Waves

1. Two identical explosions, A and B, occur simultaneously and the primary waves they produce travel through uniform ground. For simplicity we're going to ignore secondary and surface waves, even though in real life these matter. There is a pebble stuck in the ground at point P. Explosion A occurs three times farther away from P than does B, and the angle APB is 90°. The waves from A and B are exactly out of phase when they reach P. Your job will be to analyze the motion of P as the waves pass through it.

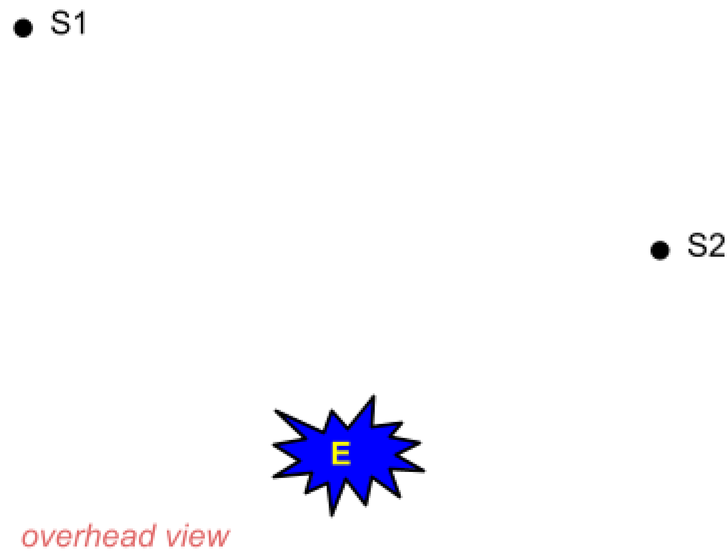
Draw a pic *to scale* that shows the way in which the pebble moves, and calculate any relevant angles. Also, use words to describe the motion of the pebble as completely and as clearly as possible. Things to keep in mind:

- Each wave on its own would shake P back and forth along a line segment in simple harmonic motion.
- The waves are identical but the sources are at different distances. Thus their amplitudes are different at P. Since we're dealing with primary seismic waves, which are not surface waves, we will assume that the wavefronts propagate in three dimensions in the ground, i.e., hemispherical wavefronts.
- As a wavefront moves away from a source, the energy it carries is spread out over an ever greater area. Determine how energy, E, and distance, d, are related. (Write it as a proportion.)
- Recall the relationship between the energy, E, in a wave and its amplitude, A.
- Combine the last two steps to write a relationship between A and d. This will give you the ratio of amplitudes of the two waves at P.
- Because the waves are out of phase at P, when B is pulling (down), A is pushing (right), and when.... Note: being out of phase in this case does NOT mean the waves are canceling each other out. They wouldn't cancel even if there magnitudes were the same, since they're acting at right angles to one another.
- Think about how vectors add in order to deduce P's overall motion.

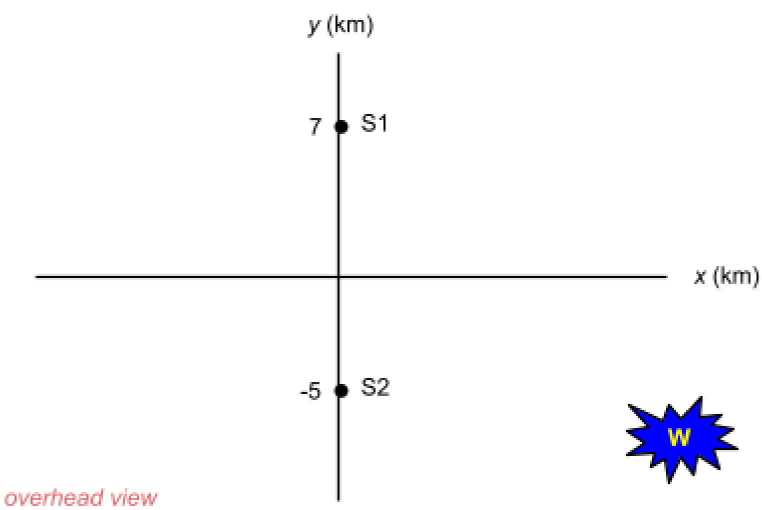
2. Waves can be used for triangulation, for navigation, say, or to pinpoint the epicenter of an earthquake. Here's a scenario. An earthquake occurs at some unknown location. (The actual location--called the hypocenter--would be underground, but we'll be interested in point directly above it at the surface--the epicenter.) There are two seismic stations, S_1 and S_2, located in different cities that detect both the P and S waves from the quake. S_2 is 100 km east and 50 km south of S_1.The speed of these waves depends, of course, on the material through which they're traveling. To keep the numbers simple, we'll assume a constant 5 km/s for P waves through earth's crust, and that S waves have only 60% of that speed, which is realistic. Station 1 measures a 24 s delay between the arrival times of S and P waves, and station 2

measures an 18 s delay. This is almost enough information to pinpoint the epicenter, E, which is your goal. Proceed as follows:

Let t be the time in seconds for P waves to reach S_1 from E. So $d_1 = 5t$ in km. In that time, the S waves have traveled $3t$ km. Δt = 24 s (given). So after the P waves arrive, the slower S waves will still be traveling for an additional 24 s. We can write $d_1 = 3(t + 24) = 5t$. Solve for t and then you can easily get d_1.



Knowing how far away the epicenter is from the first station, you can be confident that the quake occurred somewhere on a [*insert answer here; be VERY specific*]. Now repeat the above procedure for S_2. Let's use a coordinate system in which S_1 is at the origin and S_2 is at (100, -50) in km. Write an equation for the possible locations of the quake for each station (just basic algebra). To solve the system easily, type the two equations into Wolfram Alpha, separated by a comma. You'll see a graph with two possible quake sites indicated. Scroll down to see two solutions to the system. Click on Approximate Forms. What are these possible locations? What would you need in order to pinpoint the quake?



3. Schmedrick decides to take up whale tracking. Yeah, that's right...it's a perfectly valid hobby. He acquires a hydrophone in order to detect the sound waves they produce. Sound is always longitudinal, so there is only one type (only P--no S or other varieties). Thus the method in number two will not suffice. Suppose Schmed uses two "stations" (hydrophones), as in the pic on the left, to pick up a whale call. This time S_1 detects the sound 4 s after S_2. Use 1.5 km/s as the speed of sound in water to find the difference in distance: $S_1W - S_2W$.

As you learned in math class, conics sections can be defined with locus definitions. For example, the set of all points in a plane equidistant from a fixed point is a circle. An ellipse is the set of all points in a plane such that the sum of the distance from two fixed points (foci) is a constant. A hyperbola is the set all point such that the *difference* in distance is a constant. That's the situation we have here, with S_1 and S_2 as the foci. The difference in distance (which you already know) equals $2a$, where a is the parameter you learned about in math class--the distance from center to vertex. It's easy to locate the center of the hyperbola from the pic above and to find c, the distance from center to focus. Use $c^2 = a^2 + b^2$ to find b. Then find the equation of the hyperbola on which the whale lies (ignoring its depth). Only one branch of the hyperbola makes sense in this situation. Draw it.