# Triangles and Coordinate Proofs

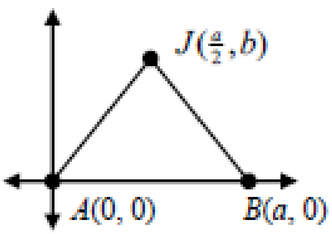
SOL G.6 (2016)

Coordinate proofs employ a combination of algebra and the coordinate plane to prove geometric concepts.

Useful hints:

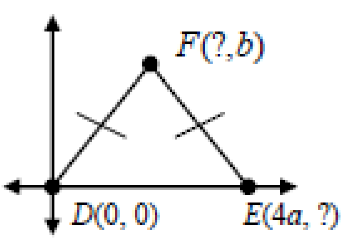
- If the $(x, y)$ coordinates are not known, variables such as $a$, $b$, $c$...may be substituted.
- Always use the origin as a vertex or center of the triangle.
- Always place at least one side of the triangle along an axis.
- Place the figure in the first quadrant whenever possible.

**Example 1: Position and label isosceles $\Delta ABJ$ on a coordinate plane so that base $\overline{AB}$ is $a$ units long.**




1. **Position vertex $A$ at (0, 0).**
2. **$\overline{AB}$ is the base, so position it along the $x$-axis. Since $\overline{AB}$ is $a$ units long, vertex $B$ will have an $x$-coordinate of $a$. This shows that it is $a$ units away from (0, 0). The $y$-coordinate is 0, because it is on the $x$-axis.**
3. **Position vertex $J$ so that the triangle appears isosceles. Since the sides are congruent, $J$'s $x$-coordinate will be exactly halfway between 0 and $a$ (*i.e.* $(\frac{a}{2}, b)$) and the $y$ coordinate will be $b$, an unknown variable.**

**Example 2: Find the missing coordinates of isosceles $\Delta DEF$.**




1. **The missing $y$-coordinate for vertex $E$ is 0, because the point is on the $x$-axis.**
2. **For vertex $F$, note that this is an *isosceles* triangle, and $F$ is the *vertex angle*. That means that its $x$-coordinate is the midpoint of $DE$. Evaluate $\frac{0+4a}{2}$ to find $F$'s $x$-coordinate.**

$E(4a, 0)$
$F(2a, b)$

## Practice

**Position and label each triangle on the coordinate plane.**

1. isosceles $\Delta FIG$ with base $\overline{FG}$ that is $c$ units long

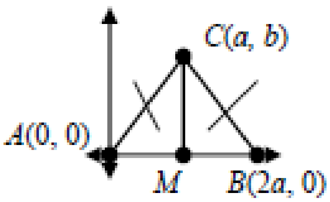
2. right $\Delta RAN$ with hypotenuse $\overline{AN}$, $RA = \frac{1}{2}NR$, and $NR$ is $a$ units long

3. equilateral $\Delta CAR$ with base $\overline{CA}$ that is $4a$ units long

## Coordinate Proof

Once a figure has been place on the coordinate plane, coordinates can be substituted into the Distance Formula, Midpoint Formula, and Slope Formula to prove statements true. Algebra is vital to this process, because most of the coordinates are expressed with variable expressions rather than real numbers.

**Example 3: Write a coordinate proof to prove that a line bisecting the vertex angle of an isosceles triangle is perpendicular to the base of the triangle.**




1. **Begin by positioning an isosceles triangle on the coordinate plane. Bisect the vertex angle with a segment that extends to the base of the triangle at *M*.**

Midpoint Formula:

$$M\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$M\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$M(a,0)$$

2. **Use the midpoint formula to evaluate the coordinates of *M*.**

Slope Formula:

$$m = \frac{y_2-y_1}{x_2-x_1}$$

$\overline{CM}$ : $m = \frac{0-b}{a-a} = \frac{-b}{0}$ (undefined)

$\overline{AB}$ : $m = \frac{0-0}{2a-0} = \frac{0}{2a} = 0$

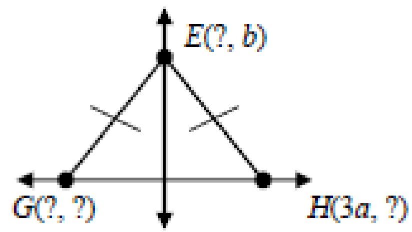
3. **If the slope of $\overline{CM}$ is the opposite reciprocal of the slope of $\overline{AB}$, then the lines must be perpendicular. Use the slope formula to prove that the segments are perpendicular.**

If the slopes of two lines are opposite inverses, then those lines are perpendicular. 0 and undefined slope *are* opposite inverses, therefore $\overline{CM} \perp \overline{AB}$.

4. **Write a brief explanation of how the algebra proves the conclusion is true.**

**Find the missing coordinates of each triangle.**

4.

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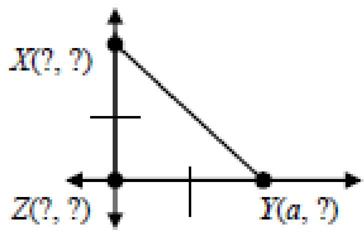


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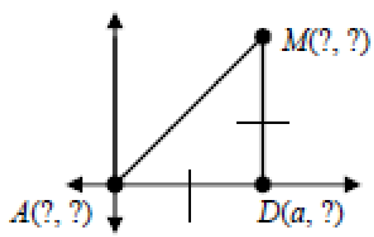


7. **Write a coordinate proof to prove that the segment joining the midpoints of the legs of an isosceles right triangle is parallel to the hypotenuse.**