the balls from the un-weighed group of four (these you know are normal balls). Also take the fourth ball from the left side of the scale and swap it with one from the right side. Making these changes together constitutes the second move, and has three outcomes:

1) If the balance becomes equal then you know the odd ball is one of the three you removed from the left side and you also know whether the odd ball is heavier or lighter (depending on whether the left side was heavier or lighter during the first weighing) - so you can select among those three balls with the last move.

2) If the balance does not become equal, but does reverse direction of which side is heavier or lighter then you know the odd ball is one of the two that were swapped so you can choose between those two with the last move.

3) If the balance does not become equal and doesn't reverse direction you know the odd ball is one of the three that stayed on the right side the whole time and you also know whether the odd ball is heavier or lighter (depending on whether the right side was heavier or lighter during the first weighing) - so you can select among those three with the last move.

There are actually many more solutions, but they are all similar..... all of them involve a left right switch - there may be solutions that do not involve bring back in reference balls.

Five-Card Trick

In this card trick the magician and her assistant can agree beforehand on a strategy they will use. The magician will leave the room and an audience member will choose five cards from the deck (no jokers) and hand them to the assistant. The assistant will look at the five cards and choose one to give back to the audience member. The magician will return and the assistant will hand her the four remaining cards one at a time (all face up, all in the same orientation). The magician will then divine the identity of the fifth card that the audience member has. What is the strategy?

Sol'n:
The magician and the assistant will agree on an absolute hierarchy for all 52 cards e.g. the Ace of spades is #1, the king of spades is #2….,the two of diamonds is #52. They will also agree on a method for partitioning the set of cards into four equal partitions of 13 cards each (a simple one would be to partition by suit). Since the audience member has picked five cards, they are guaranteed that at least one of their partitions will be represented at least twice in the set of five. The assistant is now guaranteed to be able to select two cards from the same partition, which he does as a first step. Now the cards within a single partition can be represented along a circle like a clock face (but one with thirteen hours). So if the partitioning is by suit then e.g. the King of spades would be at 12 o'clock, the Ace of spades would be at 13 o'clock and the two of spades would be at 1 o'clock etc. The two cards the assistant has picked represent two numbers on this clock face, and you will be able to get from one to the other in at most six steps (worst case scenario). The assistant and magician will have agreed on a convention for stepping either clockwise or counterclockwise, let's say they use clockwise. Then from the two cards the assistant will choose the card which can be reached by stepping clockwise from the other card in six steps or less. He hands this card back to the audience member, and presents the second card to the magician as the first of four cards. The magician now knows that the card which the audience member holds is somewhere within six clockwise steps from this first card he is given. The assistant now must use the remaining three cards to encode the number of steps, which will be a number one through six. Since they have an agreed upon an absolutely hierarchy, the remaining three cards can identified as 'highest', 'middle' and 'lowest'. There are six ways to arrange these ranks so we can map the orderings to the number of steps (the assistant and magician will have agreed on this mapping beforehand). For instance, if the assistant presents the cards in the order 'highest', then 'middle', then 'lowest' that means one step etc.

Light Switch Signals

A warden meets with 23 prisoners. He tells them the following: Each prisoner will be placed into a room numbered 1-23. Each will be alone in the room, which will be soundproof, lightproof, etc. In other words, they will NOT be able to communicate with each other. They will be allowed one planning session before they are taken to their rooms. There is a special room, room 0. In this room are 2 switches, which can each be either UP or DOWN. They cannot be left in between, they are not linked in any way (so there are 4 possible states), and they are numbered 1 and 2. Their current positions are unknown. One at a time, a prisoner will be brought into room 0. The prisoner MUST change one and only one switch. The prisoner is then returned to his cell. At any time t, for N>0, there exists a finite t_0 by which time every prisoner will have visited room 0 at least N times. (In other words, there is no fixed pattern to the order or frequency with which prisoners visit room 0, but at any given time, every prisoner is guaranteed to visit room 0 again. If you're still confused by this statement, ignore it, and you should be ok). At any time, any prisoner may declare that all 23 of them have been in room 0. If right, the prisoners go free. If wrong, they are all executed. What initial strategy is 100% guaranteed to let all go free?

Sol'n:
One prisoner is designated as the counter. He always flips the left switch 'off' if he sees it on, and for every time he sees it 'on' he increments his counter (which starts at 0). If the counter sees the left switch 'off' he toggles the right switch. Any other prisoner who enters the room follows these rules: if the left switch is 'off' and he has not already toggled it 'on' twice, then he toggles it 'on', otherwise he toggles the right switch. After the counter reaches an increment of 43 then he knows that all the other 22 prisoners have been in the room at least once each.

The Smiths Throw a Party

Mr. and Mrs. Smith throw a party inviting four other couples. After everyone arrives there is a round of introductions in which people greet one another and shake hands - of course nobody shakes hands with their own spouse or shakes anyone's hand twice, that would just be odd. After all these greetings are exchanged Mr. Smith asks each person how many hands they shook and it turns out that each person he asked shook a different number of hands! How many hands did Mr. and Mrs. Smith shake?

Sol'n:

The first thing to note is Mr. Smith himself may have shaken the same number of hands as someone else (since he doesn't ask himself). Another useful thing to note is that the act of hand shaking will always increment two peoples counters. You can further frame it as the lowest person has 1 increment to apply to another person, the second lowest person has two increments to apply to two different people, etc. So is there a way to apply the low-count people's increments to the other half of the group such that the high-half people each get a different number of increments applied to them? I'll leave it at that, but the answer is that Mr. and Mrs. Smith both shake four hands.

Polyhedron Faces

Can you make a polyhedron for which every face has a different number of edges?

Sol'n: Consider a polyhedron with n faces. No face can have 0 edges obviously. The number of edges a face has is equal to the number of OTHER faces it touches, so it can't have n faces either. Since there are n faces but only n-1 possible edge counts, there must be at least two faces which have the same number of edges.