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SECOND TERM E-LEARNING NOTE

SUBJECT: MATHEMATICS

CLASS: SS 3

SCHEME OF WORK

WEEK	TOPIC
1.	Calculation on interest on bonds and debentures using logarithm table and problems on taxes and value added tax.
2.	Coordinate Geometry of straight line: Cartesian coordinate graphs, distance between two points, midpoint of the line joining two points.
3.	Coordinate Geometry of straight lines: Gradient and Intercepts of a line, angle between two intersecting straight lines and application.
4.	Differentiation of algebraic functions: meaning of differentiation, differentiation from first principle and standard derivatives of some basic functions.
5.	Differentiation of algebraic functions: Basic rules of differentiation such as sum and difference, product rule, quotient rule and maximal and minimum application.
6.	Integration and evaluation of simple algebraic functions: Definition, method of integration: substitution, partial fraction and integration by parts, area under the curve and use of Simpson's rule.
7- 12.	Revision and mock examination.

REFERENCE TEXT

- New General Mathematics for SS book 3 by J.B Channon
- Essential Mathematics for SS book 3
- Mathematics Exam Focus
- Waec and Jamb past Questions

WEEK ONE

- Calculation on interest on bonds and debentures using logarithm table
- Problems on taxes and value added tax.

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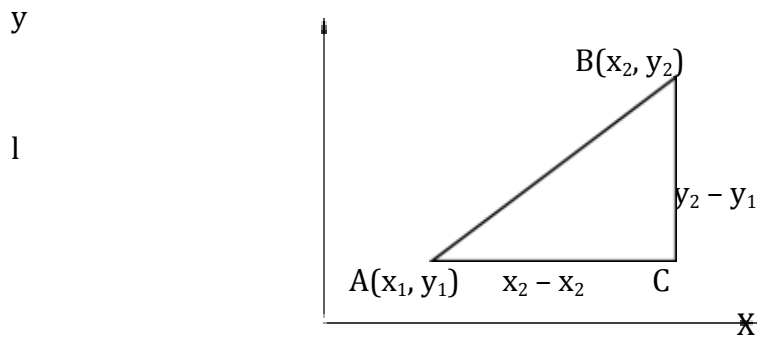
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WEEK TWO

- Coordinate Geometry of straight line: Cartesian coordinate graphs
- distance between two points
- midpoint of the line joining two points
- Coordinate Geometry of Straight line:
- Cartesian coordinate graph:

Distance between two lines:

In the figure below, the coordinates of the points A and B are (x_1, y_1) and (x_2, y_2) , respectively. Let the length of AB be l .



Using Pythagoras theorem:

$$AB^2 = AC^2 + BC^2$$

$$l^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

$$l = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Example:

Find the distance between the each pair of points: a. (3, 4) and (1, 2) b. (3, - 3) and (- 2, 5)

Solution:

$$\text{Using } l = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\text{a. } l = \sqrt{(3 - 1)^2 + (4 - 2)^2}$$

$$l = \sqrt{2^2 + 2^2}$$

$$l = \sqrt{8} = 2\sqrt{2} \text{ units}$$

$$\text{b. } l = \sqrt{(3 - (-2))^2 + (-3 - 5)^2}$$

$$= \sqrt{5^2 + (-8)^2}$$

$$= \sqrt{25 + 64} = \sqrt{89} = 9.43 \text{ units}$$

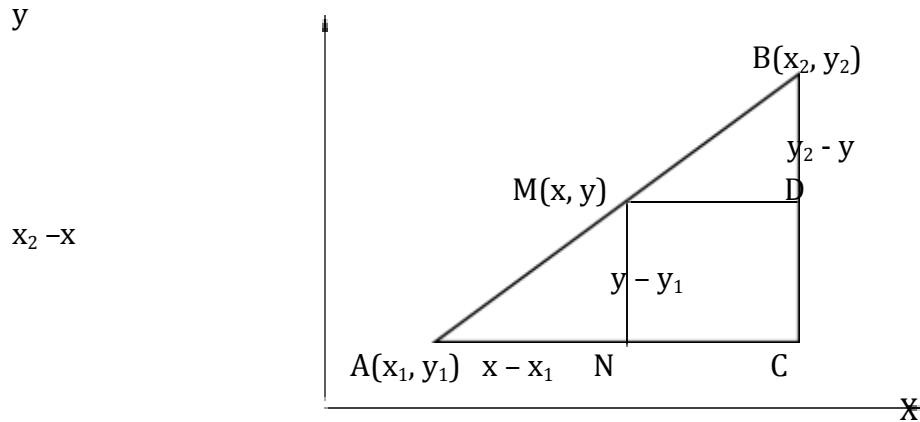
Evaluation: Find the distance between the points in each of the following pairs leaving your answers in surd form: 1. (-2, - 5) and (3, - 6) 2. (- 3, 4) and (- 1, 2)

Mid-point of a line:

The mid-point of the line joining two points:

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Triangle MAN and BMD are congruent, so $AM = MD$ and $BD = MN$

$$x - x_1 = x_2 - x$$

$$y - y_1 = y_2 - y$$

$$x + x = x_2 + x_1$$

$$y + y = y_2 + y_1$$

$$2x = x_2 + x_1$$

$$2y = y_2 + y_1$$

$$x = \frac{x_2 + x_1}{2}$$

$$y = \frac{y_2 + y_1}{2}$$

Hence, the **mid-point** of a straight line joining two is $\frac{x_2 + x_1}{2}, \frac{y_2 + y_1}{2}$

Example: Find the coordinates of the mid-point of the line joining the following pairs of points.

- a. (3, 4) and (1, 2) b. (2, 5) and (-3, 6)

Solution:

$$\text{Mid-point} = \frac{x_2 + x_1}{2}, \frac{y_2 + y_1}{2}$$

$$\text{a. Mid-point} = \frac{1 + 3}{2}, \frac{4 + 2}{2} = (2, 3)$$

$$\text{b. Mid-point} = \frac{-3 + 2}{2}, \frac{6 + 5}{2} = -\frac{1}{2}, \frac{11}{2}$$

Evaluation: Find the coordinates of the mid-point of the line joining the following pairs of points.

- a. (-2, -5) and (3, -6) b. (3, 4) and (-1, -2)

General Evaluation

- Find the distance between the points in each of the following pairs leaving your answers in surd form: 1. (7, 2) and (1, 6)
- What is the value of r if the distance between the points (4, 2) and (1, r) is 3 units?
- Find the coordinates of the mid-point (-3, -2) and (-7, -4)

Reading Assignment: NGM for SS 3 Chapter 9 page 77 – 78,

Weekend Assignment:

- Find the value of $\alpha^2 + \beta^2$ if $\alpha + \beta = 2$ and the distance between the points (1, α) and (β , 1) is 3 units.

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- The vertices of the triangle ABC are A (7, 7), B (- 4, 3) and C (2, - 5). Calculate the length of the longest side of triangle ABC.
- Using the information in '2' above, calculate the line AM, where M is the mid-point of the side opposite A.

WEEK THREE

- Coordinate Geometry of straight lines:
- Gradient and Intercepts of a line
- Angle between two intersecting straight lines and application

Gradient and Intercepts of a line

Gradient of a line of the form $y = mx + c$, is the coefficient of x , which is represented by m and c is the intercept on the y axis.

Example

- Find the equation of the line with gradient 4 and y -intercept -7.

Solution

$$m = 4, c = - 7,$$

Hence, the equation is; $y = 4x - 7$.

Evaluation:

- What is the gradient and y intercept of the line equation $3x - 5y + 10 = 0$?
- Find the equation of the line with gradient - 9 and y -intercept 4.

Gradient and One Point Form

The equation of the line can be calculated given one point (x, y) and gradient (m) by using the formula; $y - y_1 = m(x - x_1)$

Example

Find the equation of the line with gradient -8 and point(3, 7).

Solution

$$m = - 8, (x_1, y_1) = (3, 7)$$

$$\text{Equation: } y - 7 = - 8(x - 3)$$

$$y = -8x + 24 + 7$$

$$y = -8x + 31$$

Evaluation:

- Find the equation of the line with gradient 5 and point(-2, -7).
- Find the equation of the line with gradient -12 and point (3, -5).

Two Point Form:

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Given two points (x_1, y_1) and (x_2, y_2) , the equation can be obtained using the formula:

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{y - y_1}{x - x_1}$$

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{y - y_1}{x - x_1}$$

Example: Find the equation of the line passing through $(2, -5)$ and $(3, 6)$.

Solution

$$\frac{6 - (-5)}{3 - 2} = \frac{y - (-5)}{x - 2}$$

$$11 = \frac{y + 5}{x - 2}$$

$$11(x - 2) = y + 5$$

$$11x - 22 = y + 5$$

$$y - 11x + 27 = 0$$

Evaluation:

1. Find the equation of the line passing through $(3, 4)$ and $(-1, -2)$.

2. Find the equation of the line passing through $(-8, 5)$ and $(-6, 2)$.

Angles between Lines

Parallel lines:

The angle between parallel lines is 0° because they have the same gradient

Perpendicular Lines:

Angle between two perpendicular lines is 90° and the product of their gradients is -1 . Hence, $m_1 m_2 = -1$

Examples:

1. Show that the lines $y = -3x + 2$ and $y + 3x = 7$ are parallel.

solution:

$$\text{Equation 1: } y = -3x + 2, \quad m_1 = -3$$

$$\text{Equation 2: } y + 3x = 7,$$

$$y = -3x + 7, \quad m_2 = -3$$

since; $m_1 = m_2 = -3$, then the lines are parallel

2. Given the line equations $x = 3y + 5$ and $y + 3x = 2$, show that the lines are perpendicular.

solutions:

$$\text{Equation 1: } x = 3y + 5, \text{ make } y \text{ the subject of the equation.}$$

$$3y = x - 5$$

$$y = \frac{x - 5}{3}$$

$$m_1 = \frac{1}{3}$$

$$\text{Equation 2: } y + 3x = 2,$$

$$y = -3x + 2, \quad m_2 = -3$$

$$\text{hence: } m_1 \times m_2 = \frac{1}{3} \times -3 = -1$$

since: $m_1 m_2 = -1$, then the lines are perpendicular.

Evaluation: State which of the following pairs of lines are: (i) perpendicular (ii) parallel

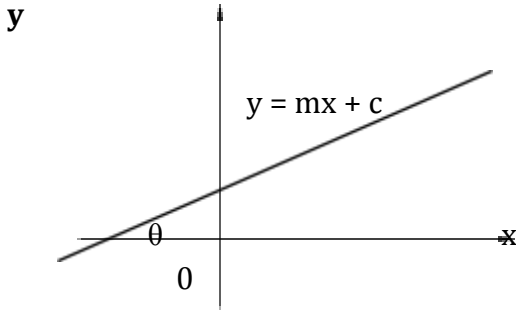
(1) $y = x + 5$ and $y = -x + 5$ (2). $2y - 6 = 5x$ and $3 - 5y = 2x$ (3) $y = 2x - 1$ and $2y - 4x = 8$

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Angles between Intersecting Lines:

The gradient of $y = mx + c$ is $\tan \theta$. Hence **$m = \tan \theta$** can be used to calculate angles between two intersecting lines. Generally the angle between two lines can be obtained using: $\tan \theta = \frac{m_2 - m_1}{1 + m_1 m_2}$

Example: Calculate the acute angle between the lines $y = 4x - 7$ and $y = x/2 + 0.5$.

Solution:

$$Y = 4x - 7, m_1 = 4, y = x/2 + 0.2, m_2 = 1/2.$$

$$\tan \theta = \frac{0.5 - 4}{1 + (0.5 \cdot 4)} = -3.5/3$$

$$\tan \theta = -1.1667$$

$$\theta = \tan^{-1}(-1.1667) = 49.4$$

Evaluation: Calculate the acute angle between the lines $y = 3x - 4$ and $x - 4y + 8 = 0$.

General Evaluation:

1. Calculate the acute angle between the lines $y = 2x - 1$ and $2y + x = 2$.
2. If the lines $3y = 4x - 1$ and $qy = x + 3$ are parallel to each other, find the value of q .
3. Find the equation of the line passing through $(2, -1)$ and gradient 3.

Reading Assignment: NGM for SS 3 Chapter 9 page 75 – 81

Weekend Assignment

1. Find the equation of the line passing through $(5, 0)$ and gradient 3.
2. Find the equation of the line passing through $(2, -1)$ and $(1, -2)$.
3. Two lines $y = 3x - 4$ and $x - 4y + 8 = 0$ are drawn on the same axes.
 - (a) Find the gradients and intercepts on the axes of each line.
 - (b) Find the equation parallel to $x - 4y + 8 = 0$ at the point $(3, -5)$

WEEK FOUR

- Differentiation of algebraic functions: meaning of differentiation
- Differentiation from first principle
- Standard derivatives of some basic functions.

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Consider the curve whose equation is given by $y = f(x)$ Recall that $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{f(x+x) - f(x)}{dx}$

$x_2 - x_1 = dx$

As point B moves close to A, dx becomes smaller and tends to zero.

The limiting value is written on $\lim_{dx \rightarrow 0} \frac{f(x+dx) - f(x)}{dx}$ and is denoted by $f'(x)$

$f'(x)$ is called the **derivative of $f(x)$** and the **gradient function of the curve**

The process of finding the derivative of $f(x)$ is called differentiation. The notations which are commonly used for the derivative of a function are $f'(x)$ read as f - prime of x , df/dx read as $dee x$ of f , $\frac{df}{dx}$ read $dee - f$ $dee - x$, $\frac{dy}{dx}$ read $dee - y$ $dee - x$

If $y = f(x)$, this $\frac{dy}{dx} = f'(x)$ (it is called the differential coefficient of y with respect to x).

Differentiation from first principle: The process of finding the derivative of a function from the consideration of the limiting value is called differentiation from first principle.

Example 1

Find from first principle, the derivative of $y = x^2$

Solution

$$y = x^2$$

$$y + y = (x + x)^2$$

$$y + y = x^2 + 2xx + (x)^2$$

$$y = x^2 + 2xx + (x)^2 - y$$

$$y = x^2 + 2xx + (x)^2 - x^2$$

$$y = 2xx + (x)^2$$

$$y = (2x + x)x$$

$$y = \frac{2x}{x} + x$$

$$x$$

$$\lim_{x \rightarrow 0} x = 0$$

$$\frac{dy}{dx} = 2x$$

$$dx$$

Example 2:

Find from first principle, the derivative of $1/x$

Solution

$$\text{Let } y = \frac{1}{x}$$

$$x$$

$$y + y = \frac{1}{x + x}$$

$$y = \frac{1}{x + x} - y$$

$$y = \frac{1}{x + x} - \frac{1}{x}$$

$$y = \frac{x - (x + x)}{(x + x)x}$$

$$y = \frac{x - x - x}{x^2 + xx}$$

$$\frac{dy}{dx} = \frac{-x}{x^2 + x}$$

$$x^2 + x$$

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$$y = \frac{-1}{x^2 + x}$$

$$\lim_{x \rightarrow 0} x = 0$$

$$\frac{dy}{dx} = \frac{-1}{x^2}$$

$$\frac{dy}{dx} = \frac{-1}{x^2}$$

Evaluation: Find from first principle, the derivatives of y with respect to x:

1. $Y = 3x^3$

2. $Y = 7x^2$

3. $Y = 3x^2 - 5x$

Rules of Differentiation: Let $y = x^n$

$$y + dy = (x + dx)^n$$

$$= x^n + nx^{n-1}dx + \frac{n(n-1)x^{n-2}(dx)^2}{2!} + \dots + (dx)^n$$

$$= x^n + nx^{n-1}dx + \frac{n(n-1)x^{n-2}(dx)^2}{2!} + \dots + (dx)^n - x^n$$

$$= nx^{n-1}dx + \frac{n(n-1)x^{n-2}(dx)^2}{2!}$$

$$\frac{dy}{dx} = nx^{n-1} + \frac{n(n-1)x^{n-2}dx}{2!}$$

$$\lim_{dx \rightarrow 0} \frac{dy}{dx} = nx^{n-1}$$

$$dx = 0$$

$$\text{Hence; } \frac{dy}{dx} = nx^{n-1} \text{ if } y = x^n$$

Example 3:Find the derivative of the following with respect to x: (a) x^7 (b) $x^{1/2}$ (c) $5x^2 - 3x$ (d) $-3x^2$ (e) $y = 2x^3 - 3x + 8$

Solution

a. Let $y = x^7$

$$\frac{dy}{dx} = 7x^{7-1} = 7x^6$$

b. Let $y = x^{1/2}$

$$\frac{dy}{dx} = \frac{1}{2}x^{1/2-1} = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$$

c. Let $y = 5x^2 - 3x$

$$\frac{dy}{dx} = 10x - 3$$

d. Let $y = -3x^2$

$$\frac{dy}{dx} = 2 \times -3x^{2-1} = -6x$$

e. Let $y = 2x^3 - 3x + 8$

$$\frac{dy}{dx} = 3 \times 2x^{3-1} - 3 + 0$$

$$= 6x^2 - 3$$

Evaluation:

1. If $y = 5x^4$, find $\frac{dy}{dx}$

2. Given that $y = 4x^{-1}$ find $\frac{dy}{dx}$

General Evaluation1. Find, from first principles, the derivative of $4x^2 - 2$ with respect to x.2. Find the derivative of the following a. $3x^3 - 7x^2 - 9x + 4$ b. $2x^3$ c. $3/x$ 3. Using idea of difference of two square; simplify $243x^2 - 48y^2$

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4. Expand $(2x - 5)(3x - 4)$ 5. If the gradient of $y = 2x^2 - 5$ is -12 find the value of y .**Reading Assignment: NGM for SS 3 Chapter 10** page 82 -88,**Weekend Assignment****Objective**

- Find the derivative of $5x^3$ (a) $10x^2$ (b) $15x^2$ (c) $10x$ (d) $15x^3$
- Find dy/dx , if $y = 1/x^3$ (a) $-3/x^4$ (b) $3/x^4$ (c) $4/x^3$ (e) $-4/x^3$
- Find $f'(x)$, if $f(x) = x^3$ (a) $3x$ (b) $3x^2$ (c) $\frac{1}{2}x^3$ (d) $2x^3$
- Find the derivative of $1/x$ (a) $1/x^2$ (b) $-1/x^2$ (c) $-x$ (d) $-x^2$
- If $y = -2/3 x^3$. Find dy/dx (a) $4/3 x^2$ (b) $2x^2$ (c) $-2x^2$ (d) $-2x$

Theory

- Find from first principle, the derivative of $y = x + 1/x$
- Find the derivative of $2x^2 - 2/x^3$

WEEK FIVE

- Differentiation of algebraic functions:
- Basic rules of differentiation such as sum and difference, product rule, quotient rule
- Maximal and minimum application.

Derivative of algebraic functionsLet f, u, v be functions such that

$$f(x) = u(x) + v(x)$$

$$f(x + x) = u(x + x) + v(x + x)$$

$$f(x + x) - f(x) = \{u(x + x) + v(x + x) - v(x + x) - u(x) - v(x)\}$$

$$= u(x + x) - u(x) + v(x + x) - v(x)$$

$$f(x + x) - f(x) = u(x + x) - u(x) + v(x + x) - v(x)$$

$$\lim_{x \rightarrow 0} \frac{f(x + x) - f(x)}{x} = U'(x) + V'(x)$$

if $y = u + v$ and u and v are functions of x , then **$dy/dx = du/dx + dv/dx$** **Examples:** Find the derivative of the following

- $2x^3 - 5x^2 + 2$
- $3x^2 + 1/x$
- $2x^3 + 2x^2 + 1$

Solution

- Let $y = 2x^3 - 5x^2 + 2$
 $dy/dx = 6x^2 - 10x$

- Let $y = 3x^2 + 1/x = 3x^2 + x^{-1}$
 $dy/dx = 6x - x^{-2} = 6x - \frac{1}{x^2}$

- Let $y = 2x^3 + 2x^2 + 1$
 $dy/dx = 6x^2 + 4x$

Evaluation: 1. If $y = 3x^4 - 2x^3 - 7x + 5$. Find dy/dx 2. Find $\frac{d}{dx}(8x^3 - 5x^2 + 6)$ D_x

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Function of a function (chain rule)

Suppose that we know that y is a function of u and that u is a function of x , how do we find the derivative of y with respect to x ?

Given that $y = f(x)$ and $u = h(x)$, what is dy/dx ?

$dy/dx =$, this is called the chain rule

Examples

Find the derivative of the following. (a) $y = (3x^2 - 2)^3$ (b) $y = (1 - 2x^3)^{1/2}$ (c) $y = 5/(6-x^2)^3$

Solution

$$\begin{aligned} 1. \quad y &= (3x^2 - 2)^3 \\ \text{Let } u &= 3x^2 - 2 \\ y &= (3x^2 - 2)^3 \Rightarrow y = u^3 \\ y &= u^3 \\ dy/du &= 3u^2 \\ du/dx &= 6x \end{aligned}$$

$$\begin{aligned} dy/dx &= 3u^2 \times 6x \\ &= 18xu^2 = 18x(3x^2 - 2)^2 \end{aligned}$$

$$\begin{aligned} 2. \quad y &= (1 - 2x^3)^{1/2} \Rightarrow (1 - 2x^3)^{1/2} \\ \text{Let } u &= 1 - 2x^3, \text{ hence } y = u^{1/2} \\ dy/dx &= \frac{1}{2} u^{-1/2} \times (-6x^2) \\ &= -3x^2 u^{-1/2} = \underline{-3x^2} \end{aligned}$$

$$\begin{aligned} u^{1/2} \\ \frac{-3x^2}{\sqrt{u}} &= \frac{-3x^2}{\sqrt{1 - 2x^3}} \end{aligned}$$

$$\begin{aligned} 3. \quad y &= \frac{5}{(6 - x^2)^3} = 5(6 - x^2)^{-3} \\ \text{Let } u &= 6 - x^2 \\ y &= 5(u)^{-3} \\ dy/du &= -15u^{-4} \\ du/dx &= -2x \\ dy/dx &= dy/du \times du/dx = -15u^{-4} \times (-2x) = 30x u^{-4} = 30x (6 - x^2)^{-4} \\ &= \underline{\frac{30x}{(6 - x^2)^4}} \end{aligned}$$

Evaluation:

$$1. \quad \text{Given that } y = \frac{1}{(2x + 3)^4} \text{ find } dy/dx$$

$$2. \quad \text{If } y = (3x^2 + 1)^3, \text{ Find } dy/dx$$

Product Rule

We shall consider the derivative of $y = uv$ where u and v are function of x .

$$\text{Let } y = uv$$

$$\text{Then } y + y = (u + u)(v + v)$$

$$= uv + uv + vu + uv$$

$$y = uv + uv + vu + uv - uv$$

$$y = uv + vu + uv$$

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$$y = \frac{uv}{x} + \frac{vu}{x} + \frac{uv}{x}$$

As $x \rightarrow 0, u \rightarrow 0, v \rightarrow 0$

$$\lim_{x \rightarrow 0} y = \lim_{x \rightarrow 0} \frac{uv}{x} + \lim_{x \rightarrow 0} \frac{vu}{x} + \lim_{x \rightarrow 0} \frac{uv}{x}$$

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Hence $\frac{dy}{dx} = U \frac{dv}{dx} + V \frac{du}{dx}$ **Examples**

Find the derivatives of the following.

(a) $y = (3 + 2x)(1 - x)$ (b) $y = (1 - 2x + 3x^2)(4 - 5x^2)$

Solution

1. $y = (3 + 2x)(1 - x)$

Let $u = 3 + 2x$ and $v = (1 - x)$

$\frac{du}{dx} = 2$ and $\frac{dv}{dx} = -1$

$$\frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$$

$$= (1 - x)2 + (3 + 2x)(-1) = 2 - 2x - 3 - 2x$$
$$\frac{dy}{dx} = -1 - 4x$$

2. $y = (1 - 2x + 3x^2)(4 - 5x^2)$

Let $u = (1 - 2x + 3x^2)$ and $v = (4 - 5x^2)$

$\frac{du}{dx} = -2 + 6x$ and $\frac{dv}{dx} = -10x$

$$\frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$$

$$= (1 - 2x + 3x^2)(-10x) + (4 - 5x^2)(-2 + 6x)$$
$$= -10x + 20x^2 - 30x^3 + (-8 + 10x^2 + 24x - 30x^3)$$
$$= -10x + 20x^2 - 30x^3 - 8 + 10x^2 + 24x - 30x^3$$
$$= 14x + 30x^2 - 60x^3 - 8$$

EvaluationGiven that (i) $y = (5+3x)(2-x)$ (ii) $y = (1+x)(x+2)^{3/2}$, find $\frac{dy}{dx}$ **Quotient Rule:**If $y = \frac{u}{v}$ v

then; $\frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$

$$v^2$$

Examples:Differentiate the following with respect to x. (a) $\frac{x^2+1}{1-x^2}$ (b) $\frac{(x-1)^2}{\sqrt{x}}$

Solution:

1. $y = \frac{x^2+1}{1-x^2}$

Let $u = x^2 + 1$ $\frac{du}{dx} = 2x$
 $v = 1 - x^2$ $\frac{dv}{dx} = -2x$

$$\frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

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$$dy/dx = \frac{(1-x^2)(2x) - (x^2+1)(-2x)}{(1-x^2)^2}$$

$$= \frac{2x - 2x^3 + 2x^3 + 2x}{(1-x^2)^2}$$

$$dy/dx = \frac{4x}{(1-x^2)^2}$$

$$2. \ y = \frac{(x-1)^2}{\sqrt{x}}$$

$$\text{Let } u = (x-1)^2 \quad du/dx = 2(x-1)$$

$$v = \sqrt{x} \quad dv/dx = 1/2\sqrt{x}$$

$$dy/dx = \frac{\sqrt{x} 2(x-1) - (x-1)^2 \cdot 1/2\sqrt{x}}{(\sqrt{x})^2}$$

$$dy/dx = \frac{\sqrt{x} 2(x-1) - (x-1)^2 \cdot 1/2\sqrt{x}}{x}$$

Evaluation: Differentiate with respect to x: (1) $\frac{(2x+3)^3}{(x^3-4)^2}$ (2) $\frac{\sqrt{x}}{\sqrt{x+1}}$

Applications of differentiation:

There are many applications of differential calculus.

Examples:

1. Find the gradient of the curve $y = x^3 - 5x^2 + 6x - 3$ at the point where $x = 3$.

Solution:

$$Y = x^3 - 5x^2 + 6x - 3$$

$$dy/dx = 3x^2 - 10x + 6$$

$$\begin{aligned} \text{where } x = 3; dy/dx &= 3(3^2) - 10(3) + 6 \\ &= 27 - 30 + 6 \\ &= 3. \end{aligned}$$

2. Find the coordinates of the point on the graph of $y = 5x^2 + 8x - 1$ at which the gradient is -2

Solution:

$$Y = 5x^2 + 8x - 1$$

$$dy/dx = 10x + 8$$

replace; dy/dx by -2

$$10x + 8 = -2$$

$$10x = -2 - 8$$

$$x = -10/10 = -1$$

3. Find the point at which the tangent to the curve $y = x^2 - 4x + 1$ at the point $(2, -3)$

Solution:

$$Y = x^2 - 4x + 1$$

$$dy/dx = 2x - 4$$

$$\text{at point } (2, -3): dy/dx = 2(2) - 4$$

$$dy/dx = 0$$

$$\text{tangent to the curve: } y - y_1 = dy/dx(x - x_1)$$

$$y - (-3) = 0(x - 2)$$

$$y + 3 = 0$$

Evaluation:

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1. Find the coordinates of the point on the graph of $y = x^2 + 2x - 10$ at which the gradient is 8.
2. Find the point on the curve $y = x^3 + 3x^2 - 9x + 3$ at which the gradient is 15.

Velocity and Acceleration

Velocity: The velocity after t seconds is the rate of change of displacement with respect to time.

Suppose; s = distance and t = time,

Then; **Velocity** = ds/dt

Acceleration: This is the rate of change of velocity compared with time.

Acceleration = dv/dt

Example:

A moving body goes s metres in t seconds, where $s = 4t^2 - 3t + 5$. Find its velocity after 4 seconds. Show that the acceleration is constant and find its value.

Solution:

$$S = 4t^2 - 3t + 5$$

$$ds/dt = 8t - 3$$

$$\text{velocity} = ds/dt = 8(4) - 3$$

$$= 32 - 3$$

$$= 29$$

$$\text{Acceleration: } dv/dt = 8.$$

Maxima and Minimal

1. Find the maximum and minimum value of y on the curve $6x - x^2$.

Solution:

$$y = 6x - x^2$$

$$dy/dx = 6 - 2x$$

$$\text{equated } dy/dx = 0$$

$$6 - 2x = 0$$

$$6 = 2x$$

$$X = 3$$

The turning point is (3, 9)

2. Find the maximum and minimum of the function $x^3 - 12x + 2$.

Solution:

$$Y = x^3 - 12x + 2$$

$$dy/dx = 3x^2 - 12$$

$$3x^2 - 12 = 0$$

$$3x^2 = 12$$

$$x^2 = 12/3$$

$$x^2 = 4 \quad x = \pm 2$$

minimum point occur when $d^2y/dx^2 > 0$

maximum point occurs when $d^2y/dx^2 < 0$

$$d^2y/dx^2 = 6x$$

$$\text{substitute } x = 2; \quad d^2y/dx^2 = 6 \times 2 = 12$$

therefore: the function is minimum at point $x = 2$ and $y = -14$

$$\text{substitute } x = -2; \quad d^2y/dx^2 = 6(-2) = -12$$

therefore: the function is maximum at point $x = -2$ and $y = 18$

Evaluation:

Name _____ Date _____

1. A particle moves in such a way that after t seconds it has gone s metres, where $s = 5t + 15t^2 - t^3$
2. Find the maximum and minimum value of y on the curve $4 - 12x - 3x^2$.

General Evaluation

Use product rule to find the derivative of

1. $y = x^2 (1 + x)^{\frac{1}{2}}$
2. $y = \sqrt{x} (x^2 + 3x - 2)^2$
3. Find the derivative of $y = (7x^2 - 5)^3$
4. Using completing the square method find t if $s = ut + \frac{1}{2}at^2$
5. If 3 is a root of the equation $x^2 - kx + 42 = 0$ find the value of k and the other root of the equation

READING ASSIGNMENT: NGM for SS 3 Chapter 10 page 90 -101,

WEEKEND ASSIGNMENT

OBJECTIVE

1. Differentiate the function $4x^4 + x^3 - 5$ (a) $4x^3 + 3x^2$ (b) $16x^2 + 3x^2$ (c) $16x^3 + 3x^2$ (d) $16x^4 + 3x^2$
2. Find d^2y/dx^2 of the function $y = 3x^5$ wrt x . (a) $15x^3$ (b) $45x^4$ (c) $60x^3$ (d) $3x^5$ (e) $12x^3$
3. If $f(x) = 3x^2 + 2/x$ find $f'(x)$ (a) $6x + 2$ (b) $6x + 2/x^2$ (c) $6x - 2/x^2$ (d) $6x - 2$
4. Find the derivative of $2x^3 - 6x^2$ (a) $6x^2 - 12x$ (b) $6x^2 - 12x$ (c) $2x^2 - 6x$ (d) $8x^2 - 3x$
5. Find the derivative of $x^3 - 7x^2 + 15x$ (a) $x^2 - 7x + 15$ (b) $3x^2 - 14x + 15$ (c) $3x^2 + 7x + 15$ (d) $3x^2 - 7x + 15$

THEORY

1. Differentiate with respect to x . $y^2 + x^2 - 3xy = 4$
2. Find the derivative of $3x^3(x^2 + 4)^2$