Questions 26-39. Read the following passage carefully before you choose your answers.

(The following passage is from a contemporary biography about a mathematician.)

For [Paul] Erdős, mathematics was a glorious combination of science and art. On the one hand, it was the science of certainty, because its conclusions were logically unassailable. Unlike biologists, chemists, or even physicists, Erdős, Graham, and their fellow mathematicians *prove* things. Their conclusions follow syllogistically from premises, in the same way that the conclusion "Bill Clinton is mortal" follows from the premises "All presidents are mortal" and "Bill Clinton is a president." On the other hand, mathematics has an aesthetic side. A conjecture can be "obvious" or "unexpected." A result can be "trivial" or "beautiful." A proof can be "messy," "surprising," or, as Erdős would say, "straight from the Book." In a good proof, wrote Hardy, "there is a very high degree of *unexpectedness,* combined with *inevitability* and *economy.* The argument takes so odd and surprising a form; the weapons used seem so childishly simple when compared with the far-reaching consequences; but there is no escape from the conclusions."

What is more, a proof should ideally provide insight into why a particular result is true. Consider one of the most famous results in modern mathematics, the Four Color Map Theorem, which states that no more than four colors are needed to paint any conceivable flat map of real or imaginary countries in such a way that no two bordering countries have the same color. From the middle of the nineteenth century, most mathematicians believed that this seductively simple theorem was true, but for 124 years a parade of distinguished mathematicians and dedicated amateurs searched in vain for a proof—and a few contrarians looked for a counterexample. "When I started at AT&T," said Graham, "there was a mathematician there named E. F. Moore who was convinced that he could find a counterexample. Each day he would bring in a giant sheet of paper, and I mean giant, two feet by three feet, on which he had drawn a map with a few thousand countries. 'I know this one will require five colors,' he'd confidently announce in the morning and volunteer to give me a dollar if it wasn't the long-sought-after counterexample. Then he'd go off and spend hours coloring it. He'd come by at the end of the day, shake his head, and hand me a dollar. The next day he'd be back with another map and we'd go through the same thing again. It was the easiest way to make a buck!"

By 1976 it was clear why Moore's quest for a five-color map had come to nought. That was the year Kenneth Appel and Wolfgang Haken of the University of Illinois finally conquered this mathematical Mount Everest. When word of the proof of the Four Color Map Theorem reached college mathematics departments, instructors cut short their lectures and broke out champagne. Some days later they learned to their dismay that Appel and Haken's proof had made unprecedented use of high-speed computers: more than 1,000 hours logged among three machines. What Appel and Haken had done was to demonstrate that all possible maps were variations of more than 1,500 fundamental cases, each of which the computers were then able to paint using at most four colors. The proof was simply too long to be checked by hand, and some mathematicians feared that the computer might have slipped up and made a subtle error. Today, more than two decades later, validity of the proof is generally acknowledged, but many still regard it as unsatisfactory. "I'm not an expert on the four-color problem," Erdős said, "but I assume the proof is true. However, it's not beautiful. I'd prefer to see a proof that gives insight into why four colors are sufficient."

Beauty and *insight*—these are words that Erdős and his colleagues use freely but have difficulty explaining. "It's like asking why Beethoven's Ninth Symphony is beautiful," Erdős said. "If you don't see why, someone can't tell you. I know numbers are beautiful. If they aren't beautiful, nothing is."

GO ON TO THE NEXT PAGE.

(A

Questions 26-39. Read the following passage carefully before you choose your answers.

(The following passage is from a contemporary biography about a mathematician.)

For [Paul] Erdös, mathematics was a glorious combination of science and art. On the one hand, it was the science of certainty, because its conclusions were logically unassailable. Unlike biologists, chemists, or even physicists, Erdös, Graham, and their fellow mathematicians prove things. Their conclusions follow syllogistically from premises, in the same way that the conclusion “Bill Clinton is mortal" follows from the premises “All presidents are mortal” and “Bill Clinton is a president.” On the other hand, mathematics has an aesthetic side. A conjecture can be “obvious” or “unexpected.” A result can be “trivial" or “beautiful.” A proof can be “messy,” “surprising,” or, as Erdös would say, “straight from the Book." In a good proof, wrote Hardy, “there is a very high degree of imexpectedness, combined with inevitability and economy. The argument takes so odd and surprising a form; the weapons used seem so childishly simple when compared with the far-reaching consequences; but there is no escape from the conclusions."

What is more, a proof should ideally provide insight into why a particular result is true. Consider one of the most famous results in modern mathematics, the Four Color Map Theorem, which states that no more than four colors are needed to paint any conceivable flat map of real or imaginary countries in such a way that no two bordering countries have the same color. From the middle of the nineteenth century, most mathematicians believed that this seductively simple theorem was true, but for 124 years a parade of distinguished mathematicians and dedicated amateurs searched in vain for a proofand a few contrarians looked for a counterexample. “When I started at AT&T,” said Graham, “there was a mathematician there named E. F. Moore who was convinced that he could find a counterexample. Each day he would bring in a giant sheet of paper, and I mean giant, two feet by three feet, on which he had drawn a map with a few thousand countries. ‘I know this one will require five colors,” he’d confidently announce in the morning and volunteer to give me a dollar if it wasn’t the long-sought-after counterexample. Then he’d go off and spend hours coloring it. He’d come by at the end ofthe day, shake his head, and hand me a dollar. The next day he’d be back with another map and we’d go through the same thing again. It was the easiest way to make a buckl”

By 1976 it was clear why Moore’s quest for a five-color map had come to nought. That was the year Kenneth Appel and Wolfgang Haken of the University of Illinois finally conquered this mathematical Mount Everest. When word of the proof of the Four Color Map Theorem reached college mathematics departments, instructors cut short their lectures and broke out champagne. Some days later they learned to their dismay that Appel and Haken’s proof had made unprecedented use of high-speed computers: more than 1,000 hours logged among three machines. What Appel and Haken had done was to demonstrate that all possible maps were variations of more than 1,500 fundamental cases, each of which the computers were then able to paint using at most four colors. The proof was simply too long to be checked by hand, and some mathematicians feared that the computer might have slipped up and made a subtle error. Today, more than two decades later, validity of the proof is generally acknowledged, but many still regard it as unsatisfactory. “Vm not an expert on the four-color problem,” Erdös said, “but I assume the proof is true. However, it`s not beautiful. I’d prefer to see a proof that gives insight into why four colors are sufficient."

Beauty and insight-these are words that Erdös and his colleagues use freely but have difficulty explaining. “lt’s like asking why Beethoven’s Ninth Symphony is beautiful," Erdös said. “lf you don’t see why, someone can’t tell you. I know numbers are beautiful. If they aren’t beautiful, nothing is.”

GO ON TO THE NEXT PAGE.