

There is a math trick in which you take any random three digit number (the first and last digits have to be different and more than one apart), reverse it, subtract the smaller from the larger, reverse that, and then add the two together. The final sum is always 1089. Here's how that trick works, breaking the explanation down as much as possible to make it easier to understand.

1) Let's first focus on the two digits on the ends, in the ones and hundreds places. Since we are subtracting the smaller three digit number from the larger three digit number, the larger number will always be on the left, in the hundreds place, on top, and thus the smaller in the ones place on top. For instance, if the three digit number is 326, then we will subtract 326 from 623 (which gives us 297). In the ones place we will always be subtracting a larger digit from a smaller one, because we will have ensured that we are subtracting the smaller digit from the larger one in the hundreds place. This means that we will have to borrow from the tens every time in the subtraction process.

2) The middle digit in both steps, both the first subtraction step and second addition step, are always the same, because when you reverse a three digit number, the middle digit stays where it is. This is particularly relevant in the first step. Since we had to borrow for the subtraction in the ones place, the top digit is now one less than the bottom in the tens place. We'll have to borrow again, and the answer will always be nine. For instance, in our example, we had a two in the tens place, in both of the three digit numbers after reversal. We had to borrow from it on top to subtract six from three, and so now have a one. We have to borrow again on top to subtract two from one, and so are subtracting two from eleven. The answer is nine, and always will be nine in the tens place, because we will always be subtracting a number from ten plus one less than it.

3) On the left we would have gotten the difference between the two outside numbers (in the ones and hundreds places), if not for borrowing. For example, if the two digits were six and three, then on the left (in the hundreds place) we would be subtracting three from six and would get three. But since we borrowed, we are subtracting three from five and get two. On the right, in the one's place, as we discussed above, they are reversed, and we will be subtracting the larger from the smaller, which means we'll have to borrow, which is essentially adding ten to the smaller number and then subtracting what had been the larger from it. So, in our example, we subtract six from the thirteen on the right and get seven. If we had not borrowed, we would have subtracted three from six on the left and gotten three.

4) If not for borrowing, subtracting the smaller from the larger and the larger from the smaller plus ten and adding the answers together always gives us ten. It's easier to see when represented numerically: $(a - b) + (b + 10 - a)$ gives us (removing the parentheses and rearranging the addends) $a - a + b - b + 10 = 10$. So in the second step of adding together the reversed results of the first step subtraction, we would always get ten, if not for the borrowing that occurred, which reduced the sum in the hundreds place by one. Because of the borrowing, the actual sum becomes nine.

5) So, after the final step of adding the second three digit number (297 in our example) to its

reversal (792), we always get nine on the right, in the ones place, always add nine to nine in the tens place and get 8 with one carried over, and always get nine again in the hundreds place but with the one carried over turning it into ten, thus always giving us an answer of 1089, no matter what the original three digits were.

(If the first and last digits are just one apart, the difference between the smaller and larger of itself and its inversion will be 99, leading to a final result of 198 instead of 1089. A separate “trick” could be arranged in anticipation of that possibility, if one wanted to do so.)