Aleph-4 Speed Round

Do not open the test until instructed to do so.

Information:

- The Speed Round contains 30 questions that you will be given 30 minutes to answer.
- Do not expect to be able to solve every question. Likewise do not be afraid to skip questions. The questions are only roughly but not strictly in order of difficulty.
- Please write legibly and in the specified form; all answers that cannot be read or are in the incorrect form will be considered wrong.

Instructions:

- You will receive one answer sheet.
- On the answer sheet, make sure to LEGIBLY write your name, Competition ID, and the school that you attend.
- Only answers on the answer sheet will be graded, not answers in the test booklet.
- Only basic writing implements (i.e. pencils, pens, erasers) are allowed. All other tools (e.g. calculators, compasses, rulers, etc…) or external help is prohibited.

Good luck and have fun! If you have any questions, raise your hand.

1. If two of the angles of a right triangle add up to 104 degrees, what is the measure of the smallest angle of the triangle, in degrees?

2. Jimmy lives 54 miles from Timmy. If Jimmy bikes to Timmy's house at an average speed of 9 miles per hour, how long does it take for Jimmy to get to Timmy's house, in hours?

3. If the intersection of the lines y = 3x + 5 and y = -4x + 19 is at the point *(a, b)*, what is *a* + *b*?

4. A fan has x distinct blades that rotate when the power is turned on. Shelly watches a point at the top of a fan and notes whenever a blade passes by. She finds herself counting a blade every ⅓ second. If the fan rotates once every 3 seconds at a constant speed, how many blades are on the fan?

5. Mr. T needs 10 more dollars to buy an apple pie. Mr. S is short as well, except by only two dollars. They try to pool together their money to buy one apple pie to share. They are still one dollar short. How much did the apple pie cost?

6. Minesweeper can have a maximum of 8 "mines" surrounding one tile, as a square can have at most 8 congruent squares adjacent or diagonal to it. A 3-dimensional minesweeper would use cubes as tiles, with every cube touching or diagonal to it possibly having mines, thus having 26 possible places for mines. What is the maximum number of mines surrounding one hypercube in a 4-dimensional minesweeper?

7. Given that 5555^2 is 30858025, what is 5556^2?

8. If A, B, C, D are distinct numbers chosen from the set {-8, -6, -4, 0, 2, 5, 7}. What is the minimum possible *real* value of $\frac{ABC}{D}$?

9. City A and B are some distance apart. Sam and Alex live in cities A and B respectively. They both leave at 11:00 AM, traveling at different but constant speeds. Sam travels towards city B and arrives at 5:00 PM(same day) while Alex travels towards city A and arrives at 2:00 PM(same day). How many hours after departing did they cross paths?

10. Find the smallest of the two positive integers, neither of which has a 0 in their base-10 representation, whose product is 100,000.

11. Let $\frac{a}{b}$ be the probability of getting the best possible starting hand in poker such that a and b are positive, coprime integers. In other words, the probability of randomly selecting two aces when drawing from a standard 52-card deck without replacement. What is $a + b$?

12. Marius is choosing what to buy from a menu. There are 5 choices for the entree, 3 choices for the appetizer, and 8 choices for the drink. However, 3 items on the menu contain nuts, and Marius is allergic to nuts. What is the maximum possible number of different ways in which Marius can buy his meal if he must buy exactly one entree, appetizer, and drink?

13. A geometric sequence has the property that it satisfies the recurrence $f_n = 12f_{n-1} - 36f_{n-2}$. What is the common ratio of this sequence?

14. Recall that traditionally there are 10 number buttons on a calculator. These buttons are 0,1,2,3,4,5,6,7,8,9. What is the minimum number of calculator buttons that one must remove so that one cannot get the number 27 through both typing and the addition function of the calculator?

15. The Fibonacci numbers are defined as follows: $F_0 = 0$, $F_1 = 1$, $F_2 = 1$, $F_n = F_{n-1} + F_{n-2}$. From this definition, compute F_{-12}

16. The World Series is a best-of-seven playoffs, continuing until one team has won four games. For instance, if the Angels played the Braves, some distinct sequences of games that could happen are AAAA, BBBB, and ABABABA. How many possible sequences of matches in the World Series are there?

17. What is $(2024^5 + 4^5 + 66^5 + 192^5)\ mod\ 10$?

18. $60log_x 8 - 3log_8 x = 8$. What is the larger of the two possible values of x?

19. Evaluate: $\frac{1}{\sqrt{9}+\sqrt{13}} + \frac{1}{\sqrt{13}+\sqrt{17}} ... \frac{1}{\sqrt{725}+\sqrt{729}}$

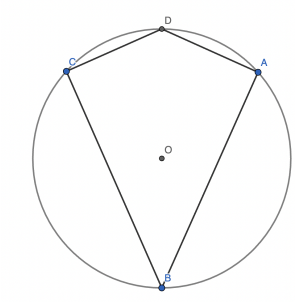
20. Determine the least positive value x(in degrees) such that $sin^2 x + tan^2 x + sec^2 x + cos^2 x = \frac{8}{3}$

21. If the volume of a regular octahedron with side length 4 can be written in the form $\frac{a}{b}\sqrt{c}$ when fully simplified, what is abc?

22. For this problem, let $x \in \mathbb{Z}$(i.e. restricted to integers). Notice that $E(x) = x \, mod \, 26$ has a range of {0,2,. . . , 25} which has a size of 26. Define $f(x) = (3x + 1) \, mod \, 26$ and $g(x) = (4x - 5) \, mod \, 26$. Let R_1 be the range of f(x) and R_2 be the range of g(x). What is the ratio between the sizes of the ranges, $\frac{|R_1|}{|R_2|}$?

23. A pâtissière is making a 50-layer ice cream cake. She has 115 flavors to choose from for each layer. There are N ways she can make this cake if all 50 layers have different flavors and the order in which she stacks the layers matters. How many trailing zeros does N have **in binary**(base-2)?

24. A quadrilateral ABCD is inscribed in a circle of diameter 25 such that $\angle A$ and $\angle C$ are both 90 degrees, AD = CD, and AB = 15. If the lengths of the diagonals of this quadrilateral are the roots of the quadratic equation: $f(x) = x^2 + ax + b$, what is the sum of the coefficients of f?



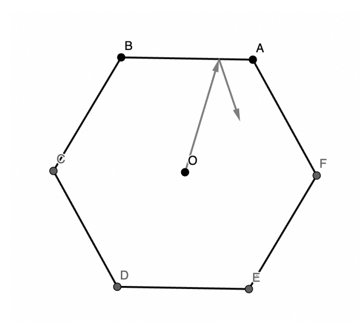
25. There are 101 tennis players registered for a tennis tournament. As is standard, this tennis tournament is held in a bracket/single-elimination style format. Since the number of people playing does not equal a power of two, byes (automatic advances) are given so that by the second round and onward, a power of two number of players is left after each round. In total, how many byes must be given?

26. An ant travels $\sqrt{3}$ m/s on the x-axis and $6\sqrt{2}$ m/s on the y-axis. It travels along the following path: It starts on the point (0,a) and travels in a straight line to the origin, then travels in a straight line to the point (b,0). If the ant's net displacement is 10 units, what y-coordinate should the ant start on to maximize the time it takes?

27. Canada has 12 books on a bookshelf. Four of them are great books, four of them are ordinary books, and the last four are terrible books. Unfortunately, he is clearing out at least three of the books to make space for new ones. How many distinct *proportions* of good, ordinary, and terrible books can he leave on the bookshelf? Define a proportion to be an ordered triple (a,b,c) where a,b,c are the percentages of great, ordinary, and terrible books on the bookshelf respectively.

28. If you inscribe a sphere in a regular tetrahedron, the fraction of the tetrahedron's volume that will be taken up by the sphere is equal to $\frac{\pi\sqrt{a}}{b}$, in simplest form. What is $a + b$?

29. A laser is shot from the center O of the regular hexagon ABCDEF which has side length of 1 unit. On its first bounce, it lands 3/16 units away from A. If $\sqrt{n}$ is the distance the laser travels before reaching another point(ABCDEFO), what is n?



30. $f(x) = \sqrt{3}sin(x) + 6cos(x) + 14sin(x + \frac{\pi}{6})$ can be rewritten as the function $A\,sin(x + \varphi)$. $sin(\varphi)$ can be expressed as the fraction $\frac{a}{b}$. What is $a + b$?