Practical work 2-3.

Work with system compute.

Purpose of the work: study of positional number systems.

Devices and accessories: computer.

1.1. Number systems used

A number system is a symbolic method of writing numbers

or a way of representing numbers using written signs called digits.

A number is some abstract entity for describing the amount of something.

Digits are the signs used to write numbers.

Since there are many more numbers than digits, a set (combination) of digits is usually used to write a number.

There are many ways to write numbers using digits.

Each such method is called a number system. The value of a number may or may not depend on the order of the digits in the entry. This property is determined by the number system and serves as the basis for the simplest classification of such systems.

This basis allows all number systems to be divided into three classes (groups): positional, non-positional and mixed.

An example of a "purely" non-positional number system is the Roman system, and a mixed one is the monetary system of units.

Positional number systems are number systems in which the value of a digit directly depends on its position in the number. For example, the number 01 represents one, 10 represents ten.

Positional number systems make it easy to perform arithmetic calculations. Representing numbers using Arabic numerals is the most common positional number system, it is called the "decimal number system". It is called the decimal system because it uses ten digits: 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9.

For compiling machine codes, it is convenient to use not the decimal, but the binary number system, which contains only two digits 0 and 1. Programmers also use the octal and hexadecimal number systems for calculations.

The number of digits used in a number system is called its "base". In the decimal system, the base is ten, in the binary system - two, and in the octal and hexadecimal systems - eight and sixteen, respectively.

In the q-number system, the number of digits is q and the digits from 0 to q - 1 are used.

To work, you need to know the representation of decimal numbers from zero to 15 in number systems with bases q = 2, 8, 16 (see Table 1.2).

Table 1.2

Representations of decimal numbers in different number systems

q=10	Q=2	Q=8	Q=16
0	0	0	0
1	1	1	1
2	10	2	2
3	11	3	3
4	100	4	4
5	101	5	5
6	110	6	6
7	111	7	7
8	1000	10	8
9	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D

14	1110	16	E
15	1111	17	F

In addition, it is useful to know the decimal values of numbers 2k from k = 0 to k = 10 (see Table 1.3).

Table 1.3

k	0	1	2	3	4	5	6	7	8	9	10
2^k	1	2	4	8	16	32	64	128	256	512	1024

Converting numbers from one number system to another

To convert an integer N with a q-th base to a decimal number, write it as a polynomial and then calculate it according to the rules of decimal arithmetic:\

$$N = a_n \bullet q^n + a_{n-1} \bullet q^{n-1} +. .. + a_2 \bullet q^2 + a_1 \bullet q^1 + a_0 \bullet q^0$$

Here a_n − are the digits of the number,

q − base system computing

n − 0, 1, 2…

Example:

$$(11001)_2 = 1 \cdot 2^4 + 1 \cdot 2^3 + 0 \cdot 2^2 + 0 \cdot 2^1 + 1 \cdot 2^0 =$$
$$= 1 \cdot 16 + 1 \cdot 8 + 0 \cdot 4 + 0 \cdot 2 + 1 \cdot 1 = (25)_{10}$$
$$(221)_3 = 2 \cdot 3^2 + 2 \cdot 3^1 \cdot 1 \cdot 3^0 = 2 \cdot 9 + 2 \cdot 3 + 1 \cdot 1 = (25)_{10}$$
$$(221)_3 = 2 \cdot 3^2 + 2 \cdot 3^1 \cdot 1 \cdot 3^0 = 2 \cdot 9 + 2 \cdot 3 + 1 \cdot 1 = (25)_{10}$$
$$(31)_8 = 3 \cdot 8^1 + 1 \cdot 8^0 = 3 \cdot 8 + 1 \cdot 1 = (25)_{10}$$
$$(534D)_{16} = 5 \cdot 16^3 + 2 \cdot 16^2 + 4 \cdot 16^1 + 13 \cdot 16^0 =$$
$$= 20480 + 512 + 64 + 13 = (21069)_{10}$$

Note: when working with different number systems, the number is written in brackets, and the base of the system is written outside the brackets.

To convert integers inversely (from the decimal number system to the system with the base q), the number N is divided by q and the remainders from the division are written until the quotient from the previous division becomes zero.

Example: convert the number 25 to the binary system:

Original number	Private	Remainder
25/2	12	1
12/2	6	0
6/2	3	0
3/2	1	1
1/2	0	1

a_4 a_3 a_2 a_1 a_0

Result: (25)10 = (11001)2

When the last quotient becomes zero, write down all the remainders in a row from the last to the first. Thus, we get a number in the binary system - $(1001)_2$. To convert mixed numbers to the binary system, you need to separately convert their integer part and fractional part. In writing the result, the integer part of the translation is separated from the fractional comma in accordance with the formula:

$$N = \pm a_n \quad a_{n-1} \quad \dots \quad a_1 \quad a_0, \quad a_{-1} \quad a_{-2} \quad \dots \quad a_{-n}$$

Basic number systems

Binary number system. In computer technology, the binary number system is mainly used. Such a system is very easy to implement in digital microelectronics, since it requires only two stable states (0 and 1).

The binary number system can be non-positional and positional. This can be implemented by the presence of some physical phenomenon or its absence.

For example: there is an electric charge or not, there is voltage or not, there is current or not, there is resistance or not, reflects light or not, is magnetized or not magnetized, there is a hole or not, etc.

Octal number system is a positional integer number system with a base of 8. It uses digits from 0 to 7 to represent numbers.

Octal number system is often used in areas related to digital devices. Characterized by easy conversion of octal numbers to binary and vice versa, by replacing octal numbers with binary triads.

Previously, this system was widely used in programming and computer documentation, but at present it has been almost completely replaced by the hexadecimal system.

To convert a binary number to octal, the original number is broken into triads to the left and right of the decimal point; the missing extreme digits are supplemented with zeros. Then each triad is written as an octal digit (see Table 1.2).

Example: illustration of converting a binary number to an octal number:

$$N = (110011{,}100010)_2 = \left(\overbrace{110}^{6}\ \overbrace{011}^{3},\overbrace{100}^{4}\ \overbrace{010}^{2}\right)_2 = (63{,}42)_8$$

The hexadecimal number system is a positional number system based on integer base 16. Typically, the hexadecimal digits are decimal digits from 0 to 9 and Latin letters from A to F to denote digits from (1010)2 to (1111)2, that is (0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F)16.

To convert a binary number to hexadecimal, the original number is divided into tetrads to the left and right of the decimal point; the missing extreme digits are supplemented with zeros. Then each tetrad is written as a hexadecimal digit (see Table 1.2).

Example: illustration of converting a binary number to a hexadecimal number:

$$N = \left(\overbrace{0111}^{7}\ \overbrace{1010}^{A}\ \overbrace{1011}^{B},\ \overbrace{1110}^{E}\ \overbrace{1111}^{F}\right)_2 = (7AB{,}EF)_{16}$$

1.2. Variants of tasks for laboratory work (see Table 1.4)

Task 1. Convert integers from the decimal number system:

– to binary;

– to octal;

– to hexadecimal.

Task 2. Convert integers from the binary number system:

– to hexadecimal.

– to octal;

– to hexadecimal;

– to decimal.

Task 3. Convert integers from the hexadecimal

number system:

– to binary;

– to octal;

– to decimal.

Task 4. Add:

– binary numbers;

– octal numbers;

– hexadecimal numbers.

Task 5. Find the difference:

– binary numbers;

– octal numbers;

– hexadecimal numbers.

Task 6. Calculate the value of the expression and represent in the decimal number system.

Table 1.4

Variants of assignments for laboratory work

	Option 1	Option 2	Option 3	Option 4	Option 5
Exercise 1	a) 2515 b) 3084 c) 9042	a) 1052 b) 1387 c) 7634	a) 2042 b) 5548 c) 2372	a) 5911 b) 6321 c) 7629	a) 3988 b) 5147 c) 1123
Exercise 2	a) 110101 b) 111010 c) 101111	a) 011001 b) 101010 c) 010101	a) 100110 b) 110011 c) 101111	a) 011001 b) 100001 c) 001001	a) 100010 b) 111000 c) 011111
Exercise 3	a) 1F52 b) 5521 c) 1101	a)1A1B b) 2350 c) 3239	a) 5EE2 b) 2682 c) 2461	a) 7B1B b) 3458 c) 6537	a) 1C2D b) 6824 c) 8673
Exercise 4	a) 1011 + + 0111 b) 573 + 325 c) F1 + E7	a) 0110 + + 1100 b) 274 + 235 c) 93 + 2C	a) 1010 + + 0101 b) 271 + 123 c) 58 + 79	a) 1101 + + 1101 b) 632++714 c) 51 + 9D	a) 1010 + + 1010 b) 521++623 c) 36 + AB

	Option 1	Option 2	Option 3	Option 4	Option 5
Exercise 5	a) 1011 – – 0111 b) 573 – 325 c) F1 – E7	a) 1110 – – 1100 b) 274 – 235 c) 93 – 2C	a) 1010 – – 0101 b) 271 – 123 c) A8 – 79	a) 1101 – – 1001 b) 732 – 714 c) B1 – 9D	a) 1010 – – 1000 b) 721 –623 c) C6 – AB
Exercise 6	23_8 + $A2_{16}$ * * 1001_2	$B1_{16}$ – – 1011_2 * * 117_8	51_8 * 21_{16} – – 45510	(59_{16} + + 1110_2) * * 456_8	25_8 * * 567_{16} – – 10101_2

1.3. Questions

1. What is called a number system?

2. Which number systems are called non-positional? Why? Give an example of such a number system and how to write numbers in it.

3. Which number systems are used in computing: positional or non-positional? Why?

4. How is a number represented in a positional number system?

5. What is called the base of a number system?

6. How can a positive integer be represented in a positional number system?

7. Which number systems are used in a computer

to represent information?

8. What are the rules for adding two positive integers?

9. What are the rules for performing arithmetic operations
in the binary number system?

10. What is the purpose of converting numbers from one number system to another?

11. Formulate the rules for converting numbers from the p-base number system to the decimal number system and vice versa: from the decimal number system to the s-base number system. Give examples.

12. How to convert numbers from the binary number system to the octal number system and vice versa? From binary Continuation of the table of the number system to the hexadecimal system and back? Give specific examples.

13. What are the rules for converting numbers from the octal to the hexadecimal number system and vice versa? Give examples.