CVMS Math League

Power - Pythagorean Theorem and Similar Triangles

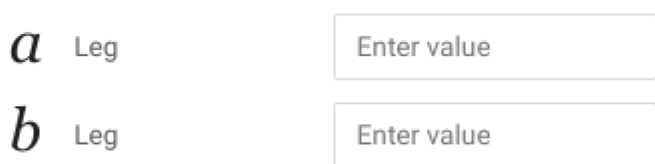
Pythagorean Theorem

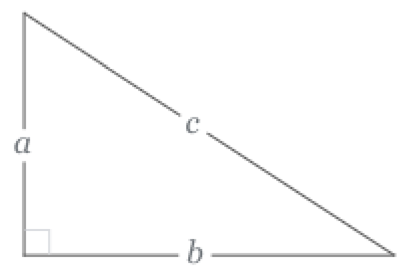
$a^2 + b^2 = c^2$

a and b are the legs of a right triangle, while c is the hypotenuse of a right triangle.

The Pythagorean Theorem only applies to right triangles.

$$c = \sqrt{a^2 + b^2}$$





Pythagorean Triples

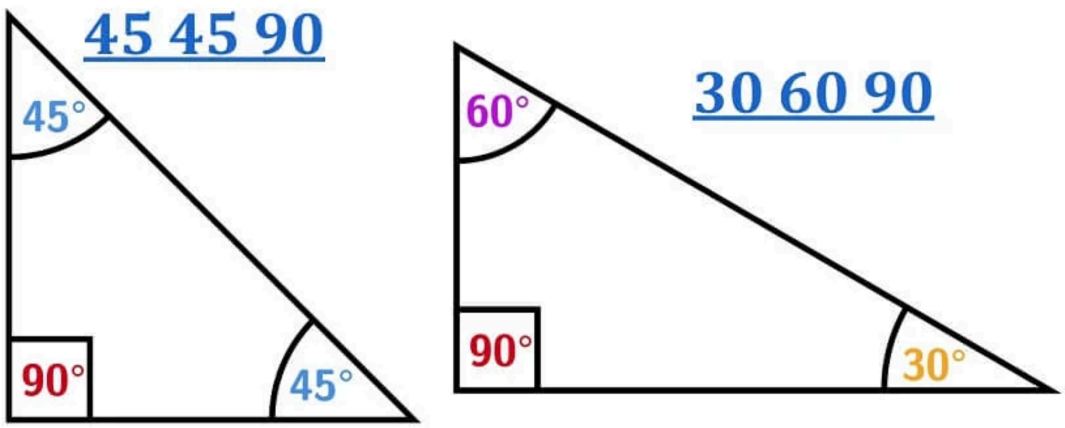
There are some cases where all three lengths are integers. We call these cases Pythagorean triples. Some examples are listed below:

a	b	c
3	4	5
5	12	13
8	15	17
7	24	25
20	21	29

These are very useful to remember. Also, notice how if $x^2 + y^2 = z^2$, then $nx^2 + ny^2 = nz^2$, where n is any real number. We can see this is true by multiplying $\frac{1}{n}$ to all of the terms. In this way, since there are an infinite number of n for which this holds, there are an infinite number of pythagorean triples.

Through the Pythagorean Theorem, we can prove two special triangles:

<u>Special Right Triangles: 45-45-90 and 30-60-90</u>



<u>Similar Triangles</u>

If two triangles are **congruent** (identical to each other), they must satisfy at least one of the following:

- All three corresponding sides are equal (SSS Congruence)
- Two corresponding sides and the angle between the two sides (also known as the included angle) are equal (SAS Congruence)
- Two corresponding angles and the side between the two angles are equal (ASA Congruence)

**SSA is NOT a valid congruence!

If two triangles are **similar** (scaled versions of each other), they must satisfy at least one of the following:

- All three corresponding sides are in the same ratio (SSS Similarity)
- Two corresponding sides are in the same ratio and the angle between the two sides are equal (SAS Similarity)
- Two corresponding angles are equal (AA Similarity)

There are certain applications and special cases.

Problems!

1. What is the smallest area of a right triangle with integer side lengths if one of the sides measures 15 units?

2. Let there be a rhombus with diagonals of length 10 and 24. What is the perimeter of the rhombus?

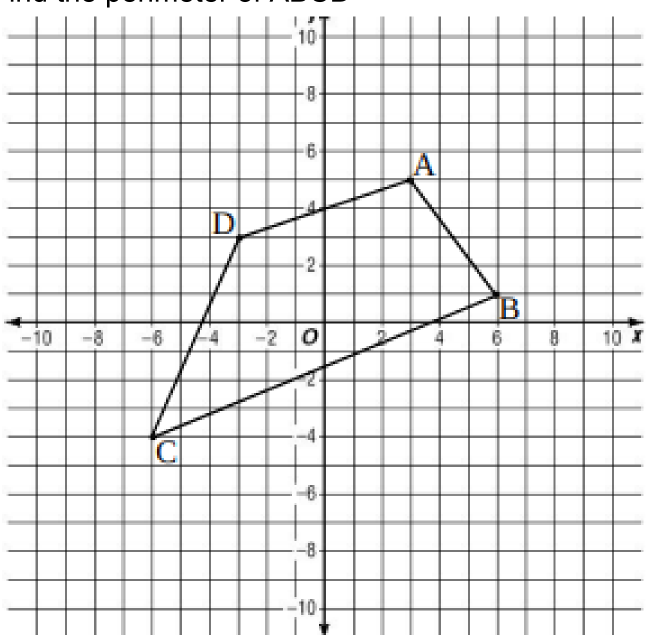
3. Find the distance between the furthest two vertices of a cube with side length 1.
    a. Can you generalize this length to any cube? (this length is called the space diagonal)

4. Find the area of an isosceles triangle with base of length 14 and two sides of length 25.

5. Find the distance between the points (4,6) and (-11, -6)
    a. Can you derive a formula for the distance between any two points (a,b) and (c,d)?

6. Quadrilateral ABCD has right angles at vertices A and B, side AB of length 15, CD of length 17, and DA of length 20. Find the area of the quadrilateral.

7. Find the perimeter of ABCD

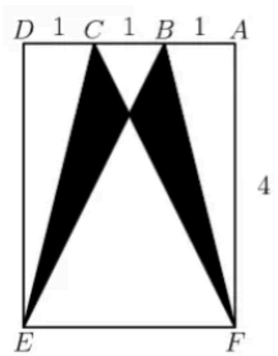


8. A 17-foot wire connects the top of a 28-foot pole to the top of a 20-foot pole. What is the shortest length of wire that you could use to attach the top of the short pole to the bottom of the tall pole? Assume all wires are straight.

Harder Problems

1. (AMC 8)

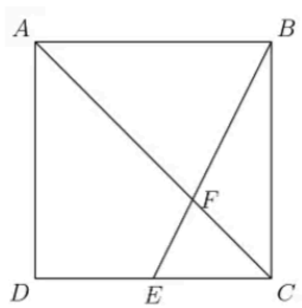
   Rectangle $DEFA$ below is a 3×4 rectangle with $DC = CB = BA$. What is the area of the "bat wings" (shaded area)?



(A) 2 (B) $2\frac{1}{2}$ (C) 3 (D) $3\frac{1}{2}$ (E) 5.

2. (AMC 8)

Point E is the midpoint of side $\overline{CD}$ in square $ABCD$, and $\overline{BE}$ meets diagonal $\overline{AC}$ at F. The area of quadrilateral $AFED$ is 45. What is the area of $ABCD$?



(A) 100 (B) 108 (C) 120 (D) 135 (E) 144.

3. (AHSME) An equilateral triangle and a regular hexagon have equal perimeters. If the area of the triangle is 2, then the area of the hexagon is

(A) 2 (B) 3 (C) 4 (D) 6 (E) 12

4. Challenge Problem! If you do get here, please ask us what an incenter is as it will be very important in solving this problem.

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Rectangle $ABCD$ is given with $AB = 63$ and $BC = 448$. Points E and F lie on AD and BC respectively, such that $AE = CF = 84$. The inscribed circle of triangle BEF is tangent to EF at point P, and the inscribed circle of triangle DEF is tangent to EF at point Q. Find PQ.

Answer Key

1. 54 units^2
2. 52 units
3. Sqrt3 units, xsqrt3 units
4. 168 units^2
5. 3sqrt41
6. 240
7. 2sqrt10+sqrt58+18
8. 25

Harder Problems

1. C) 3
2. B) 108