Hypercube unfoldings 8, 9, 56, 62, 67, 71, 83, 99, 117, 144, 177, 193, 220, and 225 comprise a family of unfoldings that can all be shown to tile space in a nearly identical manner.

From here on, we will define a cube as "at" a point if that is the point on the cube with the smallest X, Y, and Z coordinates.

Let us define an initial placement for each unfolding in coordinate space. This placement is simply the location of each of its eight component cubes. Each of these unfoldings can be placed so four of the cubes are at (0,1,1); (1,1,1); (2,1,1); and (3,1,1). Table 1 lists the placements of the four remaining cubes.

TABLE 1

Unfolding	Placements
8	(0,0,2); (0,1,2); (3,1,0); (3,2,0)
9	(1,0,2); (1,1,2); (1,2,2); (0,1,0)
56	(0,0,2); (0,1,2); (0,2,2); (2,1,0)
62	(0,0,2); (0,1,2); (0,2,2); (0,1,0)
67	(0,0,2); (0,1,2); (1,1,0); (1,2,0)
71	(0,0,2); (0,1,2); (2,1,0); (2,2,0)
83	(1,0,2); (1,1,2); (1,1,0); (1,2,0)
99	(1,0,2); (1,1,2); (1,2,2); (2,1,0)
117	(1,0,2); (1,1,2); (2,1,0); (2,2,0)
144	(0,0,2); (0,1,2); (0,2,2); (1,1,0)
177	(1,0,2); (1,1,2); (1,2,2); (1,1,0)
193	(0,0,2); (0,1,2); (0,1,0); (0,2,0)
220	(0,0,2); (0,1,2); (0,2,2); (3,1,0)
225	(1,0,2); (1,1,2); (1,2,2); (3,1,0)

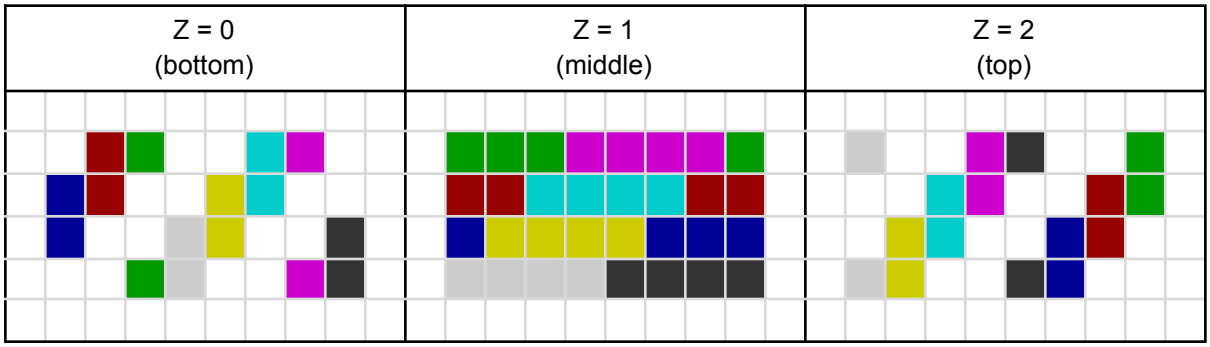
To tile space with each of these unfoldings, translate by the corresponding vector for all integer triples (a, b, c). Table 2 lists these corresponding vectors.

TABLE 2

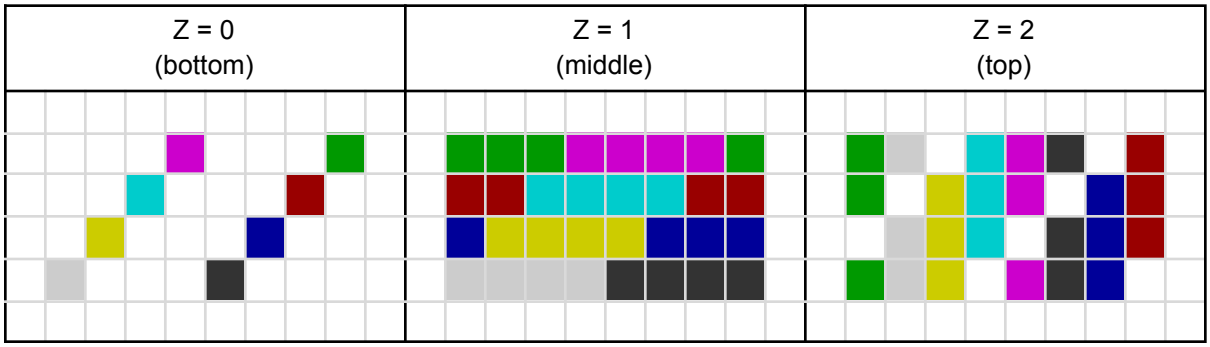
Unfolding	Translation vector
8	(4a+b, b, 2c)
9	(4a+b-c, b, 2c)
56	(4a+b, b, 2c)
62	(4a+b+2c, b, 2c)
67	(4a+b+2c, b, 2c)
71	(4a+b+c, b, 2c)
83	(4a+b-c, b, 2c)
99	(4a+b+c, b, 2c)
117	(4a+b+2c, b, 2c)
144	(4a+b+c, b, 2c)
177	(4a+b+2c, b, 2c)
193	(4a+b-c, b, 2c)
220	(4a+b-c, b, 2c)
225	(4a+b, b, 2c)

Diagrams of each tiling are shown below. The patterns can be tiled in the x direction every 8 units, in the y direction every 4 units, and in the z direction every 2 units. Cubes of the same color comprise one unfolding.

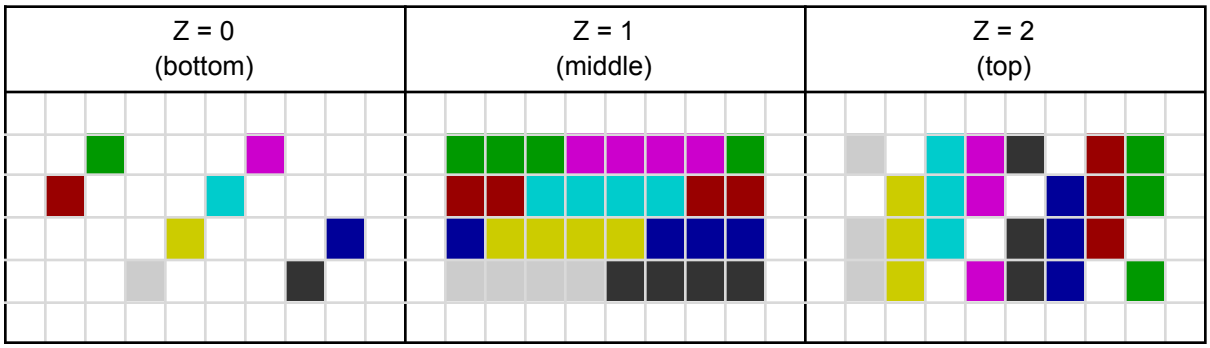
#8


Z = 0
(bottom)
Z = 1
(middle)
Z = 2
(top)

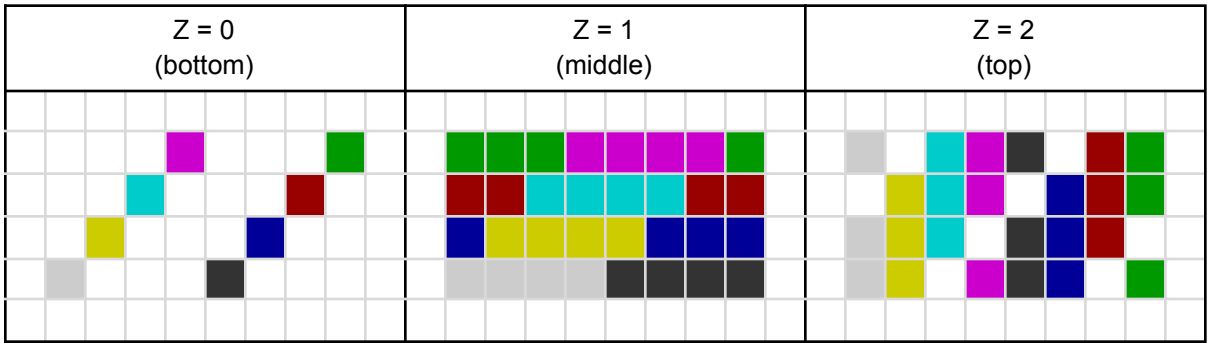
#9


Z = 0
(bottom)
Z = 1
(middle)
Z = 2
(top)

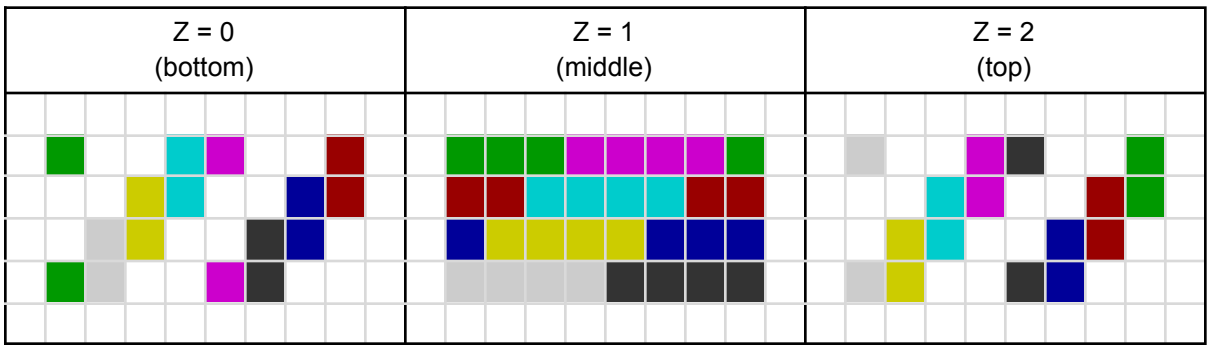
#56


Z = 0
(bottom)
Z = 1
(middle)
Z = 2
(top)

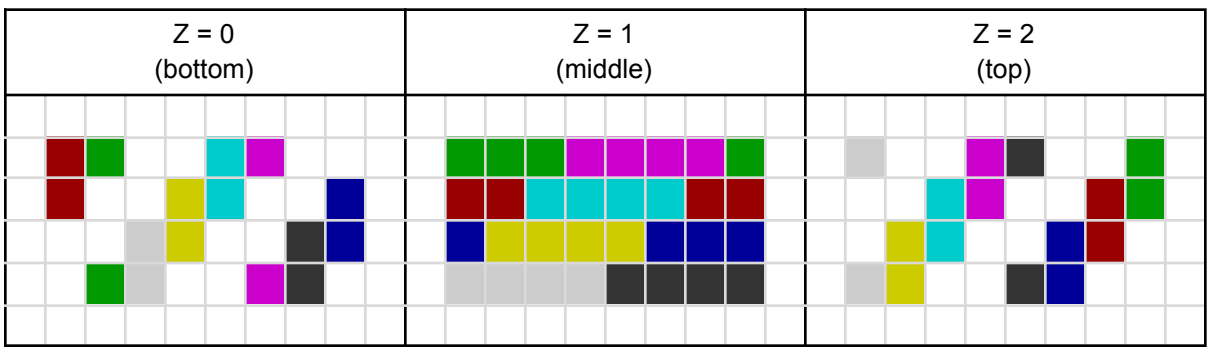
#62


Z = 0
(bottom)
Z = 1
(middle)
Z = 2
(top)

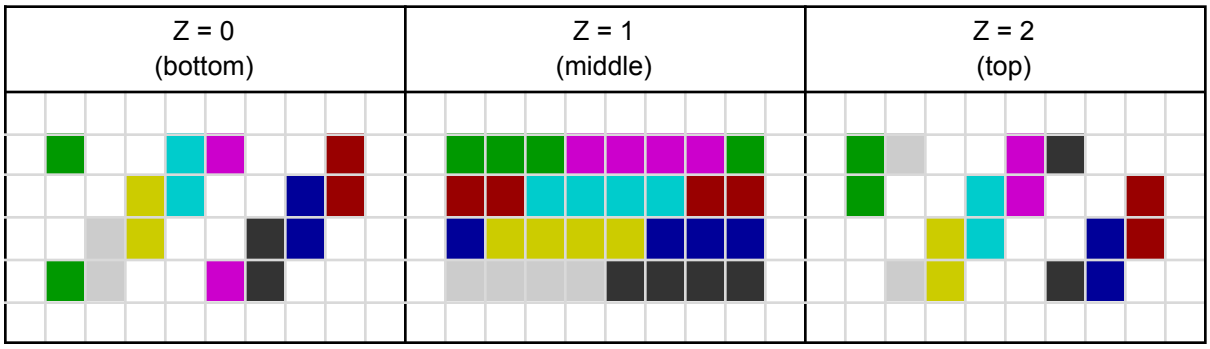
#67


Z = 0
(bottom)
Z = 1
(middle)
Z = 2
(top)

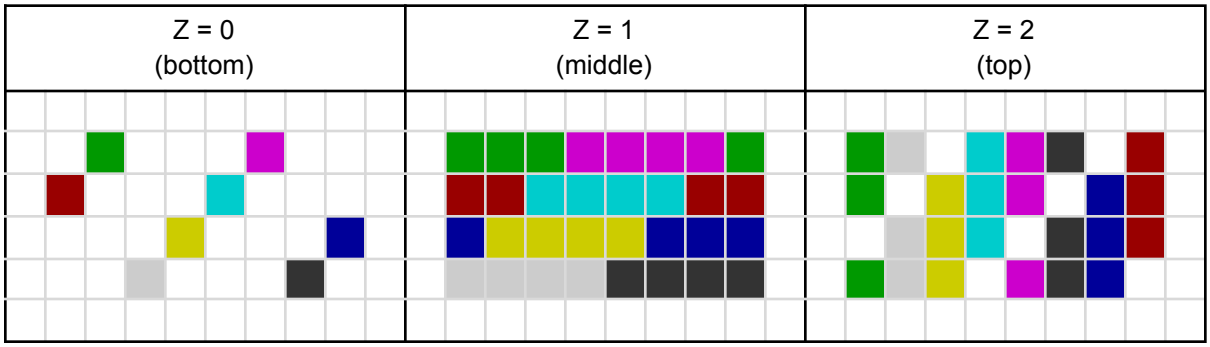
#71


Z = 0
(bottom)
Z = 1
(middle)
Z = 2
(top)

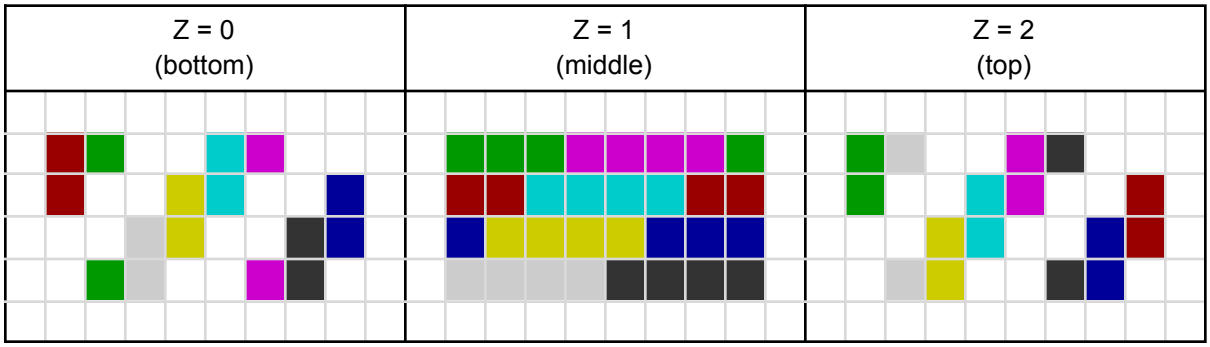
#83


Z = 0
(bottom)
Z = 1
(middle)
Z = 2
(top)

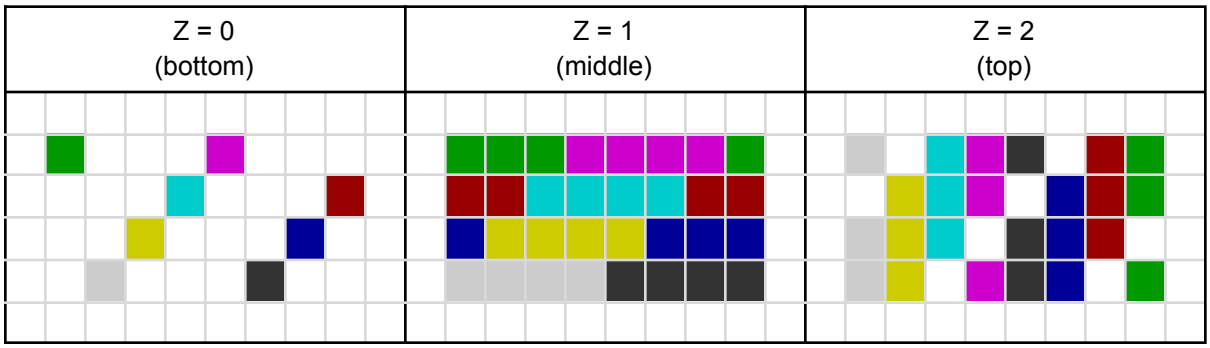
#99



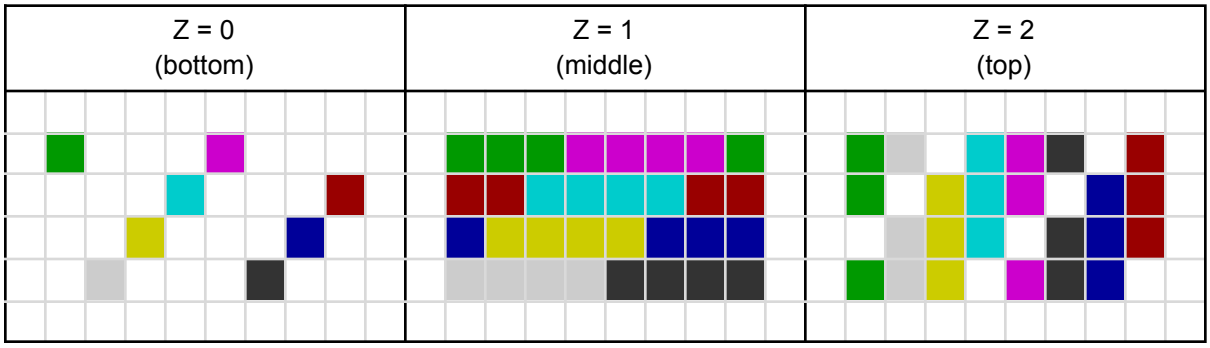
#117



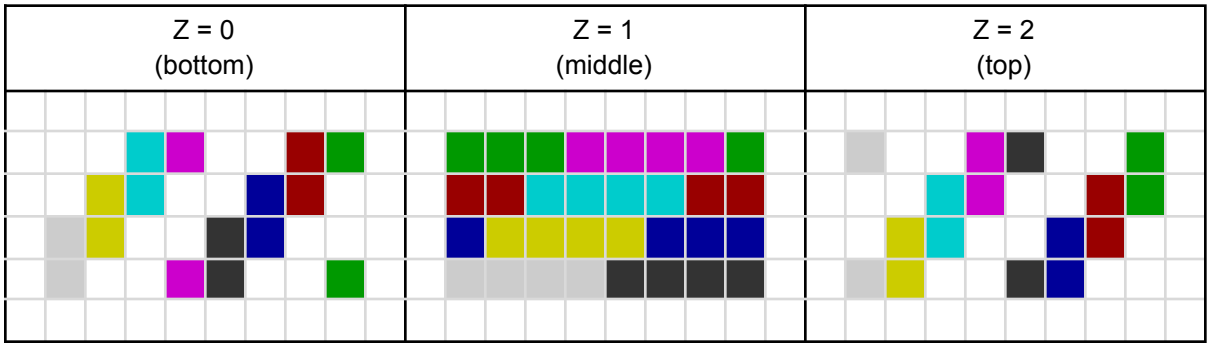
#144



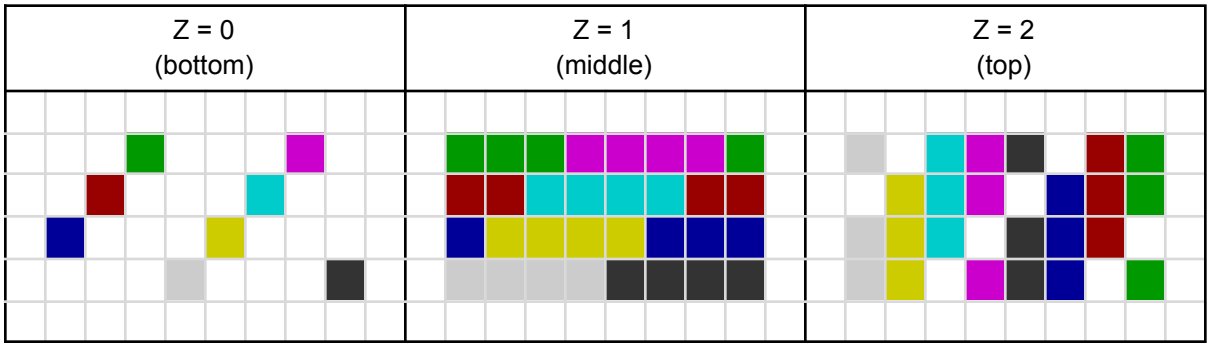
#177



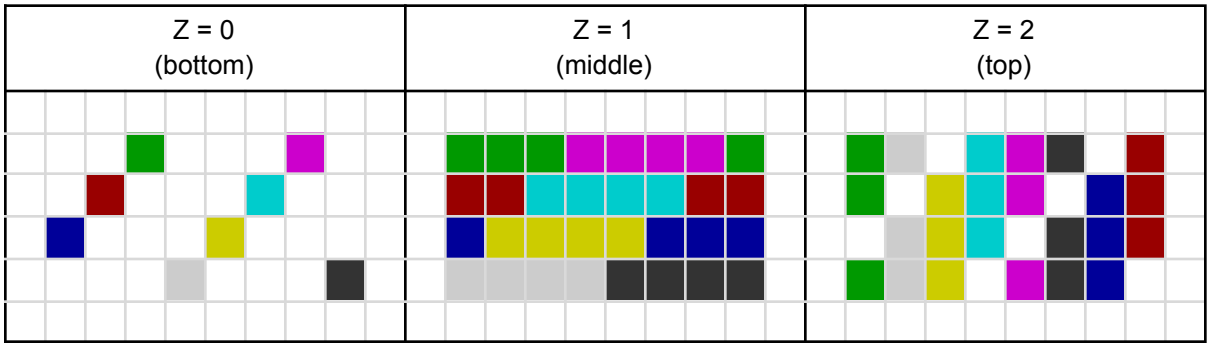
#193


Z = 0
(bottom)
Z = 1
(middle)
Z = 2
(top)

#220


Z = 0
(bottom)
Z = 1
(middle)
Z = 2
(top)

#225


Z = 0
(bottom)
Z = 1
(middle)
Z = 2
(top)