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Research Paper

The Partition Dimension of Daisy Graphs

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Abstract
The partition dimension of a graph is determined by minimum number of vertex partitions such that every vertex has different distances to the ordered partitions. A complete graph is very easy to determine its partition dimension because each vertex has the same distance to other vertices. However, what are the partition dimension if a complete graph

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1. INTRODUCTION

The concept of partition dimension arises from the study of resolving partitions, which are a generalization of resolving sets

used in metric dimension [1]. A partition $\Pi = \{P_1, P_2, \ldots, P_k\}$ of

the vertex set $V(G)$ of a graph G is said to be *resolving* if every

vertex $v \in V(G)$ has a unique vector of distances to the sets P_i. The minimum number k for which such a partition exists is called the *partition dimension* of G, denoted by pd(G) [2, 3, 4].

Daisy graphs, consisting of multiple cycles sharing a single central vertex, serve as interesting structures for studying various graph invariants, including partition dimension. These graphs resemble a daisy flower, where each cycle is a petal. Understanding the partition dimension of such graphs contributes to the broader study of network navigation, coding theory, and graph labeling problems.

2. METHODS

To determine the partition dimension of daisy graphs, we employed a combination of constructive and analytical approaches.

Consider the daisy graph D with central vertex v and n cycles

n 0

$C_{m^1}, C_{m^2}, \ldots, C_{m^n}$ such that each cycle shares only the vertex

v_0. Select a vertex u_1 from the first petal C_{m^1} and another vertex u_2 from the second petal C_m, both distinct from v_0. Construct
The methodology consists of the following steps:

3. RESULTS AND DISCUSSION

The primary result of this study is the determination of the partition dimension of daisy graphs. Through constructive and analytical methods, we have proven the following:

Theorem 3.1 *Let D_n be a daisy graph with $n \geq 2$ petals. Then*

the partition dimension of D_n is exactly 3,

that is, $pd(D_n) = 3$.

the partition:

$$\Pi = \{\{v_0\}, \{u_1\}, V(D_n) \setminus \{v_0, u_1\}\}.$$

We compute the distance vector $r(v|\Pi)$ for each $v \in V(D_n)$ and verify that all vectors are unique. The central vertex v_0 has distance zero to the first part, while any other vertex has a strictly positive distance. Vertices in the same petal have different distances to the chosen landmark u_1 or v_0. Vertices from different petals are distinguishable by their distances to $\{v_0\}$ and $\{u_1\}$. Hence, Π is a resolving partition, and $\mathrm{pd}(D_n) \leq 3$.

Furthermore, any partition with only two parts fails to distinguish all vertex pairs in general cases (especially symmetric positions on different petals), which implies $\mathrm{pd}(D_n) \geq 3$. Therefore, $\mathrm{pd}(D_n) = 3$.

This result demonstrates that despite the increasing number of petals and the overall size of the graph, the partition dimension of daisy graphs remains constant at 3. This invariance is a consequence of the structural symmetry and centralized connec- tivity of daisy graphs. The central vertex acts as a global hub, ensuring that all other vertices are within a small radius.

The key insight is that selecting landmarks (partition subsets) from different petals allows effective discrimination between vertices due to their unique shortest-path distances. The third part of the partition acts as a residual class, and only two well- chosen reference vertices (along with the central vertex) suffice to resolve the entire graph.

In contrast to other graph classes (such as grids or trees), where the partition dimension can grow with graph size, daisy graphs exhibit bounded partition dimension independent of the number of petals. This property may be advantageous in applications such as sensor networks or chemical structure analysis, where efficient localization is essential.

The result aligns with known behaviors of other graph families. For instance, the partition dimension of a wheel graph W_n is also 3 for $n \geq 6$, which is structurally similar to daisy graphs but with full edge connectivity between the center and the cycle. Daisy graphs, while sparser, retain the same resolving capability due to the central role of v_0.

4. CONCLUSIONS

We have shown that the partition dimension of any daisy graph with at least two petals is exactly three. This result contributes to the understanding of metric-based graph parameters in complex but structured graph classes. Future work may involve examining weighted daisy graphs or generalizations involving more complex petal intersections.

5. ACKNOWLEDGEMENT

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