

High-Dimensional Posterior Consistency In Multi-Response Regression Models For Error Covariance Matrix

Abstract:

The Inverse-Wishart (IW) distribution is a standard and popular choice of priors for covariance matrices and has attractive properties such as conditional conjugacy. However, the IW family of priors has crucial drawbacks, including the lack of effective choices for non-informative priors. Several classes of priors for covariance matrices that alleviate these drawbacks, while preserving computational tractability, have been proposed in the literature.

These priors can be obtained through appropriate scale mixtures of IW priors. However, the high-dimensional posterior consistency of models which incorporate such priors has not been investigated. We address this issue for the multi-response regression setting (q responses, n samples) under a wide variety of IW scale mixture priors for the error covariance matrix. Posterior consistency and contraction rates for both the regression coefficient matrix and the error covariance matrix are established in the "large q , large n " setting under mild assumptions on the true data-generating covariance matrix and relevant hyperparameters. In particular, the number of responses q_n is allowed to grow with n , but with $q_n = o(n)$. Also, some results related to the inconsistency of posterior mean for q_n/n , where $(0, \infty)$ are provided.