ESCUELA POLITÉCNICA NACIONAL

INGENIERÍA EN SISTEMAS INFORMÁTICOS Y COMPUTACIÓN

CÁLCULO VECTORIAL

NOMBRE: CHRISTIAN CHICAIZA

Actividad # 5

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TEMA: 2015 - 11 - 05

FECHA: Longitud de Arco y Curva

Determine la longitud de curva.

2) $r(t) = (2t, t^2, \frac{1}{3}t^3) \quad 0 \leq t \leq 1$

$r'(t) = (2, 2t, t^2)$

$|r'(t)| = \sqrt{2^2 + (2t)^2 + t^2} = \sqrt{(2+t^2)^2}$

$|r'(t)| = 2+t^2$

$L = \int_0^1 |r'(t)| = \int_0^1 2+t^2 dt = \left[2t + \frac{1}{3}t^3\right]_0^1 = \frac{7}{3}$

Encuentre la longitud de la curva correcta a cuatro lugares decimales

a) $r(t) = \langle \operatorname{sen} t, \cos t, \tan t \rangle \quad 0 \leq t \leq \pi/4$

$r'(t) = \langle \operatorname{sen} t, \cos t, \sec^2 t \rangle$

$|r'(t)| = \sqrt{\cos^2(t) + (-\operatorname{sen}^2 t) + \sec^4 t}$

$|r'(t)| = \sqrt{1 + \sec^4 t}$

$L = \int_0^{\pi/4} \sqrt{1+\sec^4 t} \, dt$

$L = 1,2781$

Reparametrice la curva con respecto a la longitud de arco medida desde el punto $t=0$ en la dirección en que se incrementa t.

13) $r(t) = 2t\,i + (1-3t)j + (5+4t)K$

$r'(t) = 2i + (-3)j + 4K$

$\frac{ds}{dt} : r'(t) = \sqrt{4+9+16}$

$r'(t) = \sqrt{29}$

$S = \int_0^t |r'(u)|du$

$s = \int_0^t \sqrt{29}\,du$

$t = \frac{1}{\sqrt{29}} s$

$r(t(s)) = \frac{2}{\sqrt{29}} s\,i + \left(1 - \frac{3}{\sqrt{29}}s\right)j + \left(5 + \frac{4}{\sqrt{29}}s\right)K$

14) $r(t) = e^{2t}\cos 2t\; i + 2j + e^{2t} \operatorname{sen} 2t\; K$

$r'(t) = 2e^{2t}(\cos 2t - \sin 2t) + 0j + 2e^{2t}(\cos 2t + \sin 2t)K$

$\frac{ds}{dt} = |r'(t)| = 2e^{2t}\sqrt{(\cos 2t - \sin 2t)^2 + (\cos 2t + \sin 2t)^2}$

$= 2e^{2t}\sqrt{2\cos^2(2t + 2\sin^2(2t)}$

$= 2e^{2t}\sqrt{2\,[\cancel{\cos^2 2t + \sin^2(2t)}]^{1}}$

$= 2e^{2t}\sqrt{2}$

$S = s(t) = \int_0^t |r'(u)|du = \int_0^t 2e^{2u}\sqrt{2}\,du$

$= \left[\sqrt{2}\,e^{2u}\right]_0^t$

$= \sqrt{2}(e^{2t} - 1) \Rightarrow \frac{s}{\sqrt{2}} + 1 = e^{2t}$

$t = \frac{1}{2}\ln\left(\frac{s}{\sqrt{2}} + 1\right)$

$r(t(s)) = e^{2\left(\frac{1}{2}\ln\left(\frac{s}{\sqrt{2}}+1\right)\right)} \cos 2\left(\frac{1}{2}\ln\left(\frac{s}{\sqrt{2}}+1\right)\right) i + 2j + e^{2\left(\frac{1}{2}\ln\left(\frac{s}{\sqrt{2}}+1\right)\right)} \sin 2\left(\frac{1}{2}\ln\left(\frac{s}{\sqrt{2}}+1\right)\right)K.$

Determine los vectores unitario tangente y normal unitorio $T(t)$ y $N(t)$

Aplique la fórmula 9 para calcular la curvatura $\left(k(t) = \dfrac{|T'(t)|}{|r'(t)|} \right)$

17) $r(t) = \langle 2\,\text{sen}\,t,\ 5t,\ 2\cos t \rangle$

a) $r'(t) = \langle 2\cos t,\ 5,\ -2\,\text{sen}\,t \rangle$

$$|r'(t)| = \sqrt{4\cos^2 t + 25 + 4\sin^2 t} = \sqrt{25 + 4(\cancel{\cos^2 t + \sin^2 t})}$$

$$= \sqrt{29}$$

$$T(t) = \frac{r'(t)}{|r'(t)|} = \frac{1}{\sqrt{29}} (2\cos t,\ 5,\ -2\sin t)$$

$$T'(t) = \frac{1}{\sqrt{29}} \langle -2\,\text{sen}\,t,\ 0,\ -2\cos t \rangle \Rightarrow |T'(t)| = \frac{1}{\sqrt{29}} \sqrt{4\sin^2 t + 0 + 4\cos^2 t}$$

$$|T'(t)| = \frac{\sqrt{4}}{\sqrt{29}} = \frac{2}{\sqrt{29}}$$

$$N(t) = \frac{T'(t)}{|T'(t)|} = \frac{1/\sqrt{29}}{2/\sqrt{29}} \langle -2\sin t,\ 0,\ -2\cos t \rangle$$

$$N(t) = \langle -\sin t,\ 0,\ -\cos t \rangle$$

b) $$k(t) = \frac{|T'(t)|}{|r'(t)|}$$

$$k(t) = \frac{2/\sqrt{29}}{\sqrt{29}} = 2/29$$

19) $r(t) = \langle \sqrt{2}\,t,\ e^t,\ e^{-t} \rangle$

$$r'(t) = \langle \sqrt{2},\ e^t,\ -e^{-t} \rangle$$

$$|r'(t)| = \sqrt{2 + e^{2t} + e^{-2t}} = \sqrt{(e^t + e^{-t})^2} = e^t + e^{-t}$$

$$T(t) = \frac{r'(t)}{|r'(t)|} = \left[\frac{1}{e^t + e^{-t}} \left(\sqrt{2},\ e^t,\ -e^{-t} \right) \right] \frac{e^t}{e^t}$$

$$= \frac{1}{e^{2t} + 1} \left(\sqrt{2}\,e^t,\ e^{2t},\ -1 \right)$$

$$T'(t) = \frac{1}{e^{2t} + 1} \langle \sqrt{2}\,e^t,\ 2e^{2t},\ 0 \rangle - \frac{2e^{2t}}{(e^{2t} + 1)^2} \langle \sqrt{2}\,e^t,\ e^{2t},\ -1 \rangle$$

$$= \frac{1}{(e^{2t} + 1)^2} \langle \sqrt{2}\,e^t (1 - e^{2t}),\ 2e^{2t},\ 2e^{2t} \rangle$$

$$|T'(t)| = \frac{1}{(e^{2t}+1)^2}\sqrt{2e^{2t}(1-2e^{2t}+e^{4t})+4e^{4t}+4e^{4t}}$$

$$|T'(t)| = \frac{\sqrt{2}e^t(1+e^{2t})}{(e^{2t}+1)^2} \Rightarrow |T'(t)| = \frac{\sqrt{2}\,e^t}{e^{2t}+1}$$

$$N(t) = \frac{T'(t)}{|T'(t)|} = \left(\frac{e^{2t}+1}{\sqrt{2}e^t}\right)\left(\frac{1}{(e^{2t}+1)^2}\right)\langle \sqrt{2}e^t(1-e^{2t}), 2e^{2t}, 2e^{2t}\rangle$$

$$= \frac{1}{e^{2t}+1}\langle 1-e^{2t}, \sqrt{2}\,e^t, \sqrt{2}e^t\rangle$$

b) $$\kappa(t) = \frac{|T'(t)|}{|r'(t)|} = \frac{\sqrt{2}\,e^t}{e^{2t}+1}\cdot\frac{1}{e^t+e^{-t}} = \frac{\sqrt{2}\,e^t}{e^{3t}+2e^t+e^{-t}}$$

$$\kappa(t) = \frac{\sqrt{2}\,e^{2t}}{(e^{2t}+1)^2}$$

Aplique el teorema 10 para calcular la curvatura

21) $r(t) = t^2 i + t\,k$

$r'(t) = 2t\,i + k$

$r''(t) = 2\,i$

$|r'(t)| = \sqrt{(2t)^2+0^2+1^2} = \sqrt{4t^2+1}$

$r'(t) \times r''(t) = 2\,j$

$|r'(t) \times r''(t)| = 2$

$$\kappa(t) = \frac{|r'(t)\times r''(t)|}{|r'(t)|^3} = \frac{2}{(\sqrt{4t^2+1})^3} = \frac{2}{(4t^2+1)^{3/2}}$$

23) $r(t) = 3t\ i + 4\,\text{sen}\,t\ j + 4\cos t\ K$

$r'(t) = 3i + 4\cos t\ j + (-4\,\text{sen}\,t)K$

$r''(t) = 0\,i + (-4\,\text{sen}\,t)j + (-4\cos t)k$

$|r'(t)| = \sqrt{9 + 16\cos^2 t + 16\sin^2 t} = \sqrt{9 + 16(\cancel{\cos^2 t + \text{sen}^2 t})^{1}} = \sqrt{25} = 5$

$r'(t) \times r''(t) = -16i + 12\cos t\ j - 12\sin t\ K$

$|r'(t) \times r''(t)| = \sqrt{256 + 144\cos^2 t + 144\sin^2 t} = \sqrt{256 + 144(\cancel{\cos^2 t + \sin^2 t})^{1}}$

$= \sqrt{400} = 20$

$$K(t) = \frac{|r'(t) \times r''(t)|}{|r'(t)|^3} = \frac{20}{5^3} = \frac{20}{125} = \frac{4}{25}$$

Mediante la fórmula 11 determine la curvatura.

27) $y = 2x - x^2$

$f(x) = 2x - x^2$

$f'(x) = 2 - 2x$

$f''(x) = -2$

$$k(x) = \frac{|f''(x)|}{[1 + (f'(x))^2]^{3/2}} = \frac{-2}{[1 + (2-2x)^2]^{3/2}} = \frac{2}{[4x^2 - 8x + 5]^{3/2}}$$

29) $y = 4x^{5/2}$

$f(x) = 4x^{5/2}$

$f'(x) = 10x^{3/2}$

$f''(x) = 15x^{1/2}$

$$K(x) = \frac{|f''(x)|}{[1 + (f'(x))^2]^{3/2}} = \frac{15x^{1/2}}{[1 + (10x^{3/2})^2]^{3/2}} = \frac{15\sqrt{x}}{(1 + 100x^3)^{3/2}}$$