Hand-writing answer sheets are accepted except programming codes and plots. You could provide the homework results at the beginning of the class or leave them in bldg. 5, #208.

Homework #2 (H-J-B equation & LQR)

Problem 1. H-J-B equation. Consider the example that is described by the the system equation.

$$\dot{x}(t) = u(t)$$

The control, $u(t)$, is constrained by the following equation.

$$|u(t)| < 1, \quad t \in \left[0,\ t_f\right]$$

Find out the optimal control when the cost is defined as:

1) $J = \frac{1}{2}x_f^2$
2) $J = \int_0^{t_f} \frac{1}{2}x^2 dt$
3) $J = \int_0^{t_f} \frac{1}{2}u^2 dt$
4) $J = \frac{1}{2}x_f^2 + \int_0^{t_f} \frac{1}{2}u^2 dt$

Why the optimal control of case 3) is $u(t) = 0$?

Problem 2. **LQR**. A dynamic system has a state equation, which can be expressed as:

$$\dot{x}(t) = ax(t) + bu(t)$$

An optimal control trajectory minimizes the performance that is defined as:

$$J = \frac{1}{2}h\left(x_f - x_r\right)^2 + \frac{1}{2}\int_{t_0}^{t_f}\left\{Q\left(x(t) - x_r\right)^2 + Ru^2(t)\right\}dt$$

Find the trajectory of the optimal control $u^*(t)$ that minimizes the cost, when $Q = 1,\ R = 1,\ a = 0,\ b = 1,\ h = 1$. The reference state, $x_r = 1$. For the boundary conditions, $x_0 = 0$, and $t_f = \ln ln\, 2$.

Hint. $L^{-1}\left(\frac{1}{s^2-1}\right) = \frac{e^t - e^{-t}}{2}$, $L^{-1}\left(\frac{s}{s^2-1}\right) = \frac{e^t + e^{-t}}{2}$

Problem 3. Dynamic Programming. Build a programming code to obtain the optimal trajectory of Brachistochrone problem. Provide graphical results for the final trajectory of the solution. (Use Matlab)