

1.

Points P, Q and R has coordinates $(-1, 6)$, $(2, 10)$ and $(11, 1)$ respectively.

Find

a. $\angle PRQ$

(4)

b. The area of triangle PQR

(2)

2.

a. A student suggests that for any prime number between 20 and 40, when its digits are squared and then added, the sum is an odd number.

For example, 23 has digits 2 and 3 which gives $2^2 + 3^2 = 13$, which is odd.

Show by counterexample that this suggestion is false.

(2)

b. Prove that the sum of the squares of any three consecutive positive integers cannot be divided by 3.

(3)

3.

A circle has equation $x^2 + y^2 - 2y - 24 = 0$

a. Find the coordinates of the centre of this circle and the length of its radius

(3)

b. Find the coordinates of the points when the line $x + y = 2$ intersects the circles.

(6)

4.

The number of members of a social networking site is modelled by $m = 150e^{2t}$, where m is the number of members and t is time in weeks after the launch of the site.

a. State what this model implies about the relationship between m and the rate of change of m .

(2)

b. What is the significance of the integer 150 in the model?

(1)

c. Find the week in which the model predicts that the number of members first exceeds 60 000.

(3)

d. The social networking site only expects to attract 60 000 members.

Suggest how the model could be refined to take account of this.

(1)

5.

$$f(x) \equiv x^3 - 5x^2 + 3x$$

- a. Find the exact values of the turning points on the curve $y = f(x)$

(5)

The graph $y = f(x + a)$ has a maximum point on the y - axis

- b. State the value of a

(2)

The graph $y = b + f(x)$ intersects the x - axis only once

- c. Find the range of possible values of b .

(3)

6.

Find the term independent of x in the expansion of $\left(x^2 + \frac{2}{x}\right)^6$

(6)

7.

Solve $3^{2x} + 2(3^x) = 8$

(5)

8.

$$f(x) \equiv x^3 + 6x^2 - 7x - 60$$

- a. Show $(x - 3)$ is a factor of $f(x)$

(2)

- b. Factorise $f(x)$ completely.

(2)

- c. Solve the equation $2 \log x + \log (x + 6) = \log (7x + 60)$, $x > 0$

(4)

9.

The profit (in hundreds of thousands of pounds) made by a company producing n (thousand units) with a cost of c (per thousand of units) is proportional to the product of two factors.

$$\text{So } P = kn(c - n)$$

Given the profit is £960 000 when producing 800 units and

£1 536 000 when producing 1600 units,

a. Find the values of c and k .

(4)

b. Write P in the form $P = A + B + (n + D)^2$

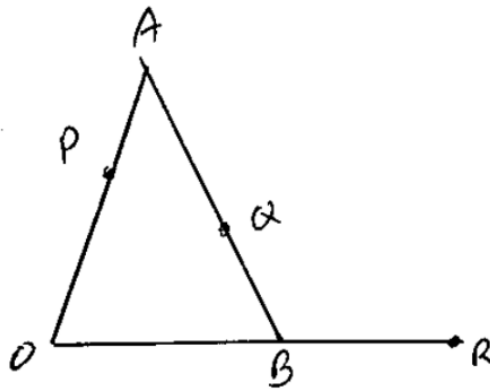
(3)

Without further calculations

c. State the maximum profit that can be achieved and the number of units produced to make this profit.

(2)

10.



Vector $\vec{OA} = \mathbf{a}$ and vector $\vec{OB} = \mathbf{b}$

P lies on OA such that $OP : PA = 2 : 1$

Q is the midpoint of AB.

R lies on OB produced such that $OR = 2OB$

a. Find in terms of $\mathbf{a}$ and $\mathbf{b}$.

i. $\vec{AB}$

ii. $\vec{OP}$

iii. $\vec{OQ}$

iv. $\vec{PQ}$

v. $\vec{QR}$

(5)

b. Explain why PQR is a straight line.

(2)

11.

Solve in the range $-180^\circ \leq \theta \leq 180^\circ$

$$\frac{4 - 5 \cos \theta}{\cos \theta} = 2 \sin \theta \tan \theta$$

(6)

12.

A curve has equation $y = kx^{\frac{3}{2}}$

Point P has x - coordinate 4

The normal to the curve at P is parallel to the line $x + 9y + 27 = 0$

a. Show $k = 3$

(5)

b. Find the equation of the normal to the curve at P.

(2)

13.

A curve has equation $y = 4e^{2x} - 1$

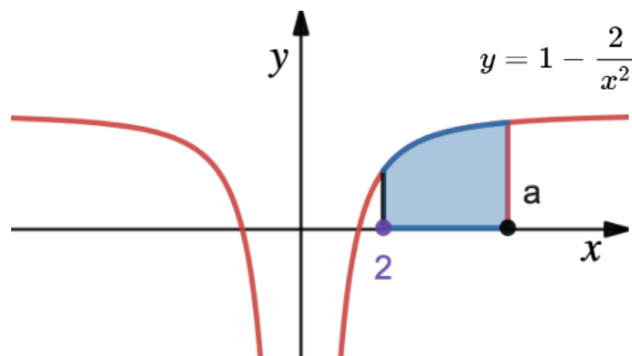
a. Find the exact value of x when $y = 8$

(3)

b. Find the value of the gradient at the point $(\ln 3, 35)$

(2)

14.



The figure shows the curve $y = 1 - \frac{2}{x^2}$

a. Write down the equations of the asymptotes to the curve

(2)

b. Find the exact coordinates of the points when the curve cuts the axes.

(2)

Given $\int_2^a \left(1 - \frac{2}{x^2}\right) dx = 2.4$, $a > 2$

c. Find the value of a .

(6)