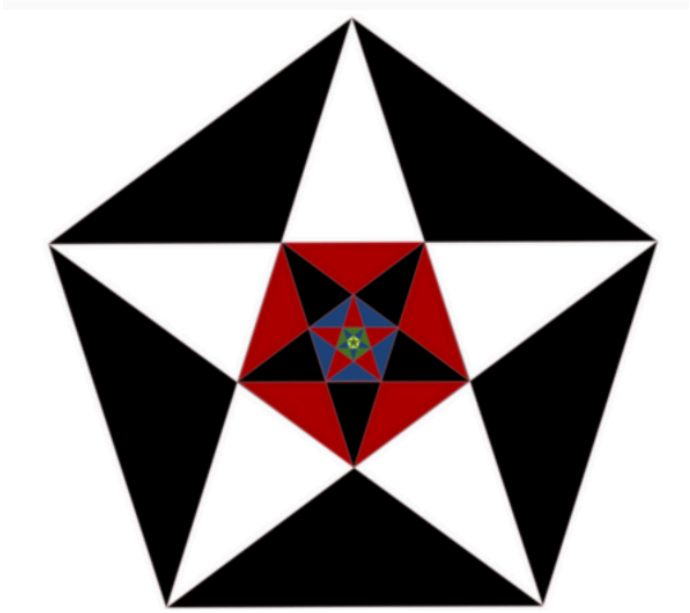
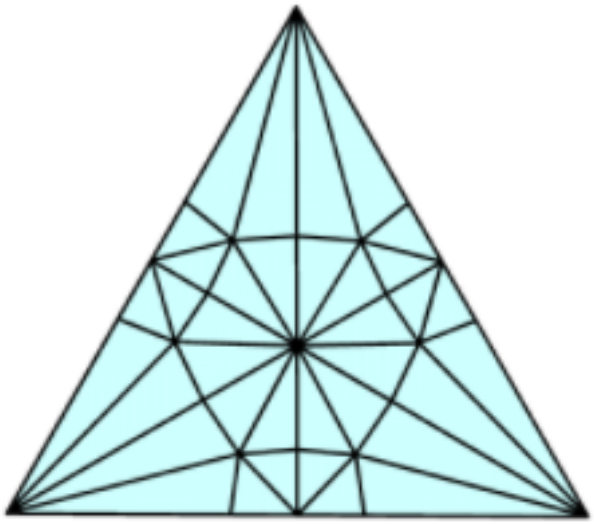
Fractal Tree Activity

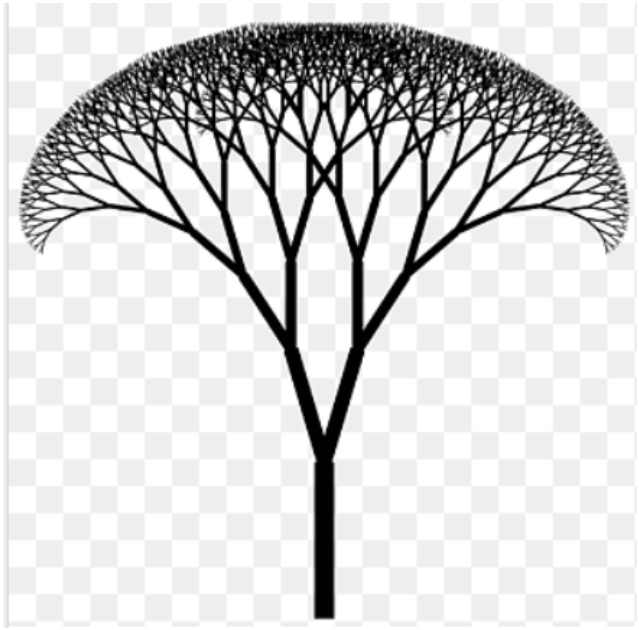
Thinking Classroom Exercise 1
Thinking Classroom Exercise 2
Thinking Classroom Exercise 3
Activity (Part 1)
Activity (Part 2)
Activity (Extension)
Grid for Fractal Tree

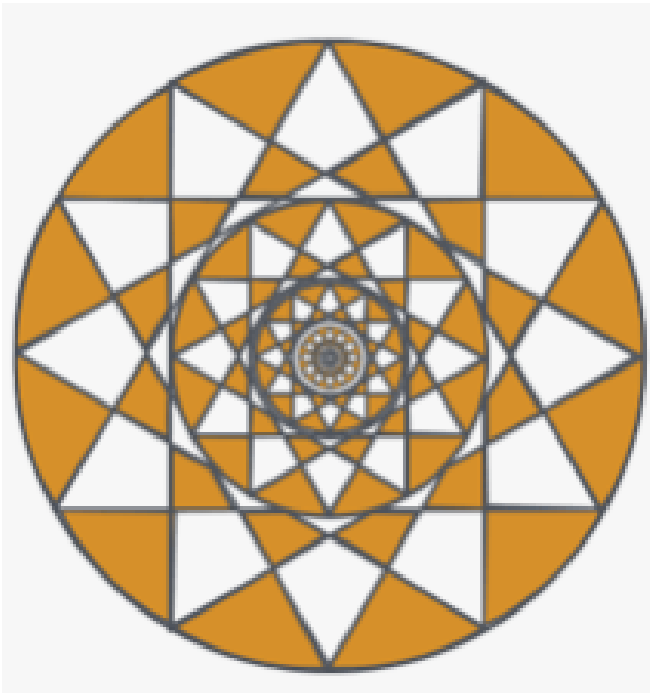
EXERCISE 1
Print out a copy for each group.

Which of the figures below are self-similar? Which are not? Explain.
If it is self-similar, circle a part of the pattern that is similar to the entire pattern.







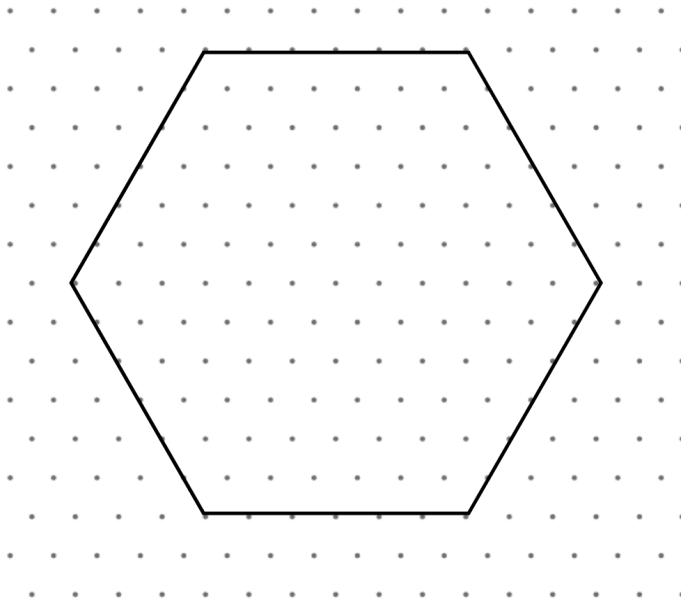


EXERCISE 2

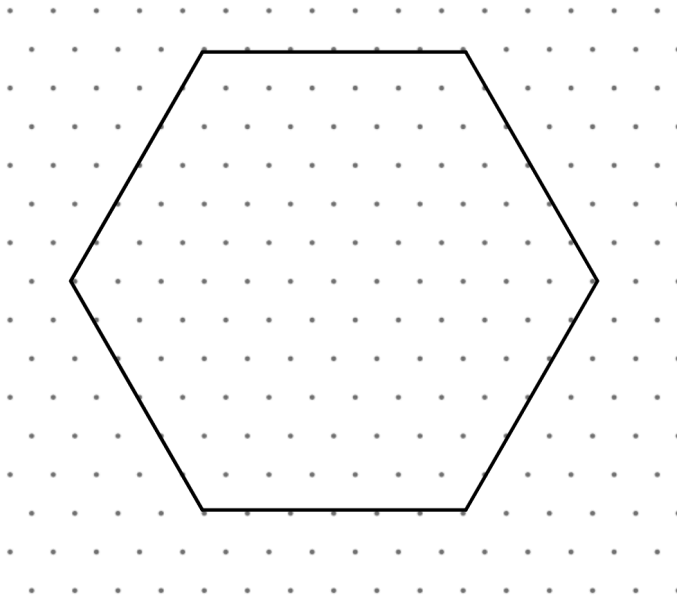
Divide hexagon 1 into six congruent triangles. Then find the measure of the angles that are made at the center of the hexagon (“central angles”).

Divide hexagon 2 into three congruent shapes. Then find the measure of the angles at the center of the hexagon (“central angles”).

HEXAGON 1



HEXAGON 2



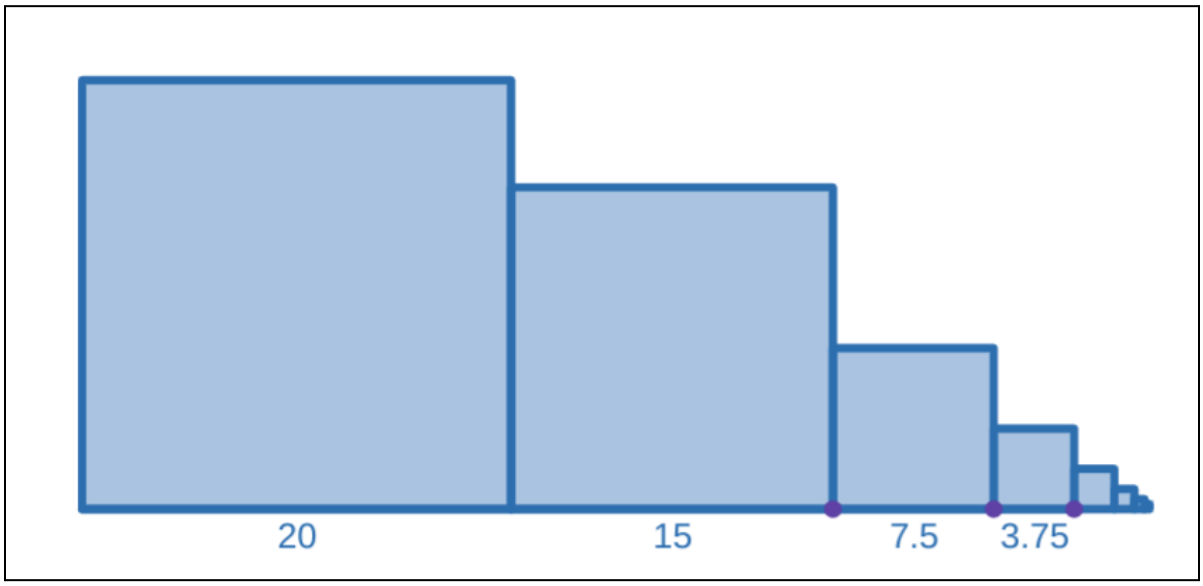
EXERCISE 3
Provide students with calculators.

You're given a branch of squares and a branch of equilateral triangles. Record this information:

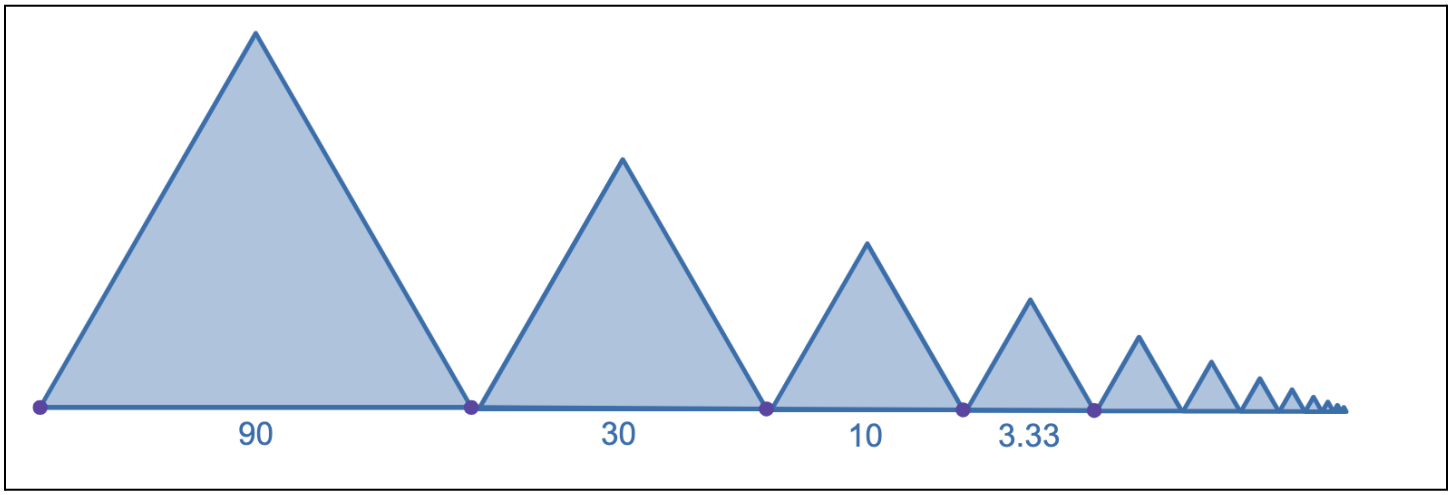
The first four stages are labeled with side lengths.

1) For each branch, find the scale factors between each of the first four stages.
    a) Round to two decimals.
2) Use this to determine which branch could be a fractal.

Squares



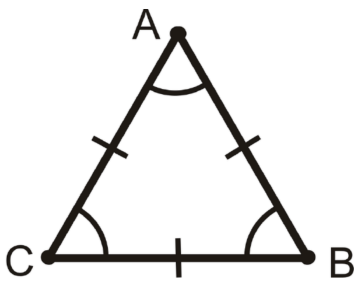
Equilateral Triangles



Fractals: Activity 1 (Part 1) **Name:**

In this lesson, we're going to explore a special type of pattern called a *fractal*. We'll draw the fractal in a dotted grid like the one below. To begin, let's figure out how to use this type of grid.

1) To the right, there is an **equilateral triangle**.



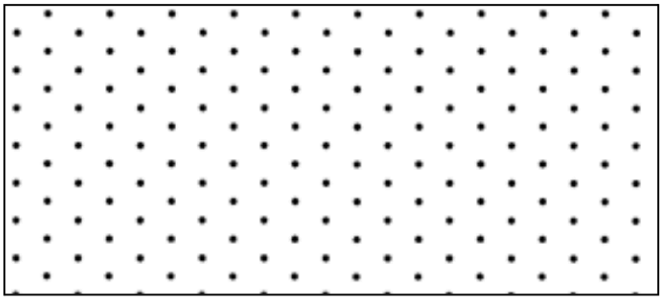
a. What do you know about the side lengths of an equilateral triangle?

b. How many degrees is each angle in an equilateral triangle?
   - Hint: The angles in any triangle add up to 180 degrees.

2) In the dotted grid, draw an equilateral triangle with side lengths of 6. Then draw another equilateral triangle with side lengths of 3.

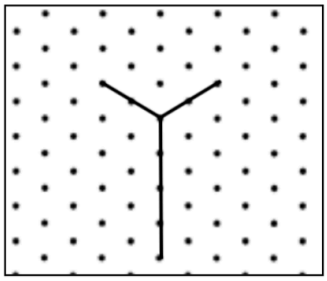
Are these triangles similar? Explain.

DOTTED GRID

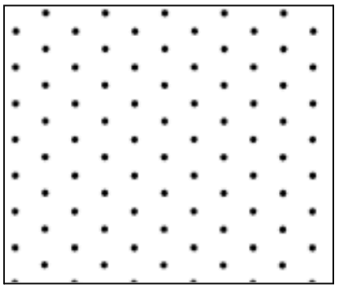


3) To the right, you're given a shape that we'll call a "branch". Draw a **congruent** branch.
   - Remember that congruent shapes have a scale factor equal to 1.

BRANCH

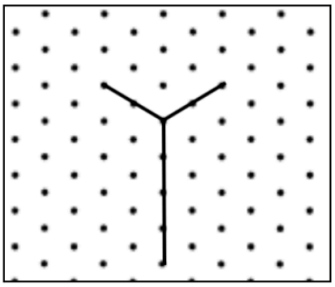


CONGRUENT BRANCH

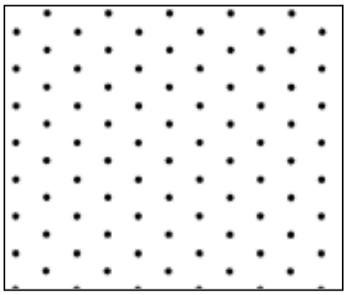


4) Now draw a **similar** branch that has a scale factor equal to ½ .

BRANCH



SIMILAR BRANCH



Fractal Tree

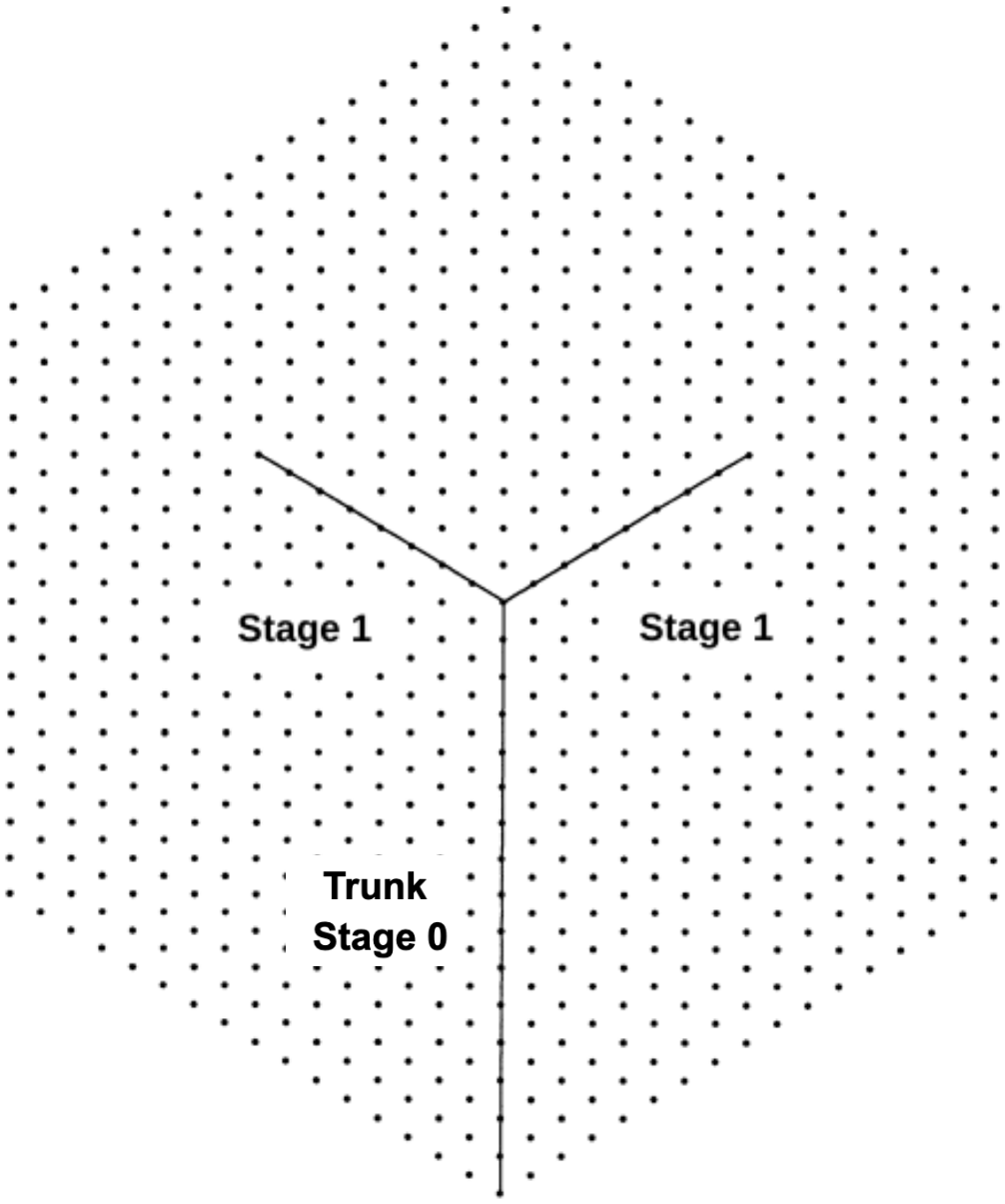
As trees grow, they branch out. From big branches grow smaller ones. From these grow smaller ones still, and so on. We'll use the dot grid below to draw a mathematical tree with some of the same properties as the real life ones.

Here is the construction pattern: *From the endpoints of each branch, draw two new branches that are half as long and that grow off at 60 degrees in opposite directions.*

Stage 0 and Stage 1 of the tree have already been drawn.

- Draw the four new branches for **Stage 2** by connecting the endpoints to the appropriate dots on the grid.
- Draw the eight new branches for **Stage 3**.
- Repeat the construction pattern again for **Stage 4**.

Tip: Endpoints should still be on the dots of the grid for each of these stages.



Reflection question: Suppose you could zoom in on the branches of this tree. Would it be possible to draw Stage 5? What about Stage 10? Will the branches ever stop growing?

Fractals: Activity 1 (Part 2) **Name:**

1) In Part 1, you drew a fractal tree. Summarize some of the **patterns** in the tree. Discuss:

- Length of branches,

- Number of branches,

- Shape of the tree.

2) What is the **scale-factor** from the length of a Stage 0 branch to a Stage 1 branch? What about Stage 1 to Stage 2? And so on.

3) Suppose that the trunk of the tree (Stage 0) has a length of **1 unit**.

a. What would the length of a Stage 1 branch be?

b. What would the length of a Stage 2 branch be?

Now imagine that the tree continues growing branches, over and over by the same process, until it is fully grown. Try to visualize this *completed tree*.

4) How many branches would have:

a. A length of $\frac{1}{4}$?

b. A length of $\frac{1}{8}$?

5) Now consider adding up the length of all the branches at each stage. What is the **sum** of the lengths of all branches that are

a. $\frac{1}{4}$ long?

b. $\frac{1}{16}$ long?

c. What is the sum of the length of all branches at any stage?

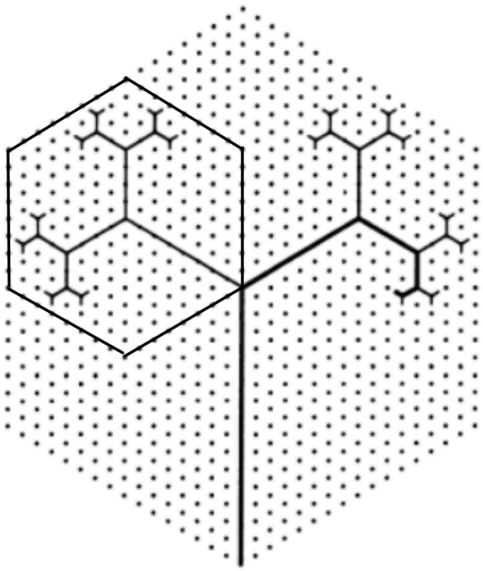
6) Suppose you added up the length of all branches at every stage of the completed tree. What would the total length be?

7) In Lesson 3, you began investigating the idea of **self-similarity**. Describe this idea. In particular, how can we tell if a shape is self-similar?

- If you're stuck, look back at Slide 2 through Slide 5 on Lesson 3.

8) Here's a useful question to ask yourself: *Are there parts of the completed tree that look like the entire tree?* This is one way to check for self-similarity.

For example, to the right, you'll see a **hexagon** drawn around a part of the tree that is an exact image of the tree itself.



a. On your fractal tree, draw a hexagon around a different part of the tree that is similar to the entire tree.

b. Draw another hexagon of a different size.

Extension: A Spiral

An interesting shape found on the completed tree is a spiral. Here's how you make it:

> *Start at the base of the tree. Turn right at the first junction point. Turn right at the next junction. Continue turning right at each and every junction point.*

Color a spiral in your tree. What is the length of this spiral?

Color a second spiral that is a reflection of the first one that you found.

Color four spirals with half the length of the original one. How many spirals in the tree have a ¼ the length of the original spiral?

Imagine that you continued on drawing one of the original spirals endlessly in a completed tree. How long would the spiral be?

Fractals: Activity 1 (Extension)

Creating Fractal Trees

The tree that you drew and analyzed in Activity 1 had these properties:

- Each branch grows into two new branches at each stage.
- The branches split at a 60 degree angle in opposite direction.
- The scale factor stays the same at each stage.

Take this link to create some different fractal trees: http://mwskirpan.com/FractalTree/

- Note: To see the fractal tree, press "SUBMIT" and then scroll down to see the fractal tree.

On this website, experiment with each of these parameters:

- Tree depth
- Line ratios
- Angles

As you experiment, think carefully about what these parameters change and control in the fractal tree.

Once you've created a new and interesting tree, take a screenshot and paste it below. Then summarize the **patterns** in the tree. Discuss:

- Length of branches,
- Number of branches,
- Scale-factor(s)
- Angles

Summary of patterns:

Screenshot of your tree:

Grid for Fractal Tree

