

Truth-tellers and liars in the Legion of Janus

There can be at most one truth-teller in each row, because if there were more, they wouldn't all be telling the truth. So there can be at most five truth-tellers.

If a liar makes such a statement, then there must be at least one truth-teller in their row or column. Thinking this through, we also see that every row must contain a truth-teller. If there were a row of only liars, then at the intersection of that row and a column that also contained only liars, we would have a soldier who is a liar and could not have made the given statement (because it would have been true).

There must be at least one such "all-liar" column, since there are 10 columns and at most 5 truth-tellers.

Therefore, the number of truth-tellers must be exactly five — no more, no less. We can even construct an example, marking the truth-tellers in orange (the image shows 10 rows by 5 columns of blue dots, where the dot in the 1st row of the 1st column is orange, the dot in the 2nd row of the 2nd column is orange, and so on up to the last orange dot on the 5th dot of the 5th column). We leave it to the reader to verify that in this configuration, every soldier could indeed have made the statement given in the problem.

