Die Berechnung durchlaufender Träger und mehrstieliger Rahmen nach dem Verfahren des Zahlenrechtecks.

Von der
Königl. Sächs. Technischen Hochschule zu Dresden
zur Erlangung der Würde eines Doktor-Ingenieurs
genehmigte Dissertation.

Vorgelegt von

Dipl.-Ing. Dr. sc. nat. **Viktor Lewe**
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Korreferent: Geheimer Hofrat Professor M. Förster.

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The Application of Matrix Calculus to Continuous Beams and Framed Structures

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From the Royal Saxon Technical College, Dresden

to obtain a doctoral-engineers

approved dissertation

Presented by

Dipl.Ing Dr.sc.nat **Viktor Lewe**

From Loningen in Oldenburg

Referee: Professor W.Gehler

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Translator’s Notes

14.10.23 – draft only. Translation is Google and requires much refinement. Engineering terms require detailed understanding before the concepts can be fully understood, consequently many words and phrases have been left unchanged and a little clumsy. Nonetheless it is clear that Lewe’s Paper proves Absolute Motion at the heart of Nuclear Energy and over-turns the Theory of Relativity.

This dissertation is a Physics paper presented to Engineering. There is no evidence that the information it contains has ever been discredited, and therefore, until proved otherwise, the paper should be considered as essentially correct, still current and still in use as reference for work carried out in both Science and Engineering.

I consider that some information has not been provided, and the manner of the omission suggests at least in some cases that the original paper may have had the rule run over it by another hand before it was published. It would appear that this guiding authority has removed what it considered would lead an inquiring mind towards the world of water, gravity and subatomic force. I have attempted to call out the anomalies where I see them and replace them, to aid clarity of understanding, but I may not have seen them all, yet. Notwithstanding these cuts, it is possible to use the information in this paper to construct an atomic explosive device, (not recommended), and therefore it is a very clear indication that German Physics was aware of the Power of Nuclear Energy in 1915, and that Lewe was about to lead them there. That the paper was allowed to be presented and published at all speaks that the ideas were still in their infancy, and had not been fully grasped. I suspect that if the full truth was known, it would not have seen the light of day at all, but we must be grateful that it was. The secrecy begun then, in 1915, still continues today. Rest mass is a deliberate mis-direction. Positive Moment is Truth.

The paper was released to Engineering, not to Physics, because it meant that the secrets it contains could be centrally controlled. I suspect there are only a handful of physicists today who actually are aware of this document, and it is this select group who have continued to develop the ideas for over a century. By releasing General Relativity in 1915, and then later, at the Solvay Conferences, Heisenberg and Born announcing that Quantum was a closed theory not subject to further modification, German Physics closed the door and laid plans. The Illinois-based Portland Cement Association’s, publication in 1942, *Circular Concrete Water Tanks without Prestressing* is based almost entirely on Lewe’s work, and yet makes no reference to him at all. This is the second break in the system. Now not just physicists are blind to Lewe’s work, but Engineers who are using it in practice, unless they are curious enough to seek the answer, have no idea where the information they are using has come from. The seed of secrecy extended to America, just in time for the Manhattan Project.

Hiding truth is an incredibly dangerous practice, and the result already is catastrophic, with worse to come. Atomic Energy and reinforced concrete both slow Earth's momentum because the energy they use is parasitic, literally acting against nature; 'gravity-defying' means that gravity locally is increased, for as long as the structure stands. At CERN's behest, and with no evidence-based justification, the metre std was changed in 1983 from a length of steel to a fraction of the speed of light. The link is then finally broken, and it is impossible to corroborate whether gravity is increasing or not. If light slows, the metre std shrinks to accommodate. So now we are all blind, except the self-selecting few.

The Atomic age of Eisen und Beton is causing a permanent drag on the planet's orbital angular momentum. As Earth slows its rate of momentum, gravity increases. Each new reinforced concrete construction, each power plant fuel cell, each explosion, brings us closer to the point of unsustainability. The magnetic pole switch is coming.

Who is more powerful, Nature's Laws or man's corruption of these laws?

Time will tell

Reynolds B Eng Oct 2023

Table of contents.

Preface.

The (theory underlying) continuous beams on simple intermediate supports has been discussed for very many years, and by various authors. The first treatises only gave methods for calculating the column pressures, such as a method by Eytelwein (1808) and later also by Navier (1826) but these calculations are very complicated. Bertot[(1)] published the first analytical relationship for the bending moments of continuous beams on simple intermediate supports at the same height through deriving the Theorem of Three-Moments for field loading. The first consideration of unequal support heights and the well-known corresponding extension of the Theorem of Three-Moments can be found in the treatise by O. Mohr, *Contributions to the Theory of Wood and Iron Construction*[(2)]. In his treatises on Technical Mechanics (2nd edition), O. Mohr, on p. 340, states that these above-mentioned equations are incorrectly attributed to Clapeyron even though they named after this author, since the equation cited by Clapeyron in the Comptes Rendus in December 1857 agrees with Bertot's. Bertot's work, however, seems to have remained quite unknown, since Mohr himself only cites Clapeyron as the discoverer of the Three-Moment law in his first relevant treatise in 1860. The influence of the design level of the supports (within the beam itself?) was first examined by Köpeke in his treatise: *On the dimensions of beam layers*[3]. However, this only examines the column pressures of continuous beams ~~and the~~ (but does show that) lowering the central columns indicates (there is) a numerical means of reducing the central column pressure. In the treatise from 1860, however, Mohr warned against relying too much on the advantages of intentionally lowering the supports, in particular after Köpcke, which had been recommended by Scheffler[4]

The first graphical treatment of the theory of continuous supports can be found in Culmann, Graph. Statics Zurich 1866, who solved the most important parts of the problem analytically. The basis of all graphic methods is the representation of the elastic line by O. Mohr[5]. W. Ritter-Zürich[6] then applied this familiar drawing process (to his method). The engineer Vianello[7] was the first to graphically treat continuous beams on spring supports. The English engineers Claxton-Fidler[8] already have a new general procedure in which both rigid and spring supports can be taken into account in the guidelines, but only take rigid supports into account(?).

This process was extended by Professor Ostenfeld[9] in Copenhagen to include spring and pinned supports. The table-based treatment of continuous beams can be found first with Winkler. The so-called Winkler numbers directly indicate the maximum and minimum moments and shear forces under field loads for continuous beams with up to five support points and taking equal field widths into account. The author (Lewe) found similar numbers for the same supports, but for line loads[10]. Tables that give the lines of influence assuming the same moments of inertia everywhere were created by Lederer and Griot[11]. Müller-Breslau states that supporting moments can be composed linearly from the ordinates of two parabolas. The composition is done according to the formula

$$M = - u . \omega_D + v . \omega_R$$

Where u and v are constant and for ω_D and ω_R, tables are specified.

The following work takes a different path in that it starts from the well-known linear equations for an n-number (-fach) statically indeterminate system and takes into account the special characteristics of this system for the so-called Clapeyron's equations to find a means of specifying a new and simple elimination procedure. Similar attempts to find a simple solution for the system of linear equations with n-static indeterminates have previously always extended to the general system. Gauss[13] has proposed the solution to the so-called Normal equations in the elimination calculation using the <u>least squares method.</u> He specified an elimination procedure, which is a precursor to determinant-theoretical procedures. Pirlet[14] applied this method to statically indeterminate systems in general and interpreted it as a <u>solution method</u> with a constant number of statically indeterminate solutions. A general <u>approximation method</u> for solving the linear system of equations is given by Hertwig

The author's intention, as indicated in the title, is to develop a purely <u>computational method</u> that can be used for all cases, the simple rules of which make it possible to quickly find the influence lines of the bending moments, transverse and abutment forces purely schematically. The method developed offers advantages over the known analytical and graphical methods in that it can be used for any design of the connection of the supports (f*ree attachment, rigid connection, differences in the stroke, the moment of inertia and the base of the supports* (British Civils terms - cantilever, fixed, trussed, continuous and simply supported beams)), even if ~~the same is~~ (these are) different for the individual sections of the treated support, deviations from the initially suggested schematic treatment of the simple running supports are easily noticeable. The table attached to the sheet contains all the ordinates (coefficients).

Sensing (influence?) lines of the cross-sectional forces are directly in the form

$$K = m . e,$$

where the resulting values of *e* can be found in the table and the multipliers *m* can be calculated using the scheme (*ie Lewe's equations derived from his theory*).

This simplification, especially in the treatment of continuous calculations with rigidly connected columns or multi-legged frames, will particularly benefit reinforced concrete construction, which will thereby be able to take into account the usual rigid support connection, through which the field moments are reduced. However, this simplification and combination of the treatment of all continuous types of beams can only be achieved by assuming that it is permissible to neglect the resulting elastic longitudinal deflections of the support points in the case of continuous beams with rigidly connected supports. The admissibility of this assumption has already been dealt with exhaustively by other authors. Professor Ostenfeld in Copenhagen also introduced this premise in his treatise on multi-stemmed frames. In the chapter "Beam Bridges" of the Handbook for Reinforced Concrete, edited by Professor Gehler, German reinforced concrete practice was also introduced to these simple formulas.

1) Comptes rendus de la Societe des Ingenieurs (*Reports of the Society of Civil Engineers)* 1855 p. 278.
2) Zeitschr. d. Arch.- u. Ing.-Ver. Hannover 1860 S. 323 u. 407.
3) Zeitschr. d. Arch.- und Ing.-Ver. Hannover 1856
4) Scheffler, Theorie der Gewolbe, Futtermauern and eisernen bracken (*Theory of vaults, lining walls and iron bridges*), Zeitschr. d. Arch.- und Ing.-Ver. Hannover 1857
5) 0. Mohr, Beitrag zur Theorie der Holz-und Eisenkonstruktionen (*Contribution to the theory of wood and iron construction*), Zeitschrift (magazine) of Arch.- und Ing.-Verein Hannover 1868 S. 19
6) Ritter, Die Elastische linie u. ihre Anwendung auf den Kontinuierlichen Balken (*The elastic line and its application to the continuous beam*) 1871, 2nd edition 1883; Die anwendungen der graph. statik (nach Culmann) (*The applications of graphical statics (according to Culmann)) (*Part) 3 Tiel: Der konsein bekanntes zeichernerisches Verfahren aufgebaut (*The consist was constructed using a well-known drawing practice?*) 1900; ferner siehe auch(*further see also*): Lippich, Theorie des Kontinuierlichen Tragers konstanten Querschnittes. (*theory of the continuous support of constant cross-section*). Elementare Darstellung der von Clayperon und Mohr begrundeten analytischen und graphischen Methoden und ihres Zusammenhanges (*Elementary presentation of the analytical and graphical methods founded by Clapeyron and Mohr and their connection*s); Förster's Bauzeitung (*construction newspaper*) 1871. Winkler, Vortage uber

Bruckenbau (*lectures on bridge construction*), Vienna 1875. Bresse, Cours de Mécanique appliqué (*Course of Applied Mechanics*) 3 Tiel. (*part*)1865. M. Levy, La Statique Graphique (*Graphical Statics*) (IIe (2nd) Partie, Paris 1886.

7) Vianello, Der Durchgehendre Trager auf elastisch senkbaren Stutzen (*The continuous beam on spring supports*), Berlin 1904
8) Claxton-Fidler, A practical treatise on bridge-construktion (sic), London 1. Aufl.1887, 3rd edition 1901.
9) A Ostenfeld, Graphische behandlung der Kontinuierlichen Trager mit festen, elastisch senkbaren oder drehbaren und elastisch senk-und drehbaren Stutzen (*Graphic treatment of continuous beams with fixed, spring, pinned and combined spring/pinned supports*), Zeitschr. F. Arch. U. ingenieurwesen (*and? Engineering*) 1905 after 1903, Issues 1 and 2.
10) Lewe, Winklersche Zahlen fur streckenlasten (*Winkler's numbers for continuous loads*), Deutsche Bauzeitung 1913. Mitt. No. 22.
11) Dr. Lederer, Analytisches Ermittlung und Anwendung von Einflusslinien (*Analytical determination and application of lines of influence*). Griot, Tabellen zum Auftragen der Einsflusslinien (*Tables for plotting the influence lines*), Zurich 1904.
12) Graphische Statik (*Graphical Statics*) Bd. (Vol) 2 Abt. (Dept) 2.
13) Börsch and Simon, Abhandlung zur Methode der Kleinsten Quadrate (*Treatises on the method of least squares*), Berlin 1887 p. 124; Bauernfeind, Vermessungskund (Surveying) 5th edition Bd. 2 s. 28.
14) Der Eisenbau 1910, 1914, 1915.
15) S. Abhandlung von Hertwig in the denkschrift fur Müller-Breslau (Leipzig, Kroners Verlag)
16) A. Ostenfeld in Zeischr. Ingenioren 1905 (Kopenhagen) No. 13 s. 83.
17) Hanbuch fur Eiesenbetonbau, 2nd Aufl (edition) Bd. 6 Kap.1 s. 222.

First section

Theoretical Consideration,

A. Treatment of the exposed support,

As with all statically indeterminate problems, one starts from the known equations, which concern the displacements on the statically-determined system as a result of the action of the loads, and then consider the effects of the statically indeterminate loads. A continuous beam with n+2 supports is annotated as positions 0,1,2…n. Any support n+1 can be determined statically ~~as soon~~ if one thinks of the beam as comprised of sections 1,2,3...n. Instead of resting on all supports, each section of beam can be assumed to rest on just its own (see Figs. 1 and 2).

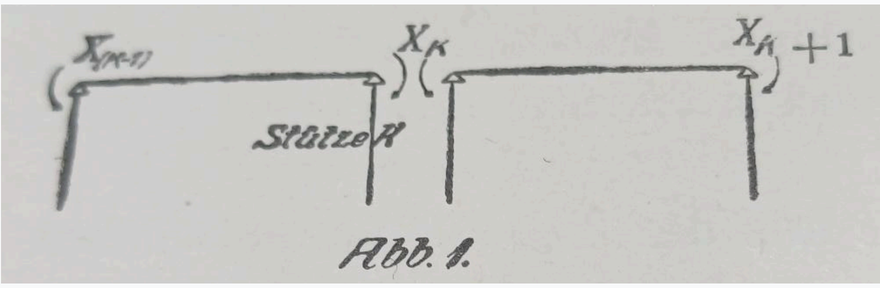


Abb. 1.

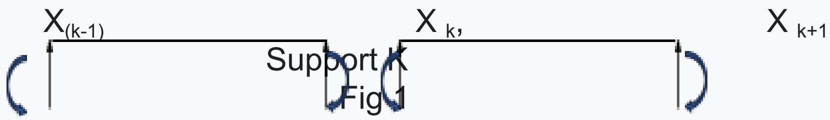
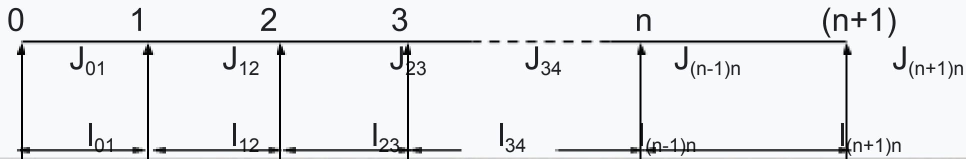


Fig 1

In order to keep everything as it was in the areas between the supports, bending moments X_1, X_2, X_3….X_n must be applied to the interfaces created above the supports, which act in pairs on the right and left ends of the beam. These supporting moments of the system are the statically indeterminate ones. The support fields and spans are considered (see Fig. 2) one after the other with l_{01}, l_{12}, l_{23},… $l_{n(n+1)}$ and correspondingly the moments of inertia of the beam, which are assumed to be constant in the individual openings, with J_{01}, J_{12}, J_{23}…$J_{n(n+1)}$

Fig.2



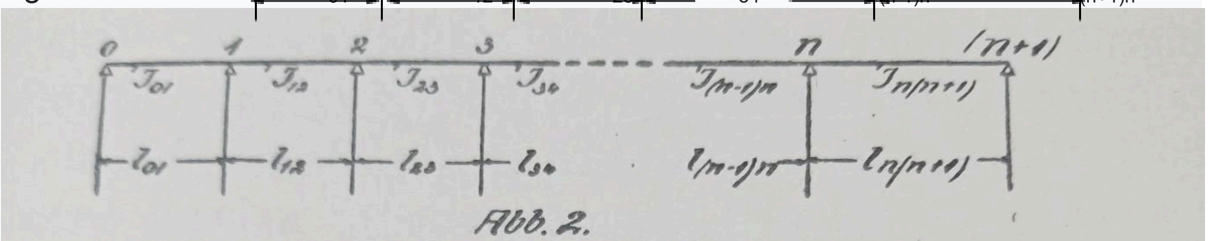
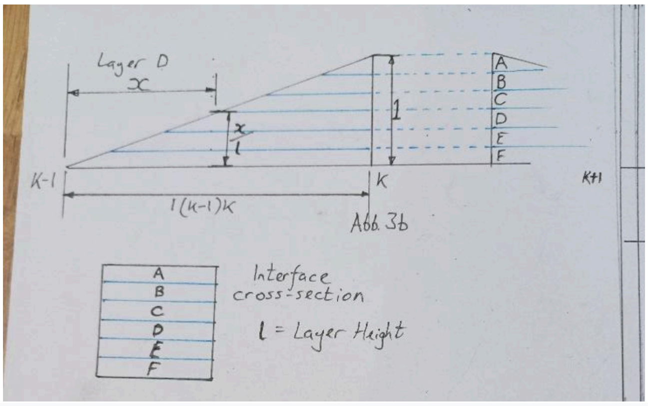
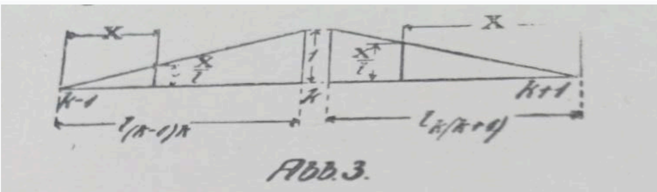


Abb. 2.

The values $\boldsymbol{\delta}^{1}$, $\boldsymbol{\delta}^{11}$, $\boldsymbol{\delta}^{111}$, $\boldsymbol{\delta}_{1}$, $\boldsymbol{\delta}_{12,}$ $\boldsymbol{\delta}_{22,}$ which will be explained in more detail below, denote the deflections and the rotations of the end cross-sections of the beam section under the effect of unit moments at the points of application of the statically indeterminate reactions, i.e. in the case that one sets the bending moments X_1=1, X_2 =1 etc. The consequence of such a moment effect is the triangular moment surface shown in Fig. 3, which only extends over the two openings adjacent to the (selected) column.





Translator Note
(Fig.) Abb. 3b added to show the laminar layers or fields . l is the height of any layer relative to the support height

At any location, x, the bending moment is

$$M_x = \frac{x}{l}$$

The differential equation of the bend line δ is

$$\frac{d^2\delta}{dx^2} = \frac{M_x}{EJ} = \frac{x}{l}\frac{1}{EJ}$$

By integration one obtains:

$$(1) \qquad \frac{d\delta}{dx} = \frac{1}{l}\frac{1}{EJ}\frac{x^2}{2} + C_1; \qquad \delta = \frac{1}{l\,EJ}\frac{x^3}{6} + C_1 x + C_2.$$

One obtains C_1, and C_2 from the condition that for x=0 and x=1 the through-bend δ = 0 becomes.

$$0 = \frac{1}{lEJ}\cdot\frac{x^3}{6} + C_1\cdot x + C_2 \text{ für } x = 0 : C_2 = 0$$

$$x = l : C_1 = -\frac{l}{6\,EJ}.$$

Then when $\xi = x/l$ is introduced:

$$(2) \qquad \ldots \quad \delta = \frac{l^2}{6\,EJ}\left(\xi^3 - \xi\right).$$

The twist angles, which are the angles a_0, and a_1, form the tangent of the bend line with the x-axis, for x = 0 and x = *l*, since in general, (according to Equation (1))

allgemein nach Gl. (1) $\alpha = \frac{d\,\delta}{d\,x} = \frac{1}{l\,EJ}\left(\frac{x^2}{2} - \frac{l^2}{6}\right)$ ist,

$$\alpha_o = -\frac{1}{EJ}\,\frac{l}{6} \quad \text{und} \quad \alpha_l = \frac{1}{EJ}\,\frac{l}{3}. \tag{3}$$

As a result of the effect of a statically indeterminate load on the statically determined main system, only 3 rotations of statically indeterminate points of attack (fracture?, concentration?) are caused, namely rotations of the cross sections individually and those of the adjacent supports, (see Fig. 4)
X=1 Moment only locally affects (the twists) $\boldsymbol{\delta}_{12}$, $\boldsymbol{\delta}_{22}$ $\boldsymbol{\delta}_{32}$, while twists at the other interfaces, eg. further on at 4 or 5: $\boldsymbol{\delta}_{42}$ $\boldsymbol{\delta}_{52}$, twists do not occur.

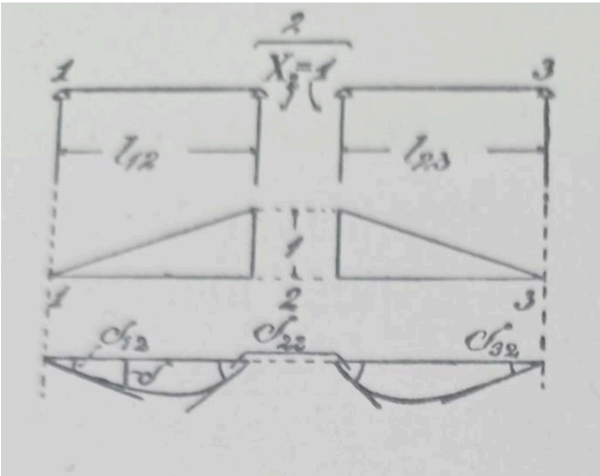


Abb. 4.

For a statically indeterminate system in which the statically indeterminate loads X_1, X_2, X_3… are $\boldsymbol{\delta}'$, $\boldsymbol{\delta}''$, $\boldsymbol{\delta}'''$, ... the displacements or rotations of each of

the attack points of application of X_1, X_2, X_3,….. as a result of the opposing loads, further $\boldsymbol{\delta}_{11}$, $\boldsymbol{\delta}_{21}$ $\boldsymbol{\delta}_{31\ldots}$ the shifts or twists of the attack points of X_1, X_2, X_3,…..as a result of the given loads X_1 = 1, $\boldsymbol{\delta}_{12}$, $\boldsymbol{\delta}_{22}$ $\boldsymbol{\delta}_{32}$… the same shifts has a result of the effect of X_2 = 1, therefore the system of equations generally applies[(1)].

$$(4)\quad \left\{\begin{array}{l} \delta' + X_1\,\delta_{11} + X_2\,\delta_{12} + X_3\,\delta_{13} + X_4\,\delta_{14} + \;\ldots\ldots = 0 \\ \delta'' + X_1\,\delta_{21} + X_2\,\delta_{22} + X_3\,\delta_{23} + X_4\,\delta_{24} + \;\ldots\ldots = 0 \\ \delta''' + X_1\,\delta_{31} + X_2\,\delta_{32} + X_3\,\delta_{33} + X_4\,\delta_{34} + \;\ldots\ldots = 0 \\ \delta^{IV} + X_1\,\delta_{41} + X_2\,\delta_{42} + X_3\,\delta_{43} + X_4\,\delta_{44} + \;\ldots\ldots = 0 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \end{array}\right.$$

Each equation applies to a point of application of a statically indeterminate load and states that the total displacement of this point disappears under the action of the loads and the statically indeterminate loads.
According to the determinant theory, the solutions according to X_1, X_2, X_3…can be written in the form

$$(5)\quad \left\{\begin{array}{l} X_1 = -(\alpha_{11}\,\delta' + \alpha_{12}\,\delta'' + \alpha_{13}\,\delta''' + \;\ldots\ldots) \\ X_2 = -(\alpha_{21}\,\delta' + \alpha_{22}\,\delta'' + \alpha_{23}\,\delta''' + \;\ldots\ldots) \\ X_3 = -(\alpha_{31}\,\delta' + \alpha_{32}\,\delta'' + \alpha_{33}\,\delta''' + \;\ldots\ldots) \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \end{array}\right.$$

where the α as a quotient of determinants appear. Is called

$$(6)\qquad N = \begin{vmatrix} \delta_{11} & \delta_{12} & \delta_{13} & \delta_{14} & \cdot & \cdot & \cdot \\ \delta_{21} & \delta_{22} & \delta_{23} & \delta_{24} & \cdot & \cdot & \cdot \\ \delta_{31} & \delta_{32} & \delta_{33} & \delta_{34} & \cdot & \cdot & \cdot \\ \delta_{41} & \delta_{42} & \delta_{43} & \delta_{44} & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{vmatrix},$$

So α_{ik} is;

$$(7)\qquad a_{ik} = (-1)^{(i+k)} \cdot \frac{1}{N} \cdot \begin{vmatrix} \delta_{11} & \delta_{12} & \cdot & \cdot & \delta_{1\,(i-1)} & \delta_{1\,(i+1)} & \cdot & \cdot \\ \delta_{21} & \delta_{22} & \cdot & \cdot & \delta_{2\,(i-1)} & \delta_{2\,(i+1)} & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \delta_{(k-1)\,1} & \delta_{(k-1)\,2} & \cdot & \cdot & \delta_{(k-1)(i-1)} & \delta_{(k-1)(i+1)} & \cdot & \cdot \\ \delta_{(k+1)\,1} & \delta_{(k+1)\,2} & \cdot & \cdot & \delta_{(k+1)(i-1)} & \delta_{(k+1)(i+1)} & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{vmatrix}$$

The determinant of the counter shown in (7) is the determinant N after eliminating the *i*-th vertical and the *k*-th horizontal row.
As stated previously, with a continuous beam only values $\boldsymbol{\delta}$ occur whose indices are the same or different by 1.
In each of the equations there are only three consecutive values $\boldsymbol{\delta}$. If one considers a field $1_{(k-1)k}$ as loaded, then in the entire system of equations (4), only two of the values can be used $\boldsymbol{\delta}', \boldsymbol{\delta}'', \boldsymbol{\delta}''', \boldsymbol{\delta}^{\mathrm{Iv},\dots}$. namely the twists $\boldsymbol{\delta}^{(k-1)}$ and $\boldsymbol{\delta}^{(k)}$ which occur on the two
adjacent supports as a result of a moving concentrated load P=1. For a loaded field (*k*-1), the *k* system of equations applies:

$$(8)\qquad \begin{cases} X_1\delta_{11} + X_2\delta_{12} = 0 \\ X_1\delta_{21} + X_2\delta_{22} + X_3\delta_{23} = 0 \\ \quad X_2\delta_{32} + X_3\delta_{33} + X_4\delta_{34} = 0 \\ \cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot \\ \delta^{(k-1)} + X_{(k-2)}\delta_{(k-1)\,(k-2)} + X_{(k-1)}\delta_{(k-1)\,(k-1)} + X_k\delta_{(k-1)\,k} = 0 \\ \delta^{(k)} + \qquad + X_{(k-1)}\delta_{k\,(k-1)} + X_k\delta_{k\,k} + X_{(k+1)}\delta_{k(k+1)} = 0 \\ \cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot \\ X_{(n-2)}\delta_{(n-1)\,(n-2)} + X_{(n-1)}\delta_{(n-1)(n-1)} + X_n\delta_{(n-1)n} = 0 \\ \qquad X_{(n-1)}\delta_{n(n-1)} + X_n\delta_{n\,n} = 0 \end{cases}$$

According to equations (5) we now have

$$\begin{aligned} X_1 &= -(a_{11}\,\delta' + a_{12}\,\delta'' + \quad \cdot\quad\cdot\quad\cdot\quad\cdot \quad + a_{1n}\,\delta^{(n)}) \\ X_2 &= -(a_{21}\,\delta' + a_{22}\,\delta'' + \quad \cdot\quad\cdot\quad\cdot\quad\cdot \quad + a_{2n}\,\delta^{(n)}) \\ &\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot \\ X_n &= -(a_{n1}\,\delta' + a_{n2}\,\delta'' + \quad \cdot\quad\cdot\quad\cdot\quad\cdot \quad + a_{nn}\,\delta^{(n)}) \end{aligned}$$

where a_{11}, a_{12}, the meaning expressed in equation (7) in this case the denominator determinant has the form:

$$(9) \quad N = \begin{vmatrix} \delta_{11} & \delta_{12} & 0 & 0 & 0 & . & 0 \\ \delta_{21} & \delta_{22} & \delta_{23} & 0 & 0 & . & 0 \\ 0 & \delta_{32} & \delta_{33} & \delta_{34} & 0 & . & 0 \\ . & . & . & . & . & . & . \\ 0 & 0 & 0 & . & \delta_{(n-1)(n-2)} & \delta_{(n-1)(n-1)} & \delta_{(n-1)n} \\ 0 & 0 & 0 & . & 0 & \delta_{n(n-1)} & \delta_{nn} \end{vmatrix}$$

From the system of equations (8) it can be seen that in the first equation one can express X_1, by X_2, and therefore in the second equation, X_2, by X_8, etc. In this way, starting from the beginning, one arrives at the replacement of (k-1) by $X_{(k-2)}$ in the $X_{(k-1)}$ equation. Likewise, starting backwards with the last equation, you can finally replace $X_{(k+1)}$ with X_k in the *k*th equation. You end up with two equations in which only $X_{(k-1)}$ and X appear and which can therefore be solved for the two unknowns. This possible solution is similar to the well-known graphical method, according to which, for example, when a field (layer) is loaded through an evenly distributed load, all supporting moments except those of the loaded field (layer) are achieved by a simple straight line using the so-called (?Clayperon again?). Fixed points J and K can be found according to Fig. 5.

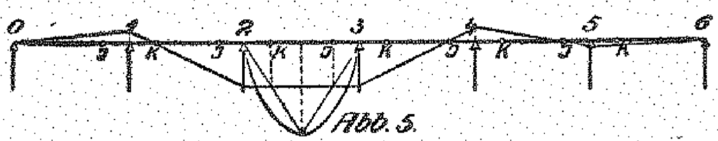


Abb. 5.

In terms of numbers, the fixed points can be replaced by numerical values *i* and *k*, where the numbers *i* represent the ratios of the supporting moments in a forwards counting direction and the numbers *k* in a backward counting direction. So i_{56} means the ratio

10) $$i_{56} = \frac{X_5}{X_6}$$

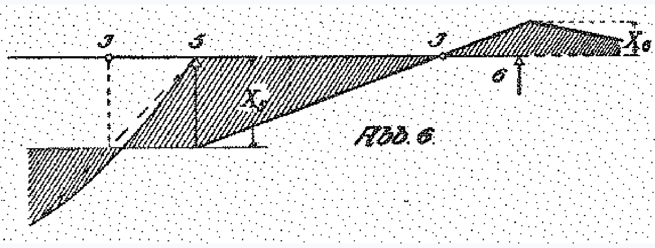


Abb. 6.

if the loaded field (Layer)precedes the column 5 (Abb. 6) and

11) $$k_{65} = \frac{X_6}{X_5}$$

if the loaded field (layer) follows support 6.

According to equations (8), in the first case the last equations, in the second case the first equations up to the next equation, for the support of the load field (layer) do not contain values ***δ', δ''***,... etc.

As shown below, all coefficients a, i, k can be expressed as subdeterminants of the denominator determinant N (see Formula 6).

Let D denote the denominator determinant

$$D_n = \begin{vmatrix} \delta_{11} & \delta_{12} & 0 & 0 & 0 & . & 0 \\ \delta_{21} & \delta_{22} & \delta_{23} & 0 & 0 & . & 0 \\ 0 & \delta_{32} & \delta_{33} & \delta_{34} & 0 & . & 0 \\ 0 & 0 & \delta_{43} & \delta_{44} & \delta_{45} & . & 0 \\ . & . & . & . & . & . & 0 \\ 0 & 0 & 0 & . & \delta_{(n-1)(n-2)} & \delta_{(n-1)(n-1)} & \delta_{(n-1)n} \\ 0 & 0 & 0 & . & 0 & \delta_{n(n-1)} & \delta_{nn} \end{vmatrix}$$

And continue in order

$$(12) \quad \left\{ \begin{array}{ll} D_0 = 1 & \text{und} \quad D'_{(n+1)} = 1 \\ D_1 = \delta_{11} & D'_n = \delta_{nn} \\ D_2 = \begin{vmatrix} \delta_{11} & \delta_{12} \\ \delta_{21} & \delta_{22} \end{vmatrix} & D'_{(n-1)} = \begin{vmatrix} \delta_{(n-1)(n-1)} & \delta_{(n-1)n} \\ \delta_{n(n-1)} & \delta_{nn} \end{vmatrix} \\ & \text{usf.} \\ D_3 = \begin{vmatrix} \delta_{11} & \delta_{12} & 0 \\ \delta_{21} & \delta_{22} & \delta_{23} \\ 0 & \delta_{32} & \delta_{33} \end{vmatrix} \ \text{usf.} & \end{array} \right.$$

so you can easily calculate the values of the simple determinants: Since due to the theorem of reciprocity of the displacements, we have: $\boldsymbol{\delta}_{12}=\boldsymbol{\delta}_{21}$ and $\boldsymbol{\delta}_{23}=\boldsymbol{\delta}_{32}$
where the denominator determinant is

$$(13) \quad \left\{ \begin{array}{l} D_0 = 1 \\ D_1 = \delta_{11} \\ D_2 = D_1 \delta_{22} - \delta_{12}^2 \cdot D_0 \\ D_3 = D_2 \delta_{33} - \delta_{23}^2 \cdot D_1 \\ D_4 = D_3 \delta_{44} - \delta_{34}^2 \cdot D_2 \quad \text{usf.} \\ \text{und bei } n = 5, \text{ z. B.} \\ D'_6 = 1 \\ D'_5 = \delta_{55} \\ D'_4 = \begin{vmatrix} \delta_{44} & \delta_{45} \\ \delta_{54} & \delta_{55} \end{vmatrix} = D'_5 \delta_{44} - \delta_{45}^2 D'_6 \\ D'_3 = \begin{vmatrix} \delta_{33} & \delta_{34} & 0 \\ \delta_{43} & \delta_{44} & \delta_{45} \\ 0 & \delta_{54} & \delta_{55} \end{vmatrix} = D'_4 \delta_{33} - \delta_{34}^2 D'_5 \end{array} \right.$$

where the denominator determinant is

$$D_n = \begin{vmatrix} \delta_{11} & \delta_{12} & 0 & 0 & 0 \\ \delta_{21} & \delta_{22} & \delta_{23} & 0 & 0 \\ 0 & \delta_{32} & \delta_{33} & \delta_{34} & 0 \\ 0 & 0 & \delta_{34} & \delta_{44} & \delta_{45} \\ 0 & 0 & 0 & \delta_{54} & \delta_{55} \end{vmatrix}$$

By applying the sets of determinants for the first 5 equations, one obtains that in a system of homogeneous, linear equations, i.e. equations without a constant term, the unknowns behave like the corresponding subdeterminants:

$$(14) \qquad k_{65} = \frac{X_6}{X_5} = -\frac{D_5}{\delta_{65} D_4}$$

und

$$(15) \qquad i_{56} = \frac{X_5}{X_6} = -\frac{D'_6}{\delta_{65} \cdot D'_7}$$

A value a_{mm} is obtained (see equation 7) by deleting the m th vertical and horizontal rows in the denominator determinant and dividing by Dn, e.g.

$$(16) \qquad \begin{aligned} a_{11} &= \frac{D_0 \cdot D'_2}{D_n} \\ a_{22} &= \frac{D_1 \cdot D'_3}{D_n} \\ a_{33} &= \frac{D_2 \cdot D'_4}{D_n} \\ a_{44} &= \frac{D_3 \cdot D'_5}{D_n} \\ a_{55} &= \frac{D_4 \cdot D'_6}{D_n} \\ a_{66} &= \frac{D_5 \cdot D'_7}{D_n}. \end{aligned}$$

The law of formation of numbers with a double index is now clear

$$(16\,a) \qquad a_{m\,m} = \frac{D_{(m-1)}\, D'_{(m+1)}}{D_n}.$$

Because of the relationship following from the well-known Maxwell's reciprocal rule $\boldsymbol{\delta}_{12}=\boldsymbol{\delta}_{21}$ must also mean $a_{12}=a_{21}$, consequently:

$$a_{34} = \frac{D_2 \cdot \delta_{34} \cdot D'_5}{D_n}$$

und

$$(17) \qquad a_{56} = \frac{D_4 \cdot \delta_{56} \cdot D'_7}{D_n}$$

$$a_{m\,(m+1)} = \frac{D_{(m-1)} \cdot \delta_{m\,(m+1)} \cdot D'_{(m+2)}}{D_n}.$$

The proof for sentences (16) and (17) is provided by the considerations of the corresponding perpetrator determinants (see equation 13). The comparison of formula (14), (15) and (17) gives the grand equation:

$$(18) \qquad \left\{ \begin{aligned} i_{56} &= -\frac{D_6'}{\delta_{65} \cdot D_7'} = -\frac{D_4 D_6'}{D_n} \cdot \frac{D_n}{D_4 \cdot \delta_{56}\, D_7'} = -\frac{a_{55}}{a_{56}} \\ k_{65} &= -\frac{D_5}{\delta_{56} \cdot D_4} = -\frac{D_5 D_7'}{D_n} \cdot \frac{D_n}{D_4 \cdot \delta_{56} \cdot D_7'} = -\frac{a_{66}}{a_{56}} \end{aligned} \right.$$

If only opening l_{56} is loaded, $\boldsymbol{\delta}^{v}$ and $\boldsymbol{\delta}^{vi}$ indicate the rotations that the attack cross sections of X_5 and X_6 experience at supports 5 and 6. The remaining $\boldsymbol{\delta}'$, $\boldsymbol{\delta}''$, $\boldsymbol{\delta}'''$...etc. disappear. From equations (5) it can be seen that for this loading case

$$
(19) \quad \left\{
\begin{array}{l}
X_1 = -(a_{15}\,\delta^{V} + a_{16}\,\delta^{VI}) \\
X_2 = -(a_{25}\,\delta^{V} + a_{26}\,\delta^{VI}) \\
\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot \\
X_5 = -(a_{55}\,\delta^{V} + a_{56}\,\delta^{VI}) \\
X_6 = -(a_{65}\,\delta^{V} + a_{66}\,\delta^{VI}) \\
\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot \\
\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot
\end{array}
\right.
$$

Since in the loaded openings when a field (layer) is loaded *m, (m+1)k* the ratio of two successive supporting moments preceding the loaded field (layer)

$$k_{65} = \frac{X_6}{X_5} = \frac{a_{6,m}\,\delta^{(m)} + a_{6,(m+1)}\,\delta^{(m+1)}}{a_{5,m}\,\delta^{(m)} + a_{5,(m+1)}\,\delta^{(m+1)}}$$

und

$$i_{56} = \frac{X_5}{X_6}, \text{ so müssen, } ($$

so, since the $\boldsymbol{\delta}$ independent variables, the ratios between two a-values, e.g.

$$k_{65} = \frac{a_{6,m}}{a_{5,m}} = \frac{a_{6(m+1)}}{a_{5(m+1)}}$$

Represent if m less than 6. General is (wenn =if or when)

$$
(18\text{ a}) \quad \left\{
\begin{array}{l}
k_{(l+1)\,l} = \dfrac{a_{(l+1)\,m}}{a_{l,\,m}}, \text{ wenn } m \geqq l+1 \\
\text{und} \\
i_{p,\,(p+1)} = \dfrac{a_{p,\,m}}{a_{(p+1),\,m}}, \text{ wenn } m \leqq p
\end{array}
\right.
$$

Example is

$$
(18\text{ b}) \quad \left\{
\begin{array}{l}
k_{54} = \dfrac{X_5}{X_4} = \dfrac{a_{55}}{a_{45}} = \dfrac{a_{56}}{a_{46}} = \dfrac{a_{57}}{a_{47}} \text{ usf.} \\
k_{43} = \dfrac{a_{45}}{a_{35}} = \dfrac{a_{46}}{a_{36}} \text{ usf.} \\
k_{32} = \dfrac{a_{35}}{a_{25}} = \dfrac{a_{36}}{a_{26}} \text{ usf.} \\
k_{21} = \dfrac{a_{25}}{a_{15}} = \dfrac{a_{26}}{a_{16}} \text{ usf.} \\
\text{und} \\
i_{67} = \dfrac{a_{66}}{a_{76}} = \dfrac{a_{65}}{a_{75}} = \dfrac{a_{64}}{a_{74}} \text{ usf.}
\end{array}
\right.
$$

So you have found the numbers of the diagonal of the denominator determinant a, a,, etc., then you can find all other numbers by dividing them with *i* and *k* values, as shown:

$$(20)\qquad \begin{array}{c|cccc|c} i_{12} & a_{11} & & & & k_{21} \\ i_{23} & & a_{22} & & & k_{32} \\ i_{34} & & & a_{33} & & k_{43} \\ & & & & a_{44} & \end{array}$$

As the above diagram suggests, divide by k on the right of the diagonal and by i on the left. This scheme will be called a number rectangle (Matrix). E.g.

$$(21)\qquad \begin{aligned} a_{34} &= a_{44} : k_{43} \\ a_{24} &= a_{34} : k_{32} \text{ usw.} \end{aligned}$$

The value a_{11} is after (16)

$$(22)\qquad \begin{aligned} a_{11} &= \frac{D'_2}{D_n} = \frac{D'_2}{\delta_{11} \cdot D_2' - \delta^2_{12} D_3'} = \frac{1}{\delta_{11} - \delta^2_{12} \frac{D'_3}{D'_2}} = \\ &= \frac{1}{\delta_{11} - \delta^2_{12} \frac{1}{\delta_{22} - \delta^2_{23} \frac{D'_4}{D'_3}}} \\ &= \frac{1}{\delta_{11} - \delta^2_{12} \frac{1}{\delta_{22} - \delta^2_{23} \frac{1}{\delta_{33} - \delta^2_{34} \frac{1}{\delta_{44} - \delta^2_{45} \frac{1}{\delta_{55} - \cdots}}}}} \\ &\qquad \cdots \frac{1}{- \delta^2_{(n-2)(n-1)} \cdot \frac{1}{\delta_{(n-1)(n-1)} - \frac{\delta^2_{(n-1)n}}{\delta_{nn}}}} \end{aligned}$$

This means that a_{11} is shown as a continued fraction from the $\boldsymbol{\delta}$ th. There according to the relationship (15)

$$\delta_{12} \frac{D'_3}{D'_2} = -\frac{1}{i_{12}}$$

will be

$$(23)\qquad a_{11} = \frac{1}{\delta_{11} + \frac{\delta_{12}}{i_{12}}}.$$

Something similar is obtained

$$(24)\qquad a_{nn} = \frac{1}{\delta_{nn} + \frac{\delta_{n(n-1)}}{k_{n(n-1)}}}$$

From (22) it follows that the terms of the continuous fraction 22 one after the other the values

$$(25)\qquad \frac{1}{i_{n(n-1)}}, \frac{1}{i_{(n-1)(n-2)}} \cdots\cdots i_{12}, a_{11}$$

and also when forming a_{nn}, represent the terms of the continuous fraction for a_{nn} one after the other.

$$(26)\qquad \frac{1}{k_{12}}, \frac{1}{k_{23}} \cdots\cdots \frac{1}{k_{(n-1)n}}, a_{nn}$$

For example, the part on the right separated by bold lines is the following continuous fraction

$$(27)\quad \cfrac{1}{\delta_{11}-\delta_{12}\cfrac{\delta_{12}}{\delta_{22}-\delta_{23}\cfrac{\delta_{23}}{\delta_{33}-\delta_{34}\cfrac{\delta_{34}}{\delta_{44}-\delta_{45}\cfrac{\delta_{45}}{\delta_{55}-\cdots\cdots \cdots\cdots-\delta_{(n-2)(n-1)}\cfrac{\delta_{(n-2)(n-1)}}{\delta_{(n-1)(n-1)}-\delta_{(n-1)n}\cfrac{\delta_{(n-1)n}}{\delta_{nn}}}}}}}}$$

The value $-\frac{1}{i_{34}}$

Result

The basic values a_{11}, a_{nn} of the numerical rectangle (matrix) are represented as finite continuous fractions with n terms according to (22). When calculating the first continuous fraction, the coefficients 1/i are obtained one after the other, and for the second continuous fraction the coefficients 1/k, which can be used as ratio values between successive values a from a_{11}, a_{nn} and all other a-values. After you have calculated all the values i and k, and the two coefficients a_{11} and a_{nn}, you can easily find the entire scheme of the values a. After solving the two continuous fractions, first write a_{11} and a_{nn}, and on the left the values 1/i on the right 1/k (the latter values each at half the height of the line) according to arrangement (20). From a_{11} and 1/i you can find a_{11}, a_{21}, a_{31}, a_{n1} and by multiplying from a_{nn} and 1/k you get $a_{(n-1)n}$...... a_{4n}, a_{3n}, a_{2n}, a_{1n}. Because of the symmetry to the number diagonal formed by a_{11}, a_{22}, a_{33}, …a_{nn} the first or last column equal to the first and last horizontal row. From the first or Last horizontal row you can now find all remaining values by further dividing by 1/i for all values to the left of the number diagonal from bottom to top and by 1/k for all values to the right of the diagonal from top to bottom. Since the ratio i or k also applies to two adjacent values a in a horizontal row, one also has controls. Knowing the values of a makes it possible to immediately specify the ordinates of the line of influence of any statically indeterminate X_m in any field $l_{k(k-1)}$. The incident line is obtained according to (19) by multiplying the curves

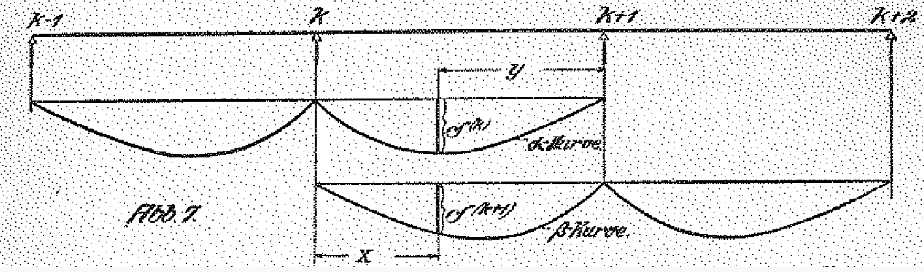
$$(28)\quad \delta^{(k+1)} = \frac{l^2{}_{k(k+1)}}{6\,EJ}(\xi^3-\xi) \text{ und } \delta^{(k)} = \frac{l^2{}_{k(k+1)}}{6\,EJ}(\eta^3-\eta),$$

$$(29)\quad \text{wo } \xi = 1-\eta \text{ und } \xi = \frac{x}{l},\ \eta = \frac{y}{l},$$

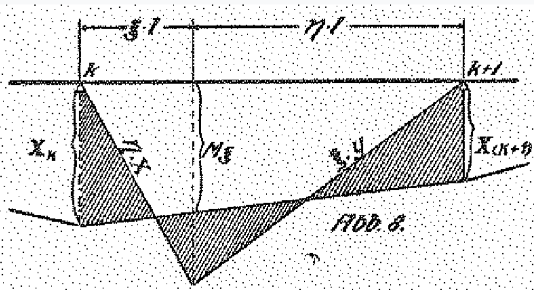
with the values $a_{m(k+1)}$ and $a_{m'k}$ and addition.

'In the following, the Greek letters a =…. indicate the corresponding curves' (see Fig. 7)

(29 a) In der Folge sollen die griechischen Buchstaben $\alpha = \eta - \eta^3$ und $\beta = \xi - \xi^3$ die entsprechenden Kurven andeuten (s. Abb. 7).



The line of influence of the field moment at a point & the opening 4+1 is obtained from X and X+1) (see Fig. 8), since 1-=n:



(30)
$$M_\xi = X_k + \xi (X_{(k+1)} - X_k) = \\ = \xi \cdot X_{(k+1)} + \eta \cdot X_k$$

Formula (30) gives the ordinate of the line of influence in all unloaded openings, in the loaded opening lx+1), however, the ordinate of the line of influence of the field moment in the beam on 2 supports must be added:

(31)
$$M_\xi = \xi \cdot X_{(k+1)} + \eta \cdot X_k + \eta \cdot x \quad \text{bezw.} + \xi \cdot y$$

$\eta \cdot x$ ist der von k, $\xi \cdot y$ der $(k+1)$ ausgehende Ast.

(n.x is the branch starting from k, ξ.y (k+1))

The line of influence of the transverse forces in an opening I $_{k(k+1).}$ is:

(32)
$$Q = -\frac{X_{(k+1)} - X_k}{l_{k(k+1)}}$$

in all unloaded openings and in the loaded opening I $_{k(k+1).}$ itself

(33)
$$Q = -\frac{X_{(k+1)} - X_k}{l_{k(k+1)}} + \eta$$

namely positive for the effect from bottom to top on the right-hand support section. The line of influence of a supporting force 4 is obtained from the difference in the transverse force values in front of and behind the support

(or)

$$A_k = Q_{k,(k+1)} - Q_{(k-1),k}$$

oder

(34)
$$A_k = \frac{X_k - X_{(k-1)}}{l_{(k-1)k}} - \frac{X_{(k-1)} - X_k}{l_{(k+1)k}} + \\ + \xi \text{ bezw. } + \eta$$

The value ξ refers to the opening in front of, and η behind, the support under consideration.

If an opening is evenly loaded with $p_{k(k+1).}$, then

$$(35)\qquad \delta^{(k)} = \delta^{(k+1)} = p_{k(k+1)} \frac{l^3_{k(k+1)}}{6EJ} \int_0^1 (\xi - \xi^3) \cdot d\xi = p_{k(k+1)} \frac{l^3_{k(k+1)}}{24EJ}$$

The supporting moment X_m becomes:

$$(36)\qquad X_m = -(a_{mk} + a_{m(k+1)}) \cdot \frac{l^3_{k(k+1)}}{24EJ} \cdot p_{k(k+1)}.$$

If p_{01}, p_{12}, p_{23}, are the uniform loads on the openings l_{01}, l_{12}, l_{23} then the supporting moment X_m as a result of this load becomes

$$(37)\qquad X_m = -\frac{1}{24E}\left\{ a_{m1} \cdot \frac{l^3_{01}}{J_{01}} \cdot p_{01} + (a_{m1} + a_{m2}) \frac{l^3_{12}}{J_{12}} p_{12} + \cdots\cdots + a_{mn} \frac{l^3_{n(n+1)}}{J_{n(n+1)}} p_{n(n+1)} \right\}$$

where J_{01}, J_{12}, etc. are the moments of inertia assumed to be constant for an opening. If there are uniform loads for only one opening, you can easily find the individual supporting moments X and at any point in an opening using formula (37):

$$(38)\qquad M_\xi = \xi \cdot X_{(k+1)} + \eta \cdot X_k + \frac{pl^2}{2}(\xi - \xi^2).$$

The integral is used for distributed loads:

$$(39)\qquad \int_\xi^1 (\xi - \xi^3)\, d\xi = \frac{1}{4}\left[2\xi^2 - \xi^4\right]_\xi^1 = \frac{1}{4}\left[1 - \xi^2(2 - \xi^2)\right]$$

The numbers in the square brackets are included in the line load table at the end (for how to use them, see page 42 (MR – pagination TBC)).

B. Application of the method to continuous beams with stiff connected supports (multi-legged frames).

In the procedure of the previous section, the beams were intended to be intersected above each support; The ends to the right and left of the interface were each supported on two support points that were close together and the bending moment pairs X were still applied at the ends to create the original state. Figure 9 shows this transformation of the continuous beam into a group of statically determined beams.

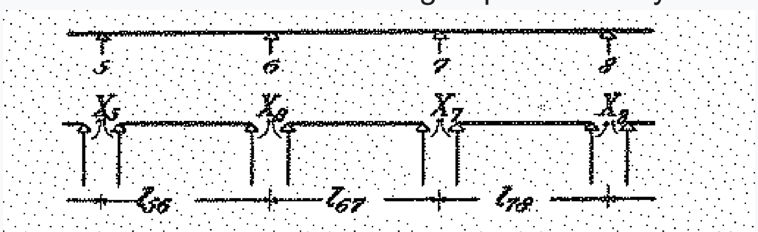


Abb. 9.

In the case of supports rigidly connected to the beams, the bending moment of the beam immediately in front of the support is different than immediately behind the support. Is the moment in

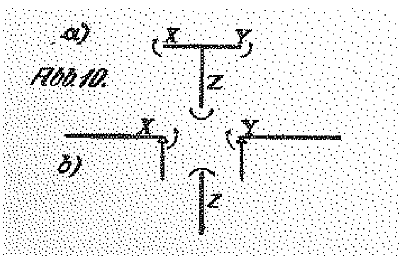


Abb. 10.

front of the support X stiff support connection (see Fig. 10a)
X-Y= Z. (40)
The continuous beam collapses again behind the support Y and at the head of the support Z, so balance requires a series of individual statically determined beams on two supports. The bending moments on each pair of supports are now no longer the same in pairs, but different by the amount (-2).
Two further equations result in the condition that applies to the connection at the column head: because of the rigid connection, the three axial directions emanating from here must have rotated by the same angle in the elastically distorted state. Additionally, all three ends will also suffer the same shift. Most authors neglect this displacement and also the elastic effect of the normal forces. This results in a neglect similar to that which occurs in the frame when the elastic effect of the horizontal thrust is neglected as a longitudinal force in the bar. In order to express the second condition using formulas, one must know the angles through which the individual arms of the supporting structure rotate. If you look at support point 5, the three bending moments (see Fig. 11) are called X_5, Y_5, Z_5. The rotations are:

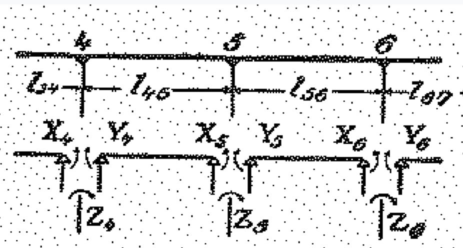


Abb. 11.

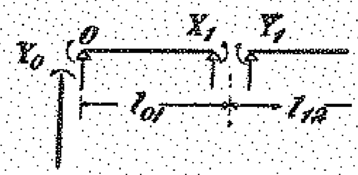


Abb. 12.

Final cross sections of the beams l_{45} and l_{56}, are similar to the previous section under the effect of $X_5=1$, $Y_5=1$:
(Rotation at 5 as a result of)

$$\left.\begin{array}{llll}
\text{Drehung bei 5 infolge} & X_5=1: & -2\,\delta_{45} = & -\dfrac{1}{EJ_{45}}\cdot\dfrac{l_{45}}{3} \\
\text{,, \quad ,, \quad 4 \quad ,,} & X_5=1: & \delta_{45} = & \dfrac{1}{EJ_{45}}\cdot\dfrac{l_{45}}{6} \\
\text{,, \quad ,, \quad 5 \quad ,,} & Y_5=1: & 2\,\delta_{56} = & \dfrac{1}{EJ_{56}}\cdot\dfrac{l_{56}}{3} \\
\text{,, \quad ,, \quad 6 \quad ,,} & Y_5=1: & -\delta_{56} = & -\dfrac{1}{EJ_{56}}\cdot\dfrac{l_{56}}{6}.
\end{array}\right\} \quad (41)$$

The rotation of the end cross-section of the support post takes place under the assumption that the support head does not experience any (horizontal) displacement. Two cases can be distinguished:

1. The support is clamped at the bottom.
Bending moment M=Z.
It is statically indeterminate. Support pressure H

$$M_x = Z - H \cdot x$$
$$\frac{d\,Mx}{d\,H} = -x$$
$$\int_0^h M_x(-x) \cdot dx = -Z\frac{h^2}{2} + H\frac{h^3}{3} = 0$$
$$H = \frac{3}{2\,h} \cdot Z \qquad \text{und} \qquad M_x = Z\left(1 - \frac{3}{2} \cdot \frac{x}{h}\right).$$

The differential equation of the laying line is:

$$-\frac{d^2\,y}{d\,x^2} = \frac{Z}{E\,J}\left(1 - \frac{3}{2} \cdot \frac{x}{h}\right)$$
$$-y = \frac{Z}{E\,J}\left(\frac{x^2}{2} - \frac{x^3}{4\,h} + C_1 \cdot x + C_2\right)$$

(and from the boundary conditions…and….=0 for….results are)

und aus den Grenzbedingungen δ und $\frac{d\,\delta}{d\,x} = 0$ für $x = h$ ergibt sich

$$-y = \frac{Z}{E\,J}\left(\frac{x^2}{2} - \frac{x^3}{4\,h} - \frac{h\,x}{4}\right).$$

The rotation angle δ under the effect of Z-1 on the support head is:

$$-\frac{d\,y}{d\,x} = \frac{1}{E\,J}\left[x - \frac{3\,x^2}{4\,h} - \frac{h}{4}\right] \text{ für } x = 0$$

$$\delta = \frac{h}{4\,E\,J}. \tag{42}$$

2. The support is hinged at the bottom.

The beam is falling on two supports, so the rotation is according to the previous (see formula 3)

$$\delta = \frac{h}{3\,\mathrm{EJ}}. \tag{43}$$

All rotations are now given in formulas (41), (42) and (43).

If we continue to look at the conditions at support point 5 (see Fig. 11) and if $\boldsymbol{\delta}_5$ denotes the angle of rotation of the support head, the condition for equality of the three angles of rotation is:

$$-Y_4 \cdot \delta_{45} - X_5 \cdot 2\,\delta_{45} = Y_5 \cdot 2\,\delta_{56} + X_6 \cdot \delta_{56} = -Z_5 \cdot \delta_5 = (X_5 - Y_5)\,\delta_5\,, \tag{44}$$

which is also a pair of two equations:

$$\left\{\begin{array}{l} Y_4 \cdot \delta_{45} + X_5\,(2\,\delta_{45} + \delta_5) - Y_5 \cdot \delta_5 = 0 \\ \qquad\qquad - X_5 \cdot \delta_5 \qquad\qquad + Y_5\,(2\,\delta_{56} + \delta_5) + X_6 \cdot \delta_{56} = 0 \end{array}\right. \tag{44 a}$$

can be written. Accordingly, the equations for support 6 are:

$$\left\{\begin{array}{l} Y_5\,\delta_{56} + X_6\,(2\,\delta_{56} + \delta_6) - Y_6 \cdot \delta_6 = 0 \\ \qquad\qquad - X_6 \cdot \delta_6 \quad + Y_6\,(2\,\delta_{67} + \delta_6) + X_7 \cdot \delta_{67} = 0. \end{array}\right. \tag{44 b}$$

(Note that if you equate the two bending moments X and Y on support 5, you get a single equation for support 5:

$$Y_4 \cdot \delta_{45} + X_5\,(2\,\delta_{45} + 2\,\delta_{56}) + X_6\,\delta_{56} = 0$$

Of the form known from the previous section on page 7 arises: because it is $\delta_{55} = 2\,\delta_{45} + 2\,\delta_{56}$. If the start or end support is also rigidly connected to the beam, the first or last equation the rotations at the first or last support. In this case, the first equations are, after already ordering as in equations (44 a) and (44 b):

$$\text{(44 c)}\quad \begin{aligned}
&Y_0\,(2\,\delta_{01} + \delta_0) + X_1 \cdot \delta_{01} = 0\\
&Y_0 \cdot \delta_{01} + X_1\,(2\,\delta_{01} + \delta_1) - Y_1 \cdot \delta_1 = 0\\
&- X_1 \cdot \delta_1 + Y_1\,(2\,\delta_{12} + \delta_1) + X_2 \cdot \delta_{12} = 0\\
&Y_1 \cdot \delta_{12} + X_2\,(2\,\delta_{12} + \delta_2) - Y_2 \cdot \delta_2 = 0\\
&- X_2 \cdot \delta_2 + Y_2\,(2\,\delta_{23} + \delta_2) + X_3 \cdot \delta_{23} = 0\\
&\ldots\ldots\ldots\ldots
\end{aligned}$$

If the opening $l_{(k-1)k}$ is loaded and the rotations of the ends of the beam on two supports (k-1) and k due to the loads are denoted by $\boldsymbol{\delta}^{(k-1)}$ and $\boldsymbol{\delta}^{(k)}$, then the four equations for the supports (k-1) and k are :

$$\text{(44 d)}\quad \begin{aligned}
&Y_{k-2} \cdot \delta_{(k-2)(k-1)} + X_{(k-1)} \cdot (2\,\delta_{(k-2)(k-1)} + \delta_{(k-1)}) - Y_{(k-1)} \cdot \delta_{(k-1)} = 0\\
&- X_{(k-1)} \cdot \delta_{(k-1)} + Y_{(k-1)}\,(2\,\delta_{(k-1)k} + \delta_{(k-1)}) + X_k \cdot \delta_{(k-1)k} + \delta^{(k-1)} = 0\\
&Y_{(k-1)} \cdot \delta_{(k-1)k} + X_k\,(2\,\delta_{(k-1)k} + \delta_k) - Y_k \cdot \delta_k + \delta^{(k)} = 0\\
&- X_k \cdot \delta_k + Y_k\,(2\,\delta_{k(k+1)} + \delta_{(k+1)}) + X_{(k+1)} \cdot \delta_{k(k+1)} = 0.
\end{aligned}$$

The number of equations (44) is as large as that of the unknowns, the form of the entire system of equations is the same as that of the simple continuous support (see formula 8 on p. 7).

The denominator determinant of the system of equations (44) has the form

$$D_n = \begin{vmatrix}
2\,\delta_{01} + \delta_0 & \delta_{01} & 0 & 0 & 0 & 0 & \cdot\\
\delta_{01} & 2\,\delta_{01} + \delta_1 & -\delta_1 & 0 & 0 & 0 & \cdot\\
0 & -\delta_1 & 2\,\delta_{12} + \delta_1 & \delta_{12} & 0 & 0 & \cdot\\
0 & 0 & \delta_{12} & 2\,\delta_{12} + \delta_2 & -\delta_2 & 0 & \cdot\\
0 & 0 & 0 & -\delta_2 & 2\,\delta_{23} + \delta_2 & \delta_{23} & \cdot\\
0 & 0 & 0 & 0 & \delta_{23} & 2\,\delta_{23} + \delta_3 & \cdot\\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot
\end{vmatrix}$$

which corresponds to the deviation of the determinant formula (9) that in the two neighboring diagonals that are symmetrically the same as the middle diagonal, the 3-values alternately have the negative sign. However, the other earlier derivations formula (10) to (22) are not affected by this circumstance. The only consequence is that the i and k values resulting from the continuous fractions formula (22) and (24) have alternating positive and negative signs.

Not only the ones discussed here, but also other related issues, e.g Viral deletion carriers lead to systems of equations whose denominator determinant has the form described above if certain neglects are made. All of these problems can be solved using matrix calculus.

The rules for applying the numerical rectangle method that follow from the theoretical treatment are compiled below and explained using examples.

Second part.
Application.

A. Continuous beam freely supported on n (e.g. 4) central supports, with variable but constant in each individual field moment of inertia.

The continuous support rests on *n*, eg, *n*=4 middle supports and two end supports over n+1= 5 spans. Use the notation in Fig. 1, where the supports are designated with the consecutive numbers 0, 1, 2,... the field widths and the constant moments of inertia in the individual fields are designated by double numbers. The supporting moments are introduced as statically indeterminate and are designated in sequence: X_1, X_2, X_3...

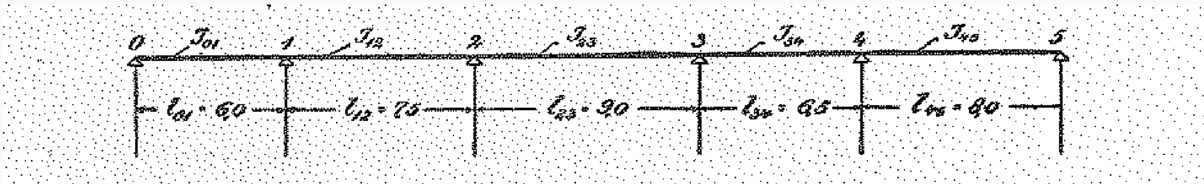


Abb. 1.

In the example in Fig. 1, the moments of inertia are:

$$J_{01} : J_{12} : J_{23} : J_{34} : J_{45} = 1 : 4{,}3 : 7{,}2 : 5{,}0 : 5{,}2 .$$

The elasticity coefficient is constant.

First create the sizes δ (twisting of the end cross sections of each opening, see Fig. 2):

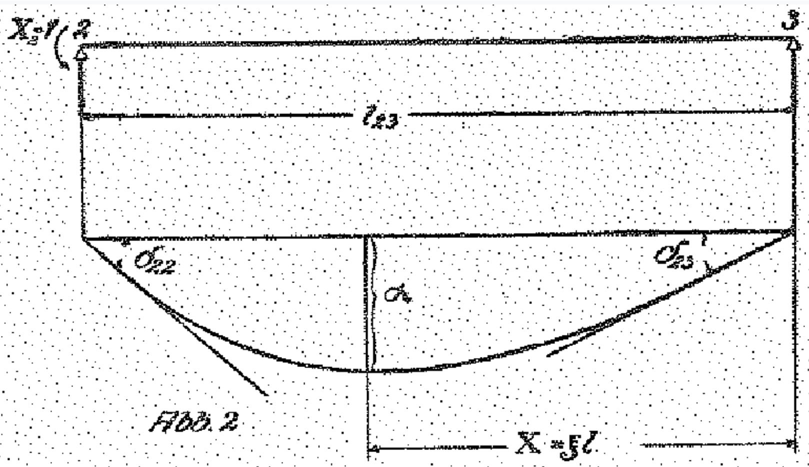


Abb. 2

$$
(1)\qquad
\begin{aligned}
\delta_{11} &= \frac{l}{3E}\left(\frac{l_{01}}{J_{01}} + \frac{l_{12}}{J_{12}}\right) \\
\delta_{12} &= \delta_{21} = \frac{l_{12}}{6EJ_{12}} \\
\delta_{22} &= \frac{l}{3E}\left(\frac{l_{12}}{J_{12}} + \frac{l_{22}}{J_{22}}\right) \\
\delta_{23} &= \delta_{32} = \frac{l_{23}}{6EJ_{23}} \quad \text{usw.}
\end{aligned}
$$

and note that the deflections in a span due to a moment X-1 at the left end cross section of the opening have the value:

$$\delta = \frac{l^2}{6EJ}(\xi - \xi^3) = \frac{l^2}{6EJ}\,\alpha$$

$$\text{wo} \quad \xi = \frac{x}{l}$$

Each of the ordinates of the first of the following curves in Fig. 3, multiplied, is $\frac{l^2}{6EJ}$. The effect of the moment X=1 on the right end cross-section of the opening creates a mirror-image deflection curve $\delta = \frac{l^2}{6EJ}\cdot\beta$, the ordinates of which are the second curve in Fig. 3

Both curves are mirror images of each other. The upper form will now be called the ***α*** form, the lower one the **β** form.

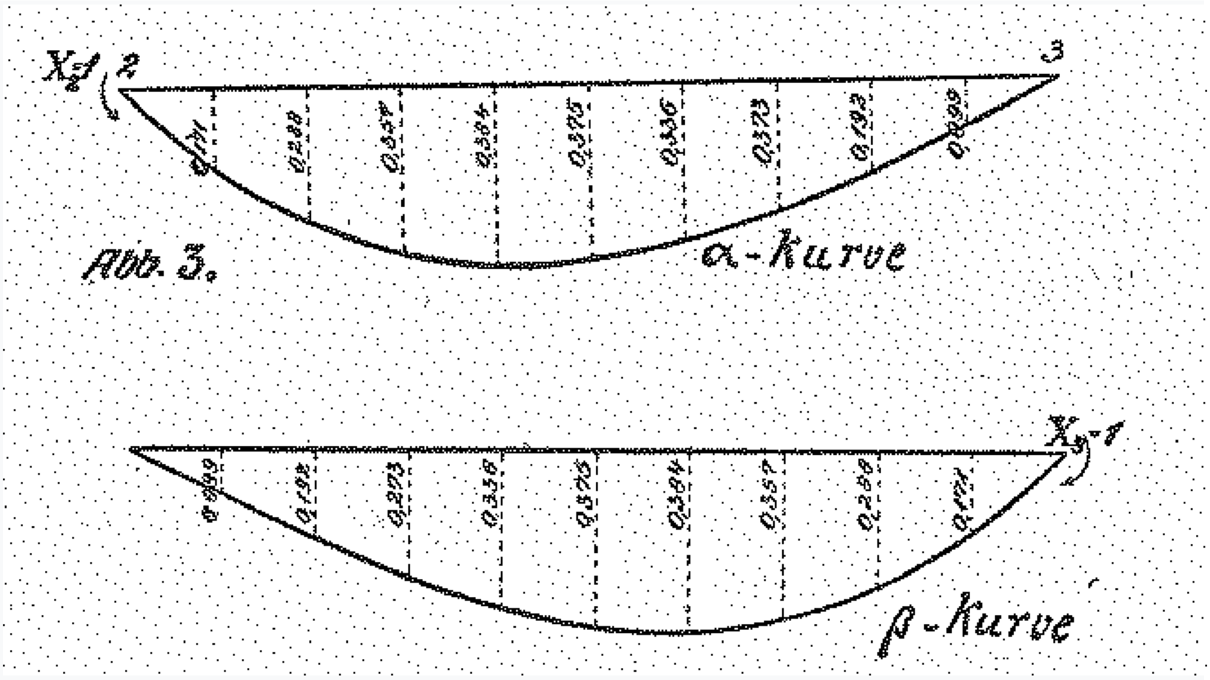


Abb. 3.

The tables at the end represent all curves $\alpha + \frac{\beta}{i}$ where i passes through the values -1<i<+1 .
Same factors in all values δ, eg. E, 3 or 6 and event. J etc. can be shortened. E.g. for the carrier shown in Fig. 1:

$$\begin{aligned}
\delta_{11} &= 2\left(\frac{6{,}0}{1{,}0} + \frac{7{,}5}{4{,}3}\right) = 15{,}48 \\
\delta_{12} &= \frac{7{,}5}{4{,}3} = 1{,}74 \\
\delta_{22} &= 2\left(\frac{7{,}5}{4{,}3} + \frac{9{,}0}{7{,}2}\right) = 5{,}98 \\
\delta_{23} &= \frac{9{,}0}{7{,}2} = 1{,}25 \\
\delta_{33} &= 2\left(\frac{9{,}0}{7{,}2} + \frac{6{,}5}{5{,}0}\right) = 5{,}10 \\
\delta_{34} &= \frac{6{,}5}{5{,}0} = 1{,}30 \\
\delta_{44} &= 2\left(\frac{6{,}5}{5{,}0} + \frac{8{,}0}{5{,}2}\right) = 3{,}08
\end{aligned}$$

Then write the schema

$$
(2)\qquad
\begin{array}{ccccccc}
\delta_{11} & \delta_{12} & 0 & 0 & 0 & 0 & . \\
\delta_{21} & \delta_{22} & \delta_{23} & 0 & 0 & 0 & . \\
0 & \delta_{32} & \delta_{33} & \delta_{34} & 0 & 0 & . \\
0 & 0 & \delta_{43} & \delta_{44} & \delta_{45} & 0 & . \\
0 & 0 & 0 & \delta_{54} & \delta_{55} & \delta_{56} & . \\
. & . & . & . & . & . & . \\
0 & 0 & 0 & 0 & \delta_{n(n-1)} & \delta_{nn} & .
\end{array}
$$

that is the coefficient of the so-called Clapeyron's equations correspond, and write accordingly in the example:

(2 a)

15,48	1,74	0	0
1,74	5,98	1,25	0
0	1,25	5,10	1,30
0	0	1,30	3,08

Starting with the last term (completely elementary), calculate the two following finite continued fractions.

$$
(3)\quad
\left\{
\begin{aligned}
\alpha_{11} &= \cfrac{1}{\delta_{11} - \delta_{12}\cfrac{\delta_{12}}{\delta_{22} - \delta_{23}\cfrac{\delta_{23}}{\delta_{33} - \delta_{34}\cfrac{\delta_{34}}{\delta_{44} - \cdots \cfrac{\cdots\cdots}{-\,\delta_{(n-1)n}\cfrac{\delta_{(n-1)n}}{\delta_{nn}}}}}}} \\
\alpha_{nn} &= \cfrac{1}{\delta_{nn} - \delta_{n(n-1)}\cfrac{\delta_{n(n-1)}}{\delta_{(n-1)(n-1)} - \cdots \cfrac{\cdots\cdots}{-\,\delta_{32}\cfrac{\delta_{32}}{\delta_{22} - \delta_{21}\cfrac{\delta_{21}}{\delta_{11}}}}}}\,.
\end{aligned}
\right.
$$

After each division into a value δ (n-1), n,……. δ_{34}, δ_{23}, δ_{12} in the first case, δ_{21}, δ_{32}, …..$\delta_{n(n-1)}$ in the second case you notice the quotient, which represents the position of the so-called. fixed point J or K within an aperture. Above an unloaded apperture, the connecting line of the supporting moments runs as a straight line, which passes through the left fixed point K when the load opening follows the right, and through the right fixed point J when the load opening moving forward. The ratio of two consecutive supporting moments X_3/X_4

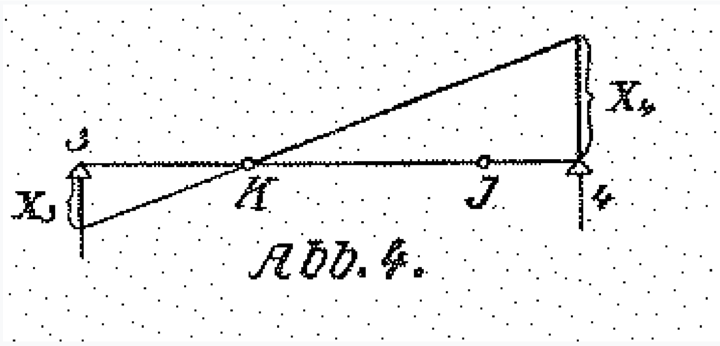


Abb. 4.

Fig.4.

or the segments 3-J/J-4 give the chain break.

$$\frac{1}{i_{34}} = -\cfrac{\delta_{34}}{\delta_{44} - \delta_{45}\cfrac{\delta_{45}}{\delta_{55} - \delta_{56}\cfrac{\delta_{56}}{\delta_{66} - \cdots \cfrac{\cdots\cdots}{- \delta_{(n-1)\,n}\cfrac{\delta_{(n-1)\,n}}{\delta_{nn}}}}}}$$

Note the minus sign on the 1/I values. Now create the number scheme

$$(4)\qquad \begin{array}{c|ccccccc|c}
 & a_{11} & a_{12} & a_{13} & \cdot & \cdot & \cdot & a_{1n} & \\
\frac{1}{i_{12}} & & & & & & & & \frac{1}{k_{21}} \\
 & a_{21} & a_{22} & a_{23} & \cdot & \cdot & \cdot & a_{2n} & \\
\frac{1}{i_{23}} & & & & & & & & \frac{1}{k_{32}} \\
 & a_{31} & a_{32} & a_{33} & \cdot & \cdot & \cdot & a_{3n} & \\
\cdot & & & & & & & & \cdot \\
 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \\
\cdot & & & & & & & & \cdot \\
 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \\
\frac{1}{i_{(n-1)\,n}} & & & & & & & & \frac{1}{k_{n\,(n-1)}} \\
 & a_{n1} & a_{n2} & a_{n3} & \cdot & \cdot & \cdot & a_{nn} &
\end{array}$$

where $a_{31}=a_{13}$ by first writing a_{11} and a_{nn} with the values resulting from the continuous fractions and then writing the values 1/k on the left (or top) half the height of a line (between the columns) and the same 1/k on the right (or bottom). Example:

(3 a)

$$a_{11} = \cfrac{1}{15{,}48 - 1{,}74\cfrac{1{,}74}{5{,}98 - 1{,}25\cfrac{1{,}25}{5{,}10 - 1{,}30\cfrac{1{,}30}{3{,}08}}}}$$

man erhält nacheinander:

$$\frac{1}{i} = -0{,}421,\ -0{,}275,\ -0{,}309 \text{ und } a_{11} = 0{,}067$$

$$a_{44} = \cfrac{1}{3{,}08 - 1{,}30\cfrac{1{,}30}{5{,}10 - 1{,}25\cfrac{1{,}25}{5{,}98 - 1{,}74\cfrac{1{,}74}{15{,}48}}}}$$

$$\frac{1}{k} = -0{,}112,\ -0{,}216,\ -0{,}269 \text{ und}$$

$$a_{44} = 0{,}366$$

(4 a) Zahlenrechteck der Einflußlinien.

	0	1	2	3	4	5
X_1		0,067				
	−0,309					−0,112
X_2						
	−0,275					−0,216
X_3						
	−0,421					−0,269
X_4					0,366	

a) Anschreiben der aus den Kettenbrüchen folgenden Zahlen.

	0	1	2	3	4	5
		−0,309	−0,275	−0,421		
X_1		**0,067**	−0,0207	0,0057	−0,0024	
	−0,309					−0,112
X_2		−0,0207	**0,186**	−0,0512	0,0213	
	−0,275					−0,216
X_3		0,0057	−0,0512	**0,237**	−0,0986	
	−0,421					−0,269
X_4		−0,0024	0,0213	−0,0986	**0,366**	
		−0,112	−0,216	−0,269		

b) fertiges Zahlenrechteck.

you get one after the other:

-0.421,0.275, -0.309 and -0.067

(3a)

1

1.30

1.25

3.08-1.30

5.10-1.25

1.74

5.98-1.74

15.48

-0.112,0.216,-0.269 and

"=0.366

(4a)

Number rectangle of influence lines.

0 1 2 3 4 5

0 1 2 3 4 5

-0.309-0.275-0.421

X_1 0.067 -0.309-

X_1 0.067 -0.0207 0.0057-0.0024

-0.112

-0.309- -0.0207 0.186

-0.112 -0.0512 0.0213

X -0.276

-0.275- ,

--0.216

-0.216

X -0.421-

X, 0.0057-0.0512

0.237-0.0986

-0.260 0.366

-0.421 -0.269 -0.0024 0.0213-0.0956 0.366

a) Write down the numbers following the continued fractions.

-0.112-0.216-0.269

b) finished number rectangle.

WORK IN PROGRESS (couple more pages from end of doc below)

Last two pages..
B. Continuous beams with any support connection, freely supported (pinned), rigidly connected (fixed), clamped at the ends, supports clamped at the bottom or articulated

The moments of inertia are:

Die Trägheitsmomente seien $J_{01} = 0{,}25$ m^4, $J_{12} = 0{,}325$ m^4, $J_{23} = 0{,}35$ m, $J_{34} = 0{,}3$ m. Dann ist $J_c = \frac{J_{01}}{l_{01}} = \frac{0{,}05 \cdot 3{,}5}{0{,}035} = 5{,}0$. Ist die Höhe $h = 3{,}5$ m und $J_h = 0{,}035$, so wird $K = \frac{0{,}05 \cdot 3{,}5}{0{,}035} = 5{,}0$.

2. Determine the continuous load po1, p12 for each opening. The influence of these loads on a statically indeterminate object is obtained by adding the horizontal loads belonging to the relevant opening in the horizontal position that applies to the statically indeterminate entity in question.

row of standing numbers and multiplication by
(bewirken ein = caused by, Zahlenrechteck is Matrix

reihe stehenden Zahlen und Multiplikation mit $\frac{1}{4} p\, l^2$. Z. B. $p_{01} = 2300$, $p_{12} = 2000$ kg/m bewirken ein (s. S. 39 Zahlenrechteck für $K = 5$):

$$Y_1 = -0{,}25\{2300(-0{,}021 + 0{,}252) \cdot 5{,}0^2 + 2000(0{,}301 - 0{,}087) \cdot 6{,}5^2\} =$$
$$= -0{,}25\{13\,300 + 27\,900\} = -10\,400 \text{ mkg.}$$

3. For individual loads, proceed as above under a, paragraph 4.

For reinforced concrete beams, the calculation of the value of K is uncertain if the upper continuous beam is a T-beam. The question then arises as to whether the plate, e.g. in false ceilings has a very large width, which must be fully taken into account when calculating the moment of inertia. If the plate is very thin compared to the web, this question should be answered in the negative. Otherwise, you can forego the calculation as a floor frame,

since the ratio K assumes such large values that the numbers $\frac{1}{i_1}, \frac{1}{i_2'}$ can practically be set equal to 1 and the image of a freely supported, running support is created (cf. the following number rectangles for K=00 on page 43. 1

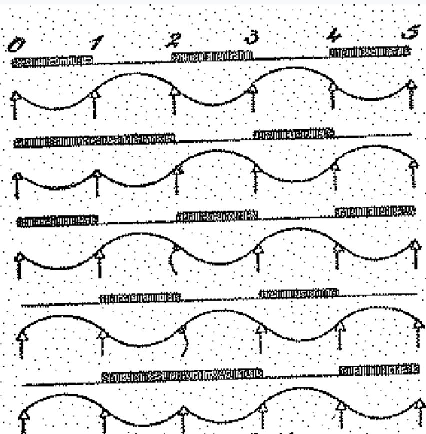
The assumption that the moments of inertia in the individual openings are proportional to the spans corresponds better to the practical implementation than the usual assumption of a constant moment of inertia everywhere. In reality, one can assume that the moments of inertia are proportional to the square of the span. The above assumption is therefore a better approximation than the usual one of constant moment of inertia everywhere.
Particularly in the case of reinforced concrete beams, the moments of inertia to be introduced into the calculation are so uncertain that one can safely use the numbers

in the schemes that quickly lead to the goal. When calculating the value K, one must be careful not to introduce a value that is too small into the calculation, because with smaller values of K the supports absorb larger proportions of the bending moments. It would therefore be a good idea to use the ratio that applies to the largest span when calculating K=.

für die Berechnung von $K = \frac{J_l \cdot h}{l \cdot J_h}$ das für die

hältnis $\frac{J_l}{l}$ der Rechnung zugrunde zu legen.

to be used as a basis for the design.
If the support is under lower restraint, the factor 1.5 must be taken into account instead of 2.0 when calculating the values do, d, etc. (see page 26). You can therefore do this even when the support is clamped at the bottom
Use the following schemes if you reduce K in the ratio 1.5/2.0 = 3/4 or approximately as the height of the support only/the one that is actually present accepts.
Regarding the distribution of the live load, it should be noted that it must be assumed that it is distributed in such a way that the greatest twisting is achieved in the case of bending moments on the cross section examined, and the greatest vertical displacement is achieved in the case of transverse and support forces. Figure 14 below shows the correct distribution at various maximum and minimum moments and forces.



Maximalmoment in 01,23 oder 45.

Maximales Stützenmoment X_1.

Rechtsdrehendes Moment für Stütze 2.

Linksdrehendes Moment für Stütze 2.

Maximales Stützmoment X_2.

Abb. 14.

Maximum torque in 01,23 or 45.
Maximum support moment X.
Clockwise rotating moment for support 2.
Counterclockwise rotating moment for support 2.
Maximum supporting moment X.
Fig.14.
If you use J_l/l to denote the stiffness of a bar, the value K represents nothing other than the ratio of the stiffness of the longitudinal beam to that of the supports K= J_l/l: J_l/h. With supports you can use proportionality between heights and moments of inertia, even more so than with beams assume because there are fewer bending moments than column pressures that determine the cross section. You can therefore

approximately use the following tables for multi-legged frames with any height of the supports, if approximately J_l/l and J_h/h is constant everywhere, where h can only be used with ¾ in the case of lower restraint.

Because the support moments usually exceed the field moments, the connection to the support is effected by nodes (sheet metal girders) or haunches (reinforced concrete frames), which means a deviation from the requirement of the moment of inertia being constant everywhere in every opening. This amplification can be approximately taken into account by forming the a, i, k or Here the K-Werke assumes the spans are reduced by the haunch reinforcement. Because at the ends of the haunch or of the node, the condition of equality of the 3 elastic angles of rotation (see page 16) will also be fulfilled. When calculating the moments, you of course have the full, or 13 to introduce.