Lab 8: AC Measurement of Magnetic Susceptibility

Jason Stein with collaboration with Miranda Etchison
TA: Han Chong
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Data collected on pages 11-14 of Even Lab Notebook

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Abstract

Discovered over 2000 years ago, iron oxides were first found to exhibit ferromagnetic properties. These magnets are still currently being experimented on in order to understand why they contain magnetic properties. Inside of all metals are electrons that have a particular amount and direction of spin. When the spins of the electrons are able to conform to each other in a spin up state, the material is able to sustain magnetization. It is determined through experimentation that materials have specific constants regarding their abilities to sustain magnetization. It is also known that these constants are only constants in nature when the temperature of the material is not changing. This phenomenon is known as the Curie-Weiss Law, and in this experiment it will be proven for a torroid exhibiting a magnetic field.

1. Purpose

The purpose of this experiment was to study the equations that govern the magnetic fields of ferromagnets. A simple ferromagnet that can be easily manipulated for experimentation is a torroid. By applying an external DC field to various torroids, as well as a small superimposed AC excitation, we were able to measure the magnetic fields as a function of time. Relating the functions about torroids and their magnetic fields, we were able to compare these measurements to the susceptibility of each torroid. Using the resulting data it was also possible to solve for the energy losses in each experiment. In a separate exercise we sought out to prove the Curie-Weiss Law for a torroid. To do this, we placed the torroid into a heater and ran the same experiment as before. By analyzing the resulting relationship between the temperature and the magnetic susceptibility it would be possible to find the critical temperature at which the susceptibility dropped to zero. We would be able to show this for both a heating up curve and for a cooling down curve. Lastly, we could repeat the first exercise except we could change temperature of the solenoid. By analyzing the results, a relationship between the magnetic susceptibility and temperature could be recovered.

2. Theory

In order to generate a magnetic field in a torroid a current must flow through the wire that wraps around the torroid. In the first experiment a small AC excitation is introduced with a field equal to,

$$H = H_0 + H_1 cos\omega t \tag{1}$$

The H_0 terms is produced by the DC current, while the H_1 term is produced by the AC current. These values are separated by a phase shift of $\frac{\pi}{2}$.

From combined Maxwell's equations for auxiliary fields we know that,

$$\vec{B} = \mu_0(\vec{H} + \vec{M}(\vec{H})) \tag{2}$$

Knowing that $M = \mathrm{X}H$, and that $\mu_r = 1 + \mathrm{X}$, we can rewrite equation 2 as,

$\vec{B} = \mu_0 \mu_r \vec{H}$ (3)

In exercise 1 we will be measuring values of $V_{lock-in}$ and I_{DC}, so for the time being, it is best to write equation 3 for μ_r as a function of the derivative of $\vec{B}$ with respect to $\vec{H}$.

$\mu_r = \frac{1}{\mu_0}\frac{d\vec{B}}{d\vec{H}}$ (4)

Since $\mu = \mu_r \mu_0$,

$\mu = \frac{d\vec{B}}{d\vec{H}}$

(5)

Using each expression for μ and X, we also know that,

X= $\frac{\mu}{\mu_0} - 1 = \frac{\frac{d\vec{B}}{d\vec{H}} - \mu_0}{\mu_0}$

(6)

To compare equation 4 to the measured quantities in the experiment, we had to examine figure 3 which showed how the circuit acted. From the measured quantity of $V_{lock-in}$, we know that,

$V_{lock-in} = - \frac{d\phi}{dt},\ where\ \phi = \vec{B}\cdot\vec{S}$ (7)

By replacing $\vec{B}$ with $\vec{H}$, we see that,

$d\phi = \mu \int \vec{H}\cdot d\vec{a}$ (8)

For a torroid,

$\vec{H}_0 = \frac{N_{p,DC} I_{DC}}{2\pi r}$, where $N_{p,DC}$ is the number of turns in the DC primary coil (9)

$\vec{H}_1 = \frac{N_{p,AC} I_p}{2\pi r}$, where $N_{p,AC}$ is the number of turns in the AC primary coil (10)

We know that $\vec{H}_0 \gg \vec{H}_1$, so

$\vec{H} \approx \frac{N_{p,DC} I_{DC}}{2\pi r}$ (11)

Plugging equation 6 into equation 6 and integrating for outer radius R_2,inner radius R_1, and height t,

$d\phi = \frac{\mu N_{p,AC} I_{AC} t}{2\pi} ln(\frac{R_2}{R_1})$ (11)

The total flux of the torroid is then the integration of equation 8 for the number of loops of the pickup coil, N_{pickup},

$\phi = \frac{\mu N_{pickup} N_{p,AC} I_p t}{2\pi} ln(\frac{R_2}{R_1})$ (12)

The inductance of the torroid is given by $L = \frac{\phi}{I}$, so for simplicity since this value is constant for each torroid,

$L_0 = \frac{\mu_0 N_{pickup} N_{p,AC} t}{2\pi} ln(\frac{R_2}{R_1})$ (13)

We can then plug equation 9 and 10 back into equation 5, and knowing that only I_{AC} depends on t,

$$V_{lock-in} = \mu_0 L_0 \frac{dI_{AC}}{dt} \quad (14)$$

Since the result we want is μ vs.$\vec{H}$, we could write equation 11 in terms of μ,

$$\mu = \frac{\mu_0 V_{lock-in}}{L_0 \frac{dI_p}{dt}} \quad (15)$$

This is the equation needed to transform the measured data of $V_{lock-in}$, $I_{DC}\ vs.\ t$ into μ for the y-axis. To transform the x-axis we just use equation 7 where $I_{DC} = I_p$ so that,

$$\vec{H} = \frac{N_p I_{DC}}{2\pi r} \quad (16)$$

To transform the y-axis the AC current needs to be taken into account

$$I_{AC} = \frac{V_{AC} sin(\omega t)}{R_{AC}}$$ where R_{AC} is the resistance of the primary coil (17)

We can use equation 14 in equation 12 and take the derivative, and the result is our expression for the y-axis,

$$\mu = \frac{\mu_0 V_{lock-in} R_{AC} t}{L_0 \omega V_{AC} cos(\omega t)} \quad (18)$$

Also, since the current is measured throughout one period, $cos(\omega t)\approx 1$, and also we know that $\omega = 2\pi f$ so,

$$\mu = \frac{\mu_0 V_{lock-in} R_{AC} h}{L_0 2\pi f V_{AC}} \quad (19)$$

An important quality that is derived from the hysteresis loops is the power lost per cycle for the solenoid. It is given by the following equation,

$$W_{loop} = V \oint HdB = V_{Solenoid} * loop\ area,$$ where $V_{Solenoid}$ is the volume of the magnetic material (20)

We know from equation 5 that by graphing equation 16 by equation 18 and by integrating for all H we find the graph of B and H. This relationship of H and B is also important to finding the power loss per cycle as given by equation 20. Substituting equations together, we see that,

$$\int \mu H dH = \frac{W_{loop}}{V_{Solenoid}} \quad (21)$$

We can simplify this expression in terms of Χ by substituting in equation 6. The result is,

$$\int \mu_o \chi H dH = \frac{W_{loop}}{V_{Solenoid}} \quad (22)$$

$$2\pi * \frac{1}{2}\mu_0 \chi H^2 = \frac{W_{loop}}{V_{Solenoid}} \quad (23)$$

If we want a relationship between Χ" and H we needed to use the correct values for this phase. The field H that is used in this calculation is H=$H_0 sin(\omega t)$, so 2π is introduced since we are calculating losses per cycle. The final simplified relation is,

$$\pi\mu_0 \chi'' H_0^2 = \frac{W_{loop}}{V_{Solenoid}} \quad (24)$$

Rearranging for the expression in terms of power losses per cycle per volume of the solenoid,

$$W_{loop} = \pi\mu_0 X'' H_0^{\ 2} = \frac{P_c}{f}$$ (25

$X' = \frac{C}{T-T_c}$, where C is the Curie constant and T_c is the Curie temperature (26)

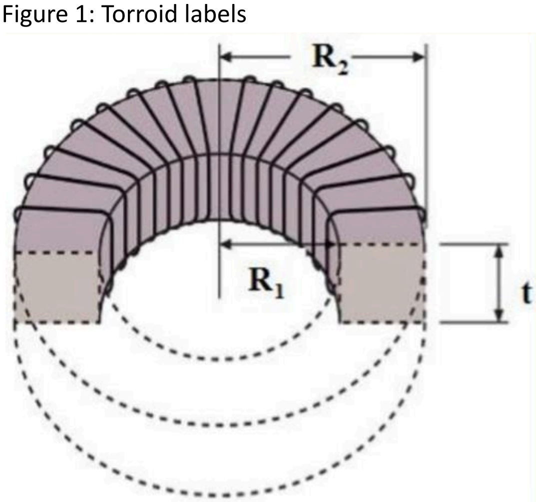
3. Experiment

3.1 Equipment

The setup for this experiment is dependent on each individual torroid. For exercise 1 we used 4 torroids. Below is a table of then quantities of each torroid.

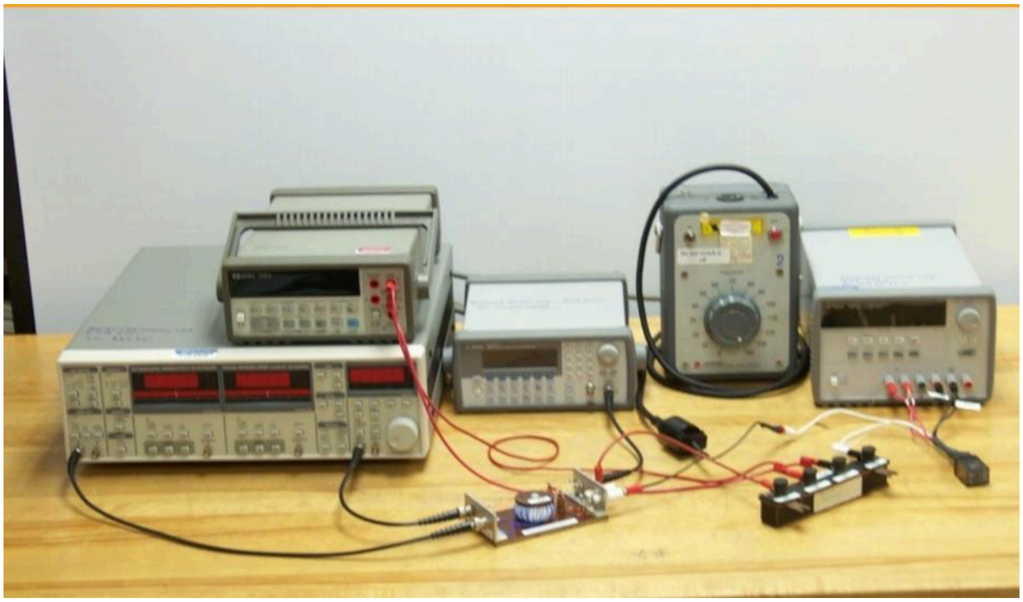
Torroid	Material	$N_{p, DC}$	$N_{p, AC}$	N_{pickup}	R_1 (m)	R_2(m)	t(m)
#7	Magnetics ZP44715TC	15	15	10	.027	.046	.015
#24	Ferroxcube T74/39/13-3C81	130	70	60	.0388	.0737	.0128
#26	Ferroxcube 4C65	30	10	10	.01340	.02242	.00672
#4	N/A	92	20	20	.0318	.0494	.0064

To understand the origin of the measurements, an image of a generic torroid is shown below.
Figure 1: Torroid labels



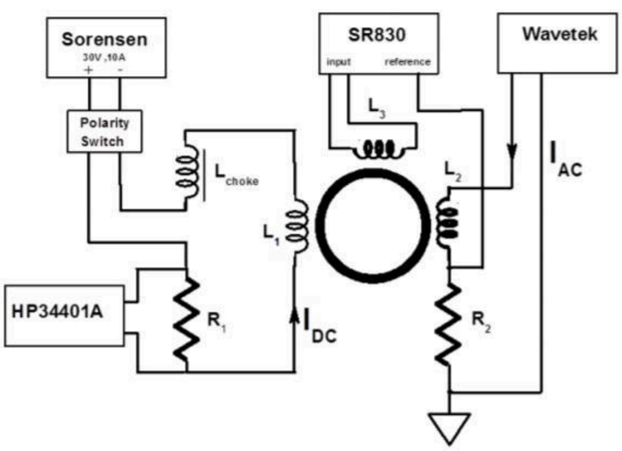
Along with each torroid, we used a WaveTek, WSMT3 Autotransformer, Regulated Power Supply, 34401A Digital Multimeter, and Model SR950P5P Lock-In Amplifier. These were connected in the arrangement shown in figure 2 below.

Figure 2: Equipment used for all exercises



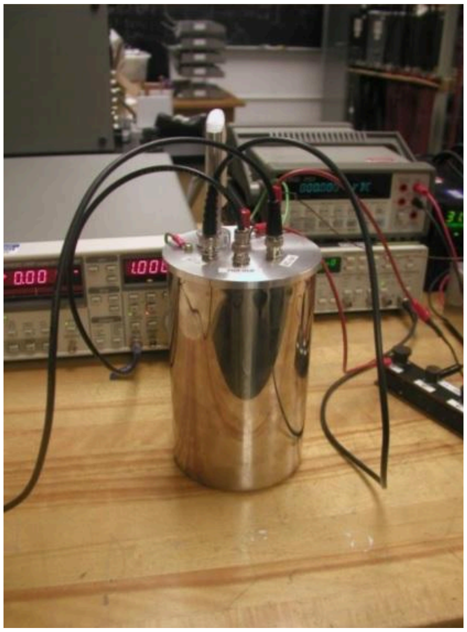
For the analysis in each exercise, the way that this equipment was connected is crucial to relating all measurements. A schematic of the total circuit is shown below.

Figure 3: Schematic of the circuit and each loop



In exercise 2, we used a heater in order to regulate the temperature of the torroids. The heater we used is shown in figure 4.

Figure 4: Heater used to regulate torroid temperatures

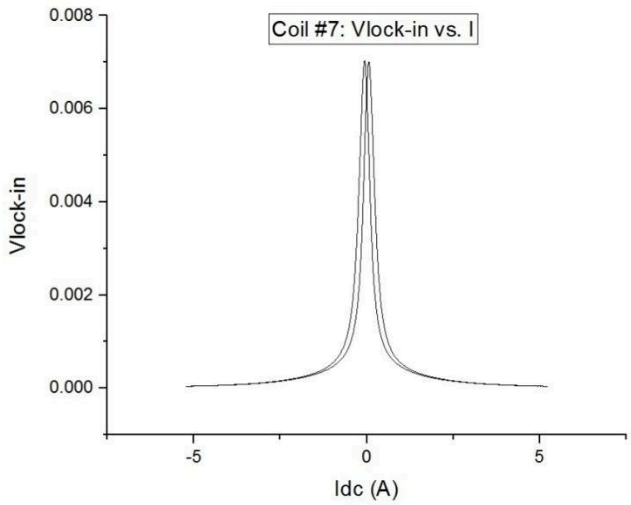


In all exercises we used the program MagneticLab to take measurements, and to analyze the data we used OriginPro.

3.2 Exercise 1: Magnetic Susceptibility and Energy Losses of Torroids

For this first exercise we sought to compare the magnetic susceptibility μ vs. magnetic field $\vec{H}$. We connected each of the four torroids in the arrangement shown in figure 3. Using the program MagneticLab we were able choose increments of $\vec{H}$ and t for various step sizes. These step sizes were dependent on a theoretical value of current that resulted in the maximum values of the magnetic susceptibility. Once acquiring this data for each torroid, we uploaded all of the data into OriginPro. The data stored included the time measurements, $V_{lock-in}$for each phase, I_{DC}, V_{AC}, $and\ f$. We also know that the value for $R_{AC} = 5000\Omega$. Below is an example graph of the raw data of $V_{lock-in}$ and I_{DC} for torroid #7.

Graph 1: $V_{lock-in}$ vs. I_{DC} for torroid #7 Magnetics ZP44715TC

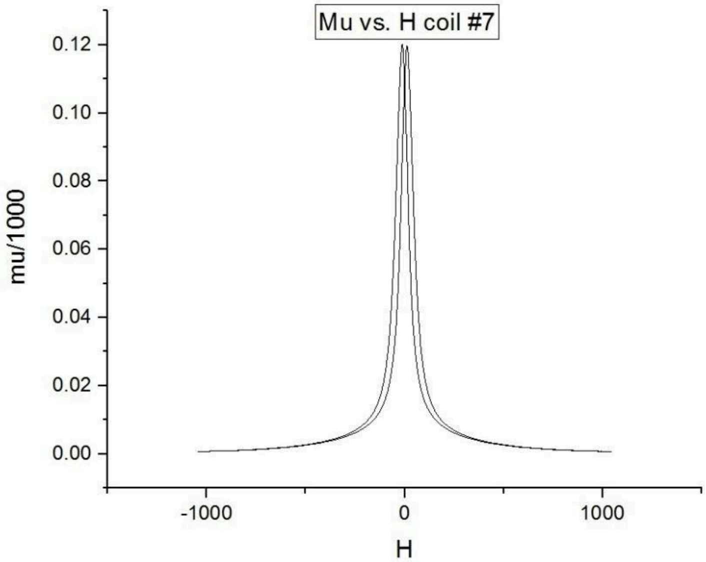


In order to fully understand the relationship of the magnetic field for each torroid, it was necessary to transform the x and y-axes. The first step in doing this was to find the value of L_0 for each torroid using equation 12. Below is a table showing this value for each case.

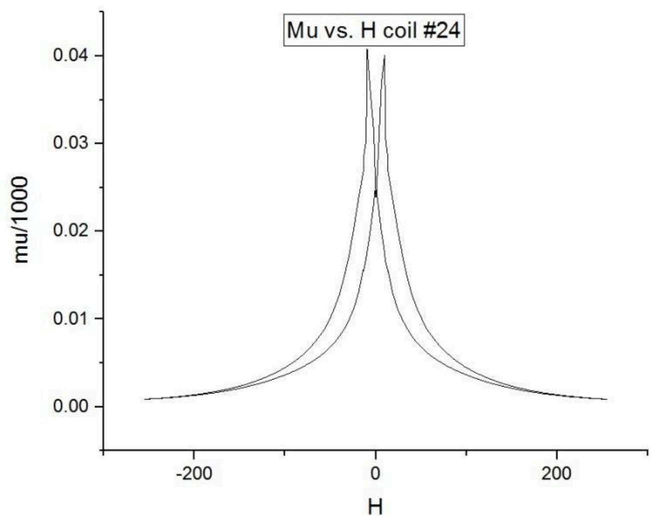
Torroid	$L_0 = \frac{\mu_0 N_{pickup} N_{p,AC}\, t}{2\pi} ln(\frac{R_2}{R_1})$
#7 Magnetics ZP44715TC	2.48E-7H
#24 Ferroxcube T74/39/13-3C81	6.90E-6H
#26 Ferroxcube 4C65	6.92E-8H
#4	2.26E-7H

Next, we could use equation 10 to transform the x-axis, and equation 18 to transform the y-axis. The result is a graph of μ vs. $\vec{H}$

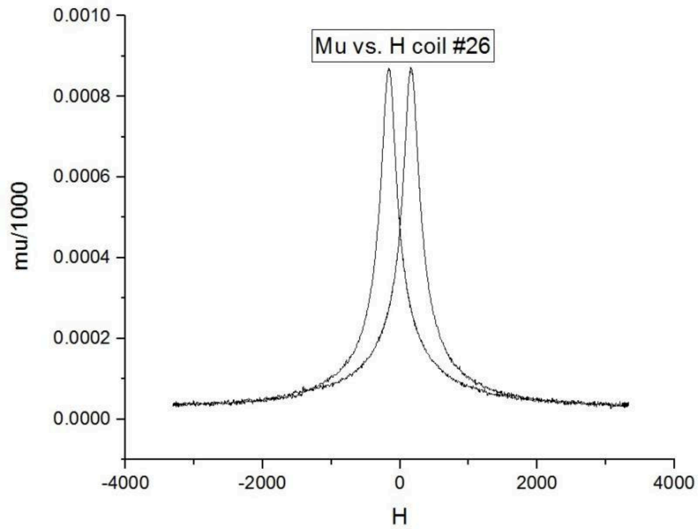
Graph 2: μ $vs.$ $\vec{H}$ for torroid #7 Magnetics ZP44715TC



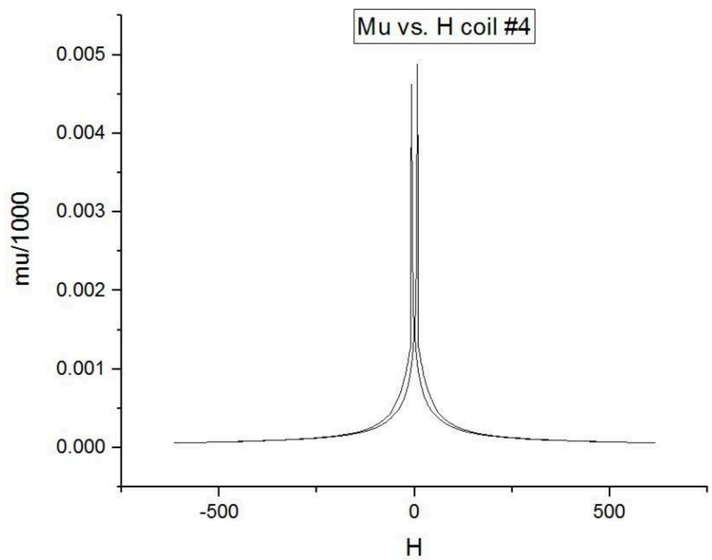
Graph 3: μ $vs.$ $\vec{H}$ for torroid #24 Ferroxcube T74/39/13-3C81



Graph 4: μ $vs.$ $\vec{H}$ for torroid #26 Ferroxcube 4C65

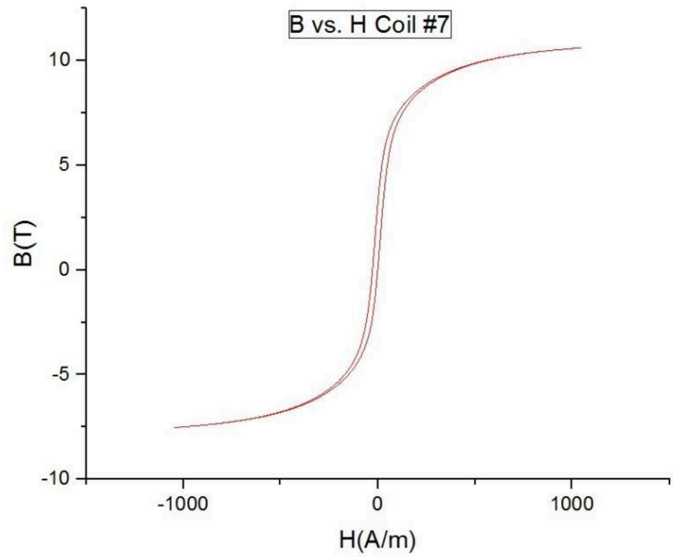


Graph 5: μ $vs.$ $\vec{H}$ for torroid #4

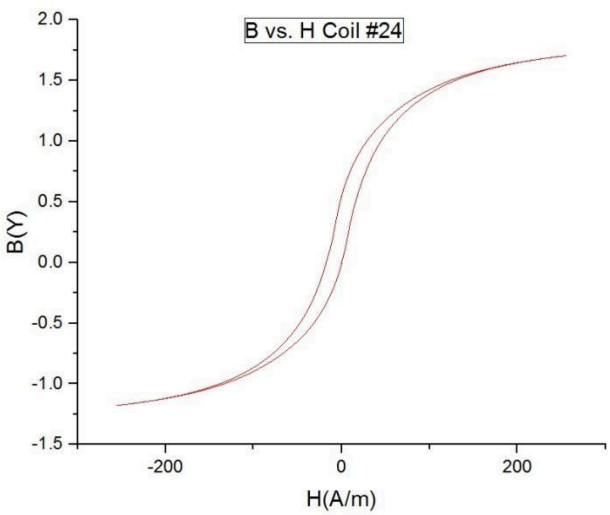


Using equation 5, by integrating graph's 2-5, we can then see the graphs of $\vec{B}$ $vs.$ $\vec{H}$ for each torroid.

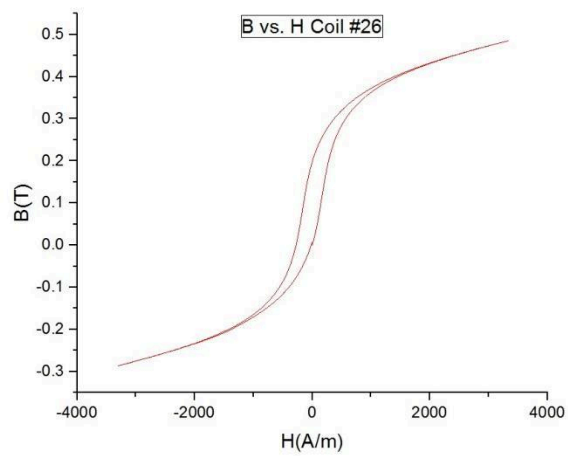
Graph 6: $\vec{B}$ $vs.$ $\vec{H}$ for torroid #7 Magnetics ZP44715TC



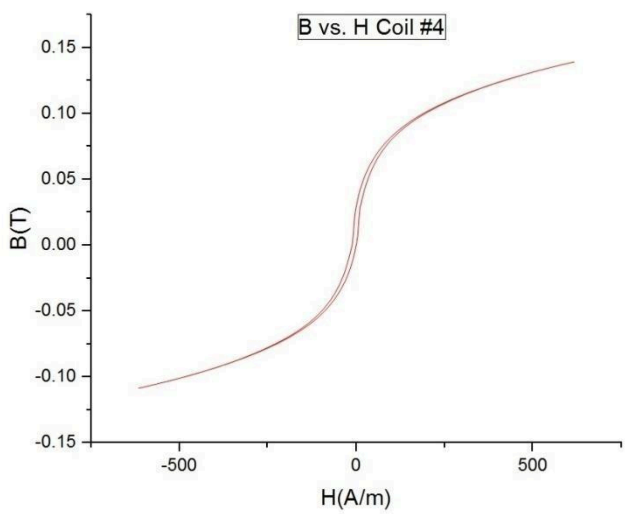
Graph 7: $\vec{B}$ $vs.$ $\vec{H}$ for torroid #24 Ferroxcube T74/39/13-3C81



Graph 8: $\vec{B}$ $vs.$ $\vec{H}$ for torroid #26 Ferroxcube 4C65



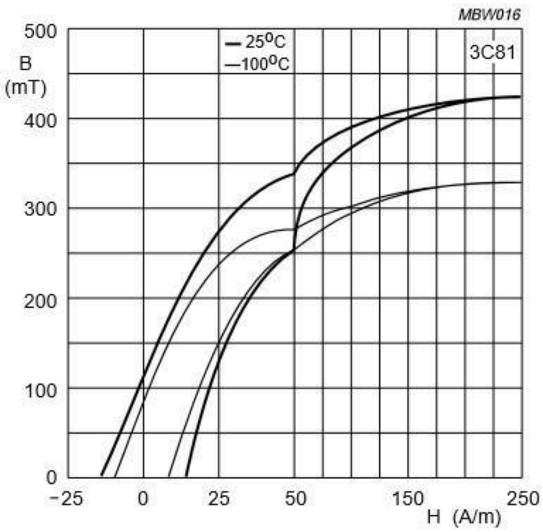
Graph 9: $\vec{B}\ vs.\ \vec{H}$ for torroid #4



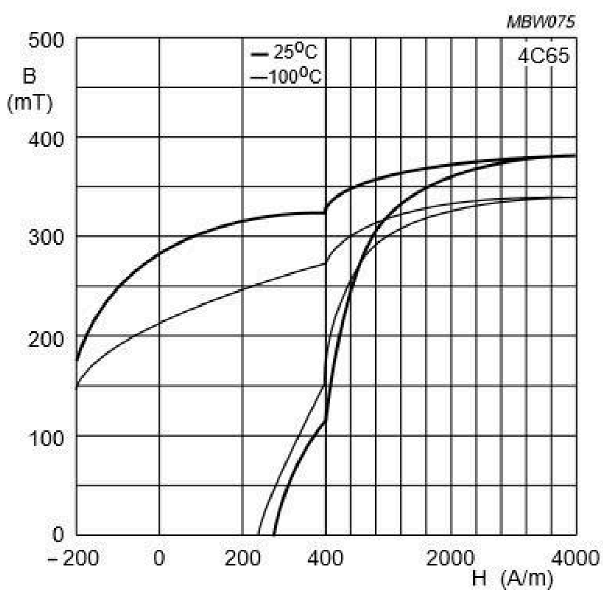
The following graphs are the torroid B vs. H curves that were available to us online.

We were able to find the BH curves for both Ferroxcube T74/39/13-3C81 and Ferroxcube 4C65.

$Graph\ 10$: $Actual\ \vec{B}\ vs.\ \vec{H}$ for torroid #24 Ferroxcube T74/39/13-3C81



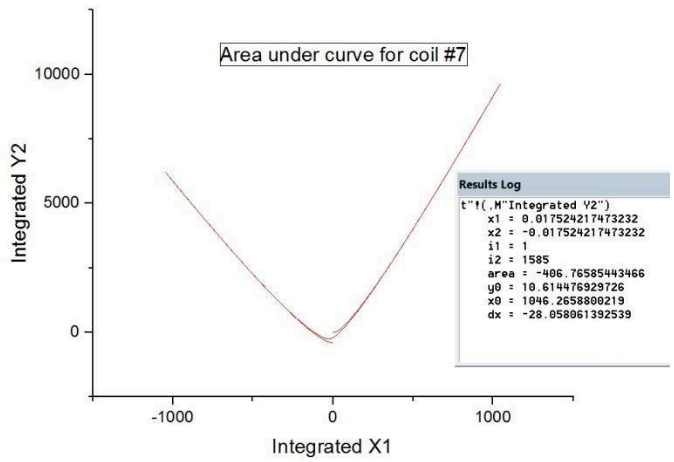
Graph 11: *Actual* $\vec{B}$ *vs.* $\vec{H}$ for torroid #26 Ferroxcube 4C65



Now that we have graphs that illustrate the relationships between the magnetic fields for each torroid, we could use equation 19 to solve for the energy losses. In this equation V is the volume of the torroid, which is given by Volume=$\frac{1}{4}\pi^2(R_1 + R_2)(R_2 - R_1)^2$

To solve for the loop area we used the analysis tool in Origin and integrated graphs 6-9. Below is a graph showing an example of the output given for loop #7

Graph 12: Integration of B vs. H for loop #7 Magnetics ZP44715TC



We repeated this process for the rest of the curves and created a table below with the volume of each torroid as well as the loop areas and power losses

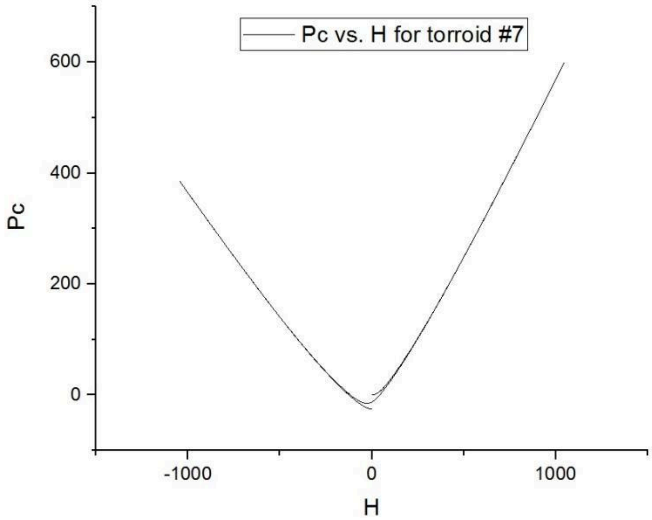
Torroid	Volume	Loop-Area	W=Vol*Loop-Area
#7 Magnetics ZP44715TC	7.22E-5m^3	$407\frac{kg}{ms^2}$	.029J
#24 Ferroxcube T74/39/13-3C81	3.38E-4m^3	$38.2\frac{kg}{ms^2}$	.013J
#26 Ferroxcube 4C65	7.19E-6m^3	$128\frac{kg}{ms^2}$	9.2E-4J
#4	6.21E-5m^3	$1.91\frac{kg}{ms^2}$	1.19E-4J

A very useful quantity in comparisons between the torroids is the power dissipated per cycle P_c. The quantity is derived by taking the energy loss and multiplying it by the frequency f. Below is a table of those values.

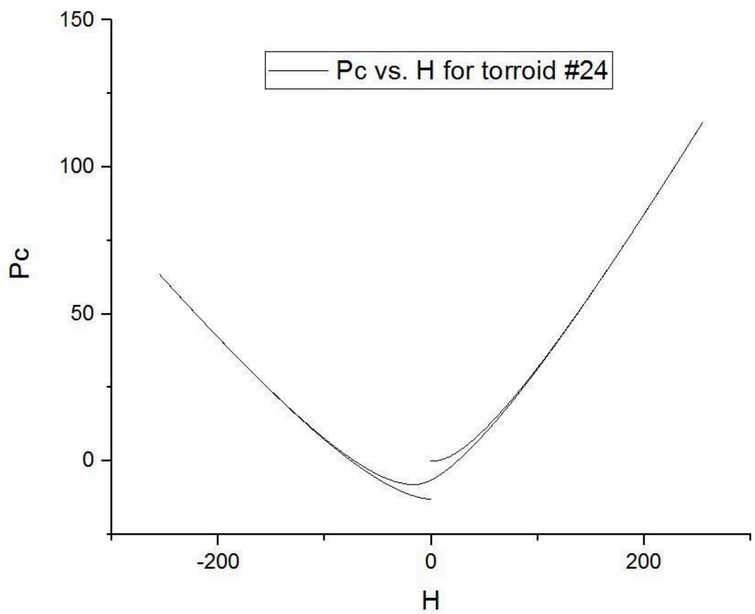
Torroid	$Total\ P_c = W * f$=W*1000Hz
#7 Magnetics ZP44715TC	29W
#24 Ferroxcube T74/39/13-3C81	13W
#26 Ferroxcube 4C65	.92W
#4	.12W

Graph 12 is a graph of the energy losses as a function of the H field. Below are these graphs for each torroid. We multiplied the y-axis of each graph by the solenoid area and frequency in order to transform the axis to P_c values.

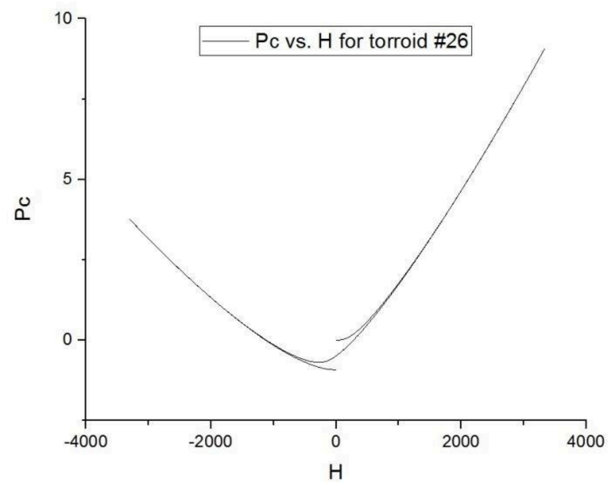
Graph 13: $P_c\ vs.\ H$ for torroid #7 Magnetics ZP44715TC



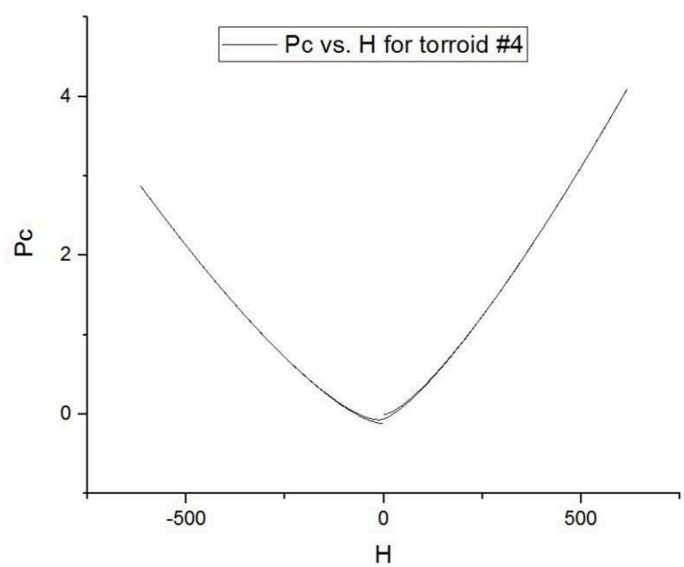
Graph 14: $P_c\ vs. H$ for torroid #24 Ferroxcube T74/39/13-3C81



Graph 15: $P_c\ vs. H$ for torroid #26 Ferroxcube 4C65



Graph 16: P_c $vs.$ H for torroid #4



3.3 Exercise 2: Temperature Dependence of the Magnetic Susceptibility

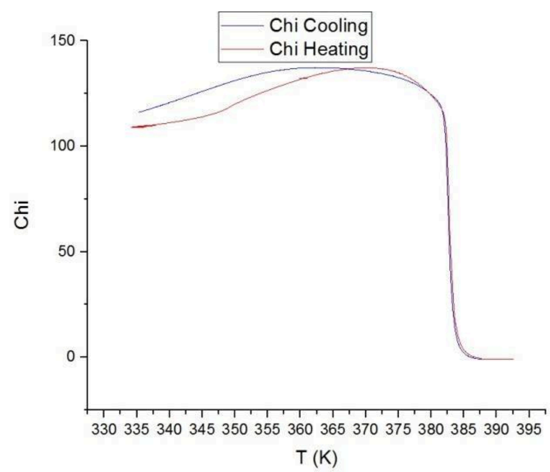
For this exercise we sought out to prove the Curie-Weiss Law. This law states that at a critical temperature, a material loses its ferromagnetic properties. To prove that this occurs, we connected a torroid in the same arrangement as before. This time, however, we used a heater in order to raise the temperature of the torroid. In the MagneticLab software there was a second setting that was able to directly measure the values of the $V_{lock-in}$ and Temperature. From before, we could use equation 18 in order to solve for the magnetic susceptibility of the torroid. In this exercise we will be using a new torroid. This one is a Ferroxcube 3E8.

Torroid	Material	$N_{p, DC}$	$N_{p, AC}$	N_{pickup}	R_1 (m)	R_2(m)	t(m)
Exercise 2 torroid	Ferroxcube 3E8	25	20	20	.01345	.02235	.00825

Using equation 12 the value for $L_0 = \frac{\mu_0 N_{pickup} N_{p,AC} t}{2\pi} ln(\frac{R_2}{R_1})$=3.35E-7H

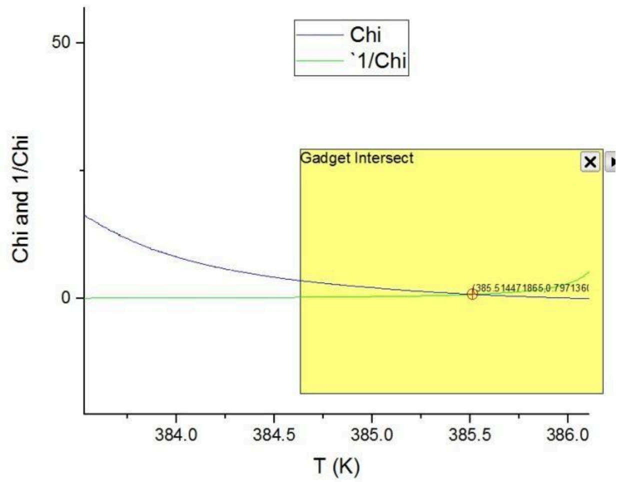
Next, we used equation 18 and 8 to plot a graph of the T values and X values.

Graph 17: X vs. T for Ferroxcube 3E8



Since we were not able to stir the vegetable oil as it was heated up, this heating process was not uniform. For this reason, the data curve collected by cooling the sample is preferred for analysis. According to Curie's law, we know that the susceptibility of a material goes to 0 at a particular temperature known as the Curie temperature. This is governed by equation 21, and it is visualized by the x intercept of graph 11. We graphed the cooling curve X and the inverse of X versus T and then solved for the intersection of the two curves. This intersection is our Curie temperature and this is known by manipulating equation 21 to eliminate C.

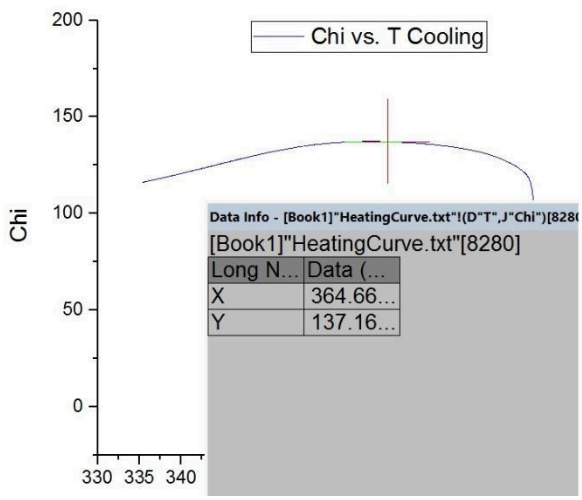
Graph 18: Cooling curves X and 1/ X vs. T for Ferroxcube 3E8



$T_c = 385.5K$

Now, by graphing just the cooling X vs. T graph and picking any data point, it would be possible to solve for the value of C. Below is this graph with a data point chosen randomly.

Graph 19: Cooling curve X vs. T for Ferroxcube 3E8

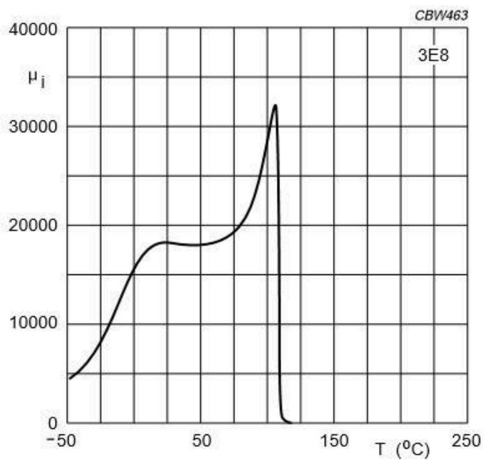


According to graph 13, for a temperature of T=364.6K, X=137.2. So knowing these values, and $T_c = 385.5K$, we find that,

C=2867.5

We were also able to find the Curie temperature for a Ferroxcube 3E8 in literature. Below is a graph of the initial magnetic permeability versus temperature for the solenoid.

Graph 20: Actual μ vs. T for Ferroxcube 3E8



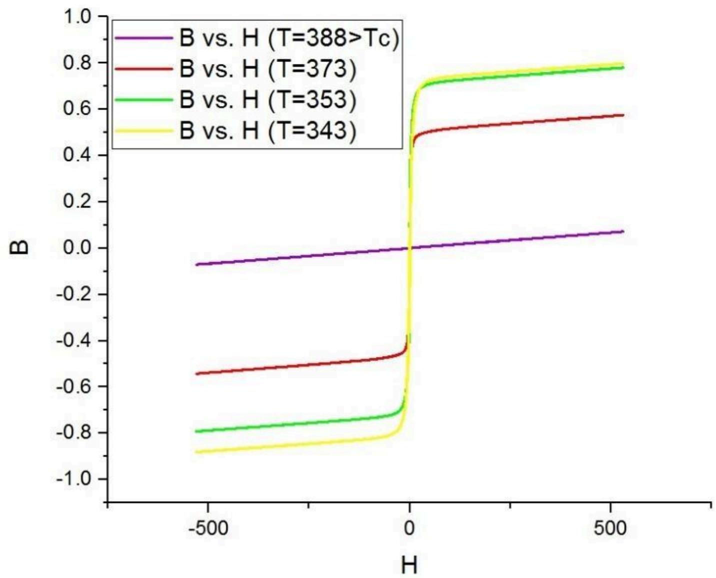
According to the accepted data in graph 16, the Curie temperature is about. $T_c \approx 113C = 386K$ This value agrees with the experimental value we solved for.

3.4 Exercise 3: Temperature Dependence of the Magnetic Susceptibility

In exercise 1 in this lab, we were able to generate B-H Hysteresis loops for different solenoids. In these cases the temperature of the solenoids was at room temperature. In exercise 2 in this lab we proved that at a temperature known as the Curie temperature, a magnetic field cannot be generated in the solenoid. By combining these two ideas and the setups from these exercises, we sought out to find the dependence of B-H Hysteresis loops on temperature. Theoretically since at temperatures above the Curie temperature a magnetic field cannot be generated in the solenoid, so the B-H Hysteresis loop should be at 0 or negligible.

The procedure for this exercise was to use the same heating method as in exercise 2. Then, using the MagneticLab software we could generate data for the values of the current and voltage like exercise 1. We used the same analytic method to convert the measured values and measured constants to result in B-H Hysteresis loops for each different temperature. The solenoid used is the same as in exercise 2 as well. Below is the plot of this all together.

Graph 21: B vs. H for Ferroxcube 3E8 at Temperature=388,373,353,343K



As predicted, when T>T_c there is no hysteresis loop. However, as the temperature decreased from the Curie temperature, the range in B induced in the solenoid decreased.

4. Conclusion

In this experiment we sought out to prove the B-H hysteresis loop for any solenoid. By inducing a current through the solenoid loops a magnetic field was generated within the solenoid. We successfully were able to measure this field, and through relevant equations, we proved the predicted result for a ferromagnet. We then cooled a solenoid and measured the response of the magnetic susceptibility to the temperature. The result was a graph that illustrated the effect of the loss of magnetism at a specific temperature. This temperature, the Curie temperature, and concurrent Curie-Weiss law were proven when compared to our raw data. In proving the relationship between the temperature and magnetic susceptibility, we could then repeat the experiment in exercise 1 while manipulating the temperature. We expected the data to resemble a line at B=0 at any temperature greater than our Curie temperature, and the magnitude of the B to decrease as temperature decreased. This prediction was proven to be true through our theoretical data. In summary, we had success in proving the responses of magnetic fields within ferromagnets.