

Leading a Math Student Circle: Guide for Facilitators

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Introduction

Math Circles originated in Eastern Europe and spread to the United States in the 1990s. Math Circles provide an alternative to traditional math classes, serving as a way to bridge the gap between the way mathematics is often taught in schools and the way the discipline is practiced by professional mathematicians. Whereas students in a traditional math class are typically passive recipients of content knowledge, copying procedures demonstrated by the teacher, participants in a Math Circle are active in constructing their own strategies, predictions, and mathematical understanding.

A Math Circle consists of one or more facilitators and a group of participants. **Participants in a Math Circle can be any age, but this guide focuses specifically on Math Circles for children aged 4 through 18.**

There are many different types of Math Circles, and many different ways to run a Math Circle. For the purposes of this guide, however, we will assume that the primary goal of a Math Circle is *to support participants in developing a mindset and an approach that allows them to be successful in a variety of different problem-solving environments*. In particular, this entails:

- Supporting participants in developing their ability to think creatively and critically when solving problems
- Supporting participants in developing their sense of agency as problem solvers
- Supporting participants in developing resilience in the face of challenge

Importantly, we do not assume that learning specific mathematical content knowledge is a primary goal of a Math Circle. This is not to say that participants do not learn math content in a Math Circle, but the emphasis is more on the process of exploration and discovery rather than the content.

Given this prioritization of the development of participants' mindset and approach to problem-solving over acquisition of content knowledge, Math Circles typically have a different structure and set of pedagogical practices than traditional math classes. In a Math Circle, a facilitator provides scaffolding for participants' exploration by posing questions, presenting challenges, or engaging participants in exploring mathematical phenomena. The facilitator gives participants the opportunity to explore these questions / challenges / phenomena until the participants arrive at a solution or proof (though the facilitator may provide guidance when appropriate). This can run for just an hour, or it can extend over multiple meetings, depending

on the participants' interest in exploring the topic.

Rather than demonstrating procedures or strategies to participants, the facilitator makes space for participants to explore, think, struggle, and learn. The facilitator aims to create an open, discovery-based environment where participants have freedom to approach and solve problems in a variety of different ways. While this may sound like a simple idea, it can be challenging to make it work. Fortunately, there are specific strategies and techniques that can help to create an environment where all participants make productive use of the space they are given and feel a sense of belonging and reward. This guide discusses some of these strategies and techniques.

How to use this guide

This guide is designed to be flexible and practical for Math Circle facilitators at any stage of experience. You can read it from start to finish for a comprehensive understanding of facilitation techniques, or you can focus on specific sections that address your immediate needs. The guide is organized around five key topics that are fundamental to the success of a Math Circle:

- *Establishing trust and connection*
- *Ensuring equitable participation*
- *Promoting agency among participants*
- *Giving guidance*
- *Avoiding and addressing disengagement and disruption*

Each category contains several general guidelines (in bold). For each of these guidelines, there is an explanation of why this guideline is important for a successful Math Circle, as well as specific examples of ways to implement these general guidelines.

Within each section, we have included a difficulty rating system to help you prioritize your learning. Practices are assigned ☆ or ☆☆ to indicate their level of complexity and difficulty. Practices marked with ☆ are accessible for new facilitators, while practices marked with ☆☆ may be somewhat more challenging to implement. (That being said, don't be afraid to try out ☆☆ practices even if you are just starting out — the more you try, the more you will learn and improve.) This structure allows you to tailor your approach to leading a Math Circle by building confidence and skill a little at a time.

This guide does not include specific math curricula or lesson plans, focusing instead on facilitation techniques to foster curiosity, collaboration, and critical thinking. By emphasizing adaptable approaches, it equips facilitators to create dynamic, participant-driven Math Circles tailored to their group's needs. The [Appendix](#) is an annotated guide to free online sources where you can find topics, manipulatives, and resources to use during Math Circles, as well as a list of books that are helpful for learning more about mathematics as a dynamic practice.

Establishing Trust and Connection

Establishing trust and connection is a crucial – and often overlooked – part of leading a Math Circle, especially when working with participants who have negative feelings about math and low self esteem regarding their ability to do math. It's important for participants to feel like the facilitator cares about them and will support them. Participants may have experience in math classes where this was not the case. Building a sense of trust and connection both with the facilitator and with each other facilitates a feeling of belonging for participants.

Here are some specific guidelines and practices that may be useful:

☆/☆☆ **Create opportunities for you to get to know the participants and for them to get to know each other**

Why is this important?

- A sense of connection and community will increase participants' level of comfort and willingness to engage and share.
- Giving participants an opportunity to share about their personal interests outside of math is a great way to foster connections.

Examples:

- ☆ Use icebreakers at the beginning of the session. For example:
 - Personal questions ("What is a hobby that you have?" "If you could be any animal, which would you be, and why?" etc.)
 - Two truths and a lie (have participants share two truths and one lie about themselves and have other participants try to figure out which is which)
 - One gotta go (present four different members of some category and have participants vote for the one they would be most willing to do without — for example, Pixar movies, different types of candy, pizza toppings, seasons, etc.)
- ☆ Learn participants' names (including how to pronounce them) and their pronouns.
- ☆☆ When searching for examples, use participants' interests or to ask them to make choices: "Tina, who is your favorite movie star?" "Johnny Depp." "Okay, Johnny Depp is driving to Hollywood at 30 miles per hour..."
- ☆☆ Ask participants to share about their lives outside the classroom (hobbies, siblings, pets, etc.). Look for opportunities to follow up with them about these things. (For example, if a participant tells you that they have a music performance coming up, make a note to ask them how it went afterwards.) You can also find and create math problems and puzzles around these interests.

☆ **Be kind, enthusiastic, encouraging, and supportive**

Why is this important?

- Especially for participants who have had negative experiences around math, it is important to bring positive energy and make them feel like you are there to support them rather than to judge them.

Specific examples:

- When a participant solves a problem or figures something out, show enthusiasm for their discovery (possibly with a high five or fist bump).
- Praise participants for their perseverance if they are struggling: "You're doing a great job of sticking with this even though it's challenging."
- At the same time, be conscious to avoid toxic positivity, and over-encouraging. Be sure your positivity is genuine, relevant, and honest.

☆☆ **Show respect for ideas that turn out to be incorrect**

Why is this important?

- Sometimes participants feel discouraged after sharing an idea that turns out to be incorrect. In order to remain willing to participate and share their ideas, participants need to trust that they won't be made to feel bad about themselves if they have an answer or idea that is incorrect.
 - This does not mean that all ideas are correct, only that it is not bad to have an idea that turns out to be incorrect.

Specific examples:

- Praise participants for sharing their ideas regardless of whether or not they are correct: "That was a good job of sharing your thinking."
- When a participant shares an idea, listen carefully and try to follow their reasoning. This can help you to understand and appreciate their thinking, and it can make it easier for you to find value in their idea even if it is not completely correct.
- If a participant shares an idea that turns out to be incorrect, focus on what they (and the rest of the group) can learn from it. For example: "Even though this idea turned out to be incorrect, it was very helpful, because now we can see ..."

Ensuring Equitable Participation

An equitable learning environment is one that works towards an ideal in which all participants are heard and respected equally. By creating extra space for participants who are less confident, have had less opportunity, or are members of marginalized groups, the facilitator promotes inclusivity and empowerment, ensures a diversity of perspective, and cultivates teamwork and open-mindedness.

Here are some specific guidelines and practices that may be useful:

☆/☆☆ **Encourage collaboration as opposed to competition**

Why is this important?

- A competitive environment may deter participants from sharing their thoughts and taking risks, while a collaborative environment will applaud and reward ideas, regardless of whether or not they turn out to be fruitful.

- Participants will see different perspectives as helpful to progressing toward a solution, rather than a threat to their original way of thinking.

Specific examples:

- ☆ Make clear to participants that they should not be racing to be the first to call out answers (and remind them of this whenever necessary).
- ☆ Use collaborative language:
 - "Can we think together to solve this problem?"
 - "Great, now we all know two different ways to solve the problem!"
- ☆☆ Let multiple participants explain one solution together; when one participant is explaining, you can have them stop after one or two points and then pass it off to another volunteer to continue.
- ☆☆ Encourage participants to discuss their thinking with each other, but make sure the discussion is not being dominated by one or two participants. (See below for more about how to prevent participants from dominating discussions.)
- ☆☆ Model the collaborative behavior you want to see from your participants: Ask questions like "What do you think?" that invite discussion and encourage all participants to share their thinking.

☆☆☆ Prevent one or a few participants from dominating the discussion

Why is this important?

- More vocal, more confident participants may tend to dominate discussions unless the facilitator takes steps to prevent this. When more vocal, more confident participants dominate discussions, it is difficult for less vocal, less confident participants to share their thinking when they want to, and it can also cause these less vocal, less confident participants to incline toward a more passive role in discussions.
- Making space for all participants to share their thinking conveys the notion that all participants' thoughts and contributions are equally valuable.

Specific examples:

- ☆ If a quieter participant is interrupted, step in and control the social situation so they have a chance to have their ideas heard.
- ☆ Thank more boisterous participants for their enthusiasm, as a way to segue into giving space for everyone to share. (Often these participants don't realize that they are dominating the discussion.) "I love your enthusiasm, and I really want to hear your idea, but right now is an opportunity to listen to and learn from your teammates, and then you'll get to share after."
- ☆ If possible, having multiple facilitators can make this process easier. If one participant really wants to continue engaging, then one facilitator can carry on the discussion with that participant, while another facilitator continues the activity with the group.
- ☆ If you're asking a "yes or no" question, have participants show their answers with their thumbs (up for "yes," down for "no," to the side for "not sure") so that all participants can share their thoughts at the same time. (Similarly, if you are asking a question like "Who do you think will win this game, Player 1 or Player 2?" you can have participants

show their answer with their fingers, i.e., one finger for Player 1 or two fingers for Player 2.)

- ☆ When asking questions to a group of participants, try to avoid asking for volunteers — when you ask for volunteers, the most vocal and confident participants will be more likely to answer, and quieter, less confident participants will be less likely to be involved.
 - When posing questions to a group, you could call on specific participants rather than asking for volunteers, making a particular effort to call on participants who haven't shared recently.
 - You could also call on participants in a specified order.
- ☆☆ That being said, there is value in creating an environment where participants can share in a more free-flowing manner, since this creates a greater sense of agency for participants. Pay attention to the dynamics of your group to see if you feel like it would work to call on volunteers (or even let participants share without being called on) without one or a few participants dominating the discussion.

☆☆ **Provide opportunities for participants to approach problems in different ways**

Why is this important?

- This allows participants to find the way to engage that feels the most natural to them, and will also help all participants discover new connections and strengthen their understanding of the concepts.

Specific examples:

- Use a whiteboard to present the problem and solution visually, using drawing, diagrams, figures, graphs, and charts.
- Explore different modalities, including:
 - Computer programs: Geogebra, Polypad, Python
 - Physical manipulatives: blocks, algebra tiles, Cuisenaire rods, paper and scissors, pattern blocks, colored tiles
 - Applied math: science, technology, engineering, economics, psychology
 - Math wrangles, discussions, storytelling, or looking at math through a historical context
- Present something relevant to the problem in order to get the participants thinking about the concept in a new way, such as a book or a piece of artwork.
- Encourage participants to use different strategies to solve problems, and compare these strategies at the end.

☆☆ **Attend to participants' needs as learners and try to accommodate them**

Why is this important?

- We all have different learning styles and needs, and a variety of accommodations can be used to make sure these are met for everyone.

- Make accommodations for all participants' needs as a regular part of the circle in order to normalize and celebrate diversity among participants.

Specific examples:

- Give a short amount of time (30 seconds or 1 minute) after the question is posed but before discussion starts for participants to think about the problem on their own.
- "Daniel and Pamela, it looks like you two like to think out loud. Why don't you two go to a different table and work on this together in order to give others a chance to process it on their own?"
- Prior to the first session, you could ask participants' families (or school staff, if the Math Circle is being offered for students at a specific school), "Is there anything it would be useful for us to know so we can help this student to have a positive experience in this Math Circle?"
- If you are experiencing issues with a participant's engagement or behavior that you don't know how to address, you could reach out to their family (or school staff, if the Math Circle is being offered for students at a specific school) to see if they can provide any insight or suggestions.

☆☆ Read the room and integrate feedback into your approach

Why is this important?

- Facilitator flexibility gives the participants agency to explore their topics of interest, and tackle problems in the ways that make the most sense to them. The facilitator might begin with a plan, but they need to be able to make adjustments based on the needs of the participants.
- Feedback will inform the facilitator about how best to proceed next: Was this topic too difficult? Are we moving too slowly? Are participants losing interest?

Specific examples:

- Monitor the body language of the participants who aren't talking. Are they making eye contact, or are they zoning out? Are they thinking critically in order to follow along, or are they doodling? If the group seems to be losing interest, you can pivot towards a different point of discussion in order to re-engage them.
- If participants seem to be engaged, but are not talking and are confused, then the problem might be too difficult. The facilitator can check in with the participants to see if this is accurate, in which case, they might present an easier version of the same problem or concept.
- A facilitator can ask for direct feedback as well. A private Google form or slip of paper at the end of a meeting works fine, and can offer participants a space to anonymously share which needs of theirs are or are not being met.

☆☆ Pay special attention to participants who are members of underrepresented groups in mathematics

Why is this important?

- Underrepresented groups in mathematics include girls, children of minority groups in the US, those who live at or below the poverty line, non-native English speakers, Native

Americans, and recent immigrants. These participants are less likely to benefit from societal structures and may arrive to the discussion with unmet needs which prevent them from realizing their academic potential.

- By drawing on these participants' diverse and unique experiences and perspectives, the discussion becomes richer, more impactful, and more multidimensional.

Specific example:

- If a participant seems hesitant or not fully comfortable, a facilitator can check in with them privately to ask how they are doing and if there is anything that they need.
- Try to call on these participants when they volunteer to contribute, especially if it is their first time doing so. When they are sharing, firmly shut down any attempted interjections or interruptions.
- Be mindful to the fact that you may give preferential treatment to strategies and solution paths that you have thought about previously. Consciously give equal weight to all ideas, treating them with the same importance when they are proposed.

Promoting Agency Among Participants

For participants in a Math Circle, having agency means that the participants see themselves as capable of (and inclined towards) doing the thinking required to answer questions, solve problems, and explore phenomena without depending on guidance from an expert. As described in the Introduction, supporting participants in developing this sense of agency is a core part of the primary goal of a Math Circle. Since traditional math classes often do not promote a sense of agency among students, participants may hesitate to take an active role in learning and discovery without explicit expectation-setting and encouragement from the facilitator. Thus, a facilitator must not only create space for participants to explore, the facilitator must convince participants that this space is theirs to take, and they must take care not to unintentionally revoke this space when participants experience challenge and uncertainty.

Moving from the well-established roles of teachers and students — where the teacher is the one with the content knowledge, and it's their job to transfer this knowledge to the students — is one that can be a challenge at first, for both participants and facilitators.

Here are some specific guidelines and practices that may be useful:

☆ **Be explicit in establishing expectations and norms for how participants will engage with the material**

Why is this important?

- Participants who are not familiar with Math Circles may assume that a Math Circle is like a math class. To the extent that you want your Math Circle to be different from a typical math class, it is important to communicate these expectations explicitly and clearly. Here are some expectations that it might be especially important to include, since they are different from the norms of traditional math classes:

- The facilitator will provide questions or problems for participants to explore, but participants will have space to explore these questions or problems and to come up with their own strategies and solutions, rather than having the facilitator show them these strategies and solutions.
- It is useful to try out ideas, even if they might be wrong — it is good to try, and it is not bad to be wrong.
- Participants will be expected to check their own answers and solutions. Their goal should be to arrive at answers that they are confident in rather than giving these answers to the facilitator to check.
- It is useful for participants to share their thinking with others so that they have opportunities to learn from each other.
- Explicitly and clearly establishing expectations can be especially important for participants who have not had similar enrichment experiences in the past, since they may be less likely to understand or intuit the norms for the space.

Specific examples:

- Give an introduction on the first day where you explain the expectations and norms for the Math Circle. For example: "This Math Circle will probably be different from your math classes. In this Math Circle, I will give you problems to explore, but instead of showing you how to solve them, I will give you space to think and make discoveries on your own and with your peers."
- As part of your introduction, it may also be useful to explicitly name some practices that will be useful to participants — for example, trying things out even if they might be wrong, sharing their thinking, etc.
- Reinforce these norms during participants' exploration: Praise them for engaging in ways that are aligned with these norms, remind them of the expectations when necessary, and point out situations where using particular practices (trying something out, checking their solution, sharing their thinking with others, etc.) could be helpful.

☆/☆☆ Treat all ideas the same, regardless of whether they are correct or incorrect

Why is this important?

- In order for participants to have agency, they – not the facilitator – need to have primary responsibility for determining which ideas and solutions are on the right track toward a solution. If the facilitator responds differently to correct ideas vs. incorrect ideas, participants tend to use this response to guide the paths that they take, which deprives the participants of agency.

Specific examples:

- ☆ If a participant asks you if an answer is correct, instead of telling them, turn the question back to them: "How could you check this yourself?" or "How confident are you?"
- ☆ If a participant has recorded an answer to a particular problem, you could ask them "How confident are you? 50%? 75%? 100%?" Be sure to use the same language and tone regardless of whether participants' ideas are correct or incorrect.

- Asking for a confidence level is useful when participants are working on problems where verifying the answer to the problems is not straightforward — for example, if the participants are deciding whether or not a particular problem is impossible, when they're looking for the minimum number of steps required to solve a problem, etc.
- ☆ If a participant makes a prediction or conjecture, instead of telling them whether it is correct or incorrect, you could ask: "How confident are you?" If they are very confident, you could ask them to explain their reasoning. If they are not very confident, you could ask them, "How could you test this to see whether or not it's correct?"
- ☆☆ You can also write predictions or conjectures on the board (regardless of whether or not they are correct) and ask the other participants in the group: "What do you think about this idea? Do you agree or disagree, or are you not sure?"
- ☆☆ Resist the temptation to immediately correct incorrect answers or solutions. (See the "Giving Guidance" section for more about how to address incorrect answers or solutions.)

☆☆ **Create opportunities for participants to interact with each other, instead of just with the facilitator**

Why is this important?

- Discussion among participants will allow them to take on even more leadership within the activity.
- The discussion will be longer and richer if the facilitator helps participants respond to one another rather than responding to participants themselves.
- There are, however, some pitfalls to watch out for when having participants interact and collaborate with each other instead of through the facilitator:
 - When participants interact directly with each other it can reinforce status hierarchies and disempower participants who are perceived as having lower status in the classroom. For example, participants who are more vocal and confident may dominate interactions. If some participants are consistently figuring things out more quickly than other participants, the participants who aren't as quick may get discouraged, especially if they already have relatively lower self esteem.
 - Direct interaction between participants can deprive participants of the space to think for themselves, since participants who have solved specific problems may show their solutions to participants who haven't yet solved the problems, or who have misconceptions or mistakes in their solutions.

Specific examples:

- Write all ideas – correct and incorrect – on the board, and ask the group to comment on the validity of the idea or answer. Use phrases such as "What do other people think about this idea? Do you agree or disagree, or are you not sure?" or "Good question! Does anyone have an idea about how we can address this?"
- When a participant shares an idea during a group discussion, instead of responding to the participant's idea yourself, ask other participants what they think.

- Use structured *pair-shares* where participants who have been exploring on their own check in with a neighbor to compare answers before sharing their ideas with the whole group.
- At times, explicitly set the expectation that participants will be checking in with a partner or with the larger group to compare answers and share ideas as they explore.

Giving Guidance

Since participants in Math Circles typically explore problems that they haven't seen before and haven't been shown how to solve, it is to be expected that some participants may end up with incorrect answers or get stuck, so you should be ready to provide support and guidance when necessary.

When deciding when and how to give guidance, it can be tricky to find the right balance between giving too little and giving too much. On the one hand, when participants experience failure and struggle during their exploration, they may disengage, and you obviously want to prevent this from happening. On the other hand, it's important to keep in mind that the long-term goal is for participants to develop the skills and confidence to be effective mathematical thinkers and problem solvers; giving guidance in a way that makes participants feel like they can't solve challenging problems without your help can undermine this goal.

Here are some specific guidelines and practices that may be useful:

☆ Encourage participants to share what they have done and what they are thinking before giving specific hints

Why is this important?

- Sharing ideas encourages participants to reflect on and think more carefully about what they have tried, and it may be enough to help them get unstuck on their own.
- Having participants share their thinking also gives the facilitator a better understanding of the thoughts behind the ideas, so that hints can build on the thinking they've already done.

Specific examples:

- If a participant seems stuck, start by asking: "What have you tried? What have you noticed? What have you learned?"

☆/☆☆ Give participants ways to identify incorrect answers other than checking their work for them

Why is this important?

- It is sometimes useful for participants to see that they have incorrect answers so that they don't go too far down the wrong path in their exploration, but if you tell participants directly when they have incorrect answers, they may feel like they should expect you to check their work, which undermines their sense of agency.

Specific examples:

- ☆ Look for situations where different participants have different answers for a specific problem — for example, if one participant thinks a particular problem is impossible and another participant thinks they have solved that problem, you could announce to the whole group, "I'm seeing some disagreement. Some people think this problem is impossible and other people think it's possible. If you think it's impossible, you might want to check again and see if you can find a way to do it. If you think it's possible, you might want to go back and check your solution to make sure you didn't break any of the rules."
 - Just to be clear, when doing this, don't have participants compare answers with each other. Making participants aware that there is disagreement without encouraging them to compare answers can motivate them to review their solution while still giving them space to think for themselves.
- ☆☆ Encourage participants to compare answers or solutions with each other, but be careful to avoid the following pitfalls:
 - Having participants share answers can reinforce status hierarchies and disempower participants who are perceived as having lower status in the classroom.
 - Having participants share answers can deprive participants of the space to think for themselves, since participants who have solved specific problems may show their solutions to participants who haven't yet solved the problems, or who have misconceptions or mistakes in their solutions.

Having participants compare answers with each other works best in a Math Circle with a strong culture of collaboration (as described in the [Ensuring Equitable Participation](#) section).

☆/☆☆ **Err on the side of giving too little rather than too much**

Why is this important?

- You can always give more information later if necessary, but you can never give less.
 - It might be helpful to think of a participant who is stuck as a turtle that has ended up on its back: You don't need to carry the turtle to its destination; all you need to do is to help the turtle to flip itself over.

Specific examples:

- ☆ Rather than asking, "Why don't you try X?" start by asking "What else could you try?"
- ☆ Rather than asking, "Do you notice a pattern?" start by asking "What do you notice?"
- ☆ If a participant asks you if an answer is correct, instead of telling them, turn the question back to them: "How could you check this yourself?" or "How confident are you?"
- ☆☆ Slowly ramp up the level of guidance as necessary: Start with smaller hints, moving to larger hints only when the smaller hints do not suffice to help the participant get unstuck.

☆☆ Adjust the level and type of guidance for each participant

Why is this important?

- Different participants may benefit from different levels and types of guidance based on where they fall along the following spectra:
 - Tolerance for struggle / resilience
 - Mathematical level and readiness for specific problems
 - Patience for exploration without a clear endpoint in sight
- For example:
 - A participant who has less tolerance for struggle may be more likely to get frustrated and disengage when they experience difficulty, so they may require more (and quicker) guidance than a participant who has more tolerance for struggle.
 - A participant who is less ready to think about and explore specific problems may benefit from having some of the problem-solving process modeled for them, while a more advanced participant may not need this type of scaffolding.
 - A participant who has more patience for exploration without a clear endpoint in sight can be given more challenging and/or more open-ended problems to explore than a participant with less patience for this type of exploration.

Specific examples:

- First, try to get a sense of where participants fall along the spectra described above.
- Pay particular attention to participants with relatively low tolerance for struggle, since they are likely to get frustrated and disengage when they experience difficulty. Be ready to offer guidance to these participants sooner than you might for other participants in order to keep them from giving up.
- Participants with more tolerance for struggle can be allowed to explore for longer, even if they start to go astray, without intervention.
- If you give participants a challenging problem that could take them a long time to solve, you may want to provide more guidance to participants who seem less ready for this type of problem or who have less patience for exploration without a clear endpoint, perhaps by giving them an easier entry point or by helping them break the problem down into smaller subproblems.
 - For example, if the problem is "How many different ways can you make a 2 x 10 rectangle out of dominoes?" more advanced participants could be allowed to explore this problem in whatever way they want, while less advanced participants could be encouraged to start by exploring this question for smaller rectangles (2 x 1, 2 x 2, 2 x 3, 2 x 4, etc.), building up slowly to 2 x 10 and noticing patterns along the way.

☆☆ Don't be too quick to rescue participants

Why is this important?

- Reflecting on, analyzing, and critiquing one's own thinking is an important part of having agency in problem solving. If the facilitator is too quick to rescue a participant whenever they make a mistake, participants will miss out on opportunities to develop these skills.

- That being said, there are situations where it is good to step in and provide assistance to participants who have gone a bit astray — in particular, this type of assistance can be helpful to prevent participants from getting frustrated and disengaging, or to prevent participants from wasting too much time on unproductive exploration.

Specific examples:

- Rather than jumping in and cutting off participants whenever they make a mistake, allow them to explore ideas to completion. Once the participant begins to notice that something is wrong, ask them how and where they notice the inconsistency.
- But do adjust your guidance to specific participants as described above. In particular, it is often a good idea to be quicker to step in to support participants who have less tolerance for struggle or who are less advanced.

☆☆ After giving guidance, encourage participants to reflect on how you helped them to get unstuck

Why is this important?

- Reflection can help participants to see how they might be able to get themselves unstuck in the future, without needing guidance from you.

Specific examples:

- After you have helped a participant to get unstuck, ask, "What did we do that helped us to get unstuck?" (Help the participant to answer if necessary: Did you suggest that they consider what else they could try? Did you suggest that they record some information? etc.)

Avoiding and Addressing Disengagement and Disruption

Math Circles thrive on open discussion and peer interaction. A disruptive participant can easily affect group dynamics, making it harder for other participants to fully engage or feel comfortable sharing their ideas. It is challenging to be proactive in minimizing the opportunity for disengagement and to be prepared to address any disruptions that do arise, but this makes for a higher level of engagement and a smoother-flowing circle.

☆ Develop a consistent routine that you will use in order to get the group's attention

Why is this important?

- A predictable routine establishes clear expectations efficiently and respectfully.
- After an established cue, participants are less likely to continue disruptive behavior as they know that other participants will refocus.

Specific examples:

- A simple call-and-response can bring everyone back together in unison; the facilitator might say “One, two, three...” and participants respond with “eyes on me!”
- A clapped rhythm or a unique sound, like a bell or chime, can also be a distinct indicator that it’s time to refocus.
- The facilitator says, “I need your attention in 3, 2, 1.”

☆ **Don’t hand out materials until you want the participants to start interacting with them**

Why is this important?

- If participants receive materials too early, they’re likely to focus on exploring the new items rather than listening to your instructions or engaging in initial discussions.

Specific examples:

- Explain the activity, how to use the materials, and any rules or guidelines before handing anything out. Clear instructions allow participants to dive right into the activity when they receive the materials.

☆/☆☆ **Redirect and reengage participants who are distracted**

Why is this important?

- If a participant is disengaged, it may be a sign that they are finding the activity too easy or too difficult. Focusing on redirection and reengagement can help the facilitator to determine if this is the case, which can open the door to successful reengagement. Focusing specifically on the distracting or disruptive behavior does not typically provide any information about what could help the participant to reengage successfully.
- Redirection towards productive behavior generally yields better results than focusing specifically on the distracting or disruptive behavior.
- Giving a disruptive participant something else to focus on can channel their energy productively. This will also prevent them from disrupting the rest of the participants.

Specific examples:

- ☆ If a participant seems disengaged, check in with them to assess their understanding of the activity: “Can you show me what you’re exploring right now?” If they are confused or stuck, then provide additional support; if they find it too easy, then try to ask questions to take the concept further or make the problem more difficult.
- ☆☆ Give a participant who is engaging in distracting or disruptive behavior a specific role such as recording ideas on the board, helping to keep track of a statistic, or leading a small part of the session.

☆☆ **Use non-verbal communication to discourage disruptive behaviors**

Why is this important?

- Non-verbal cues are quiet and unobtrusive, allowing you to regain attention or shift focus without breaking the momentum of a discussion.

- This also allows participants an opportunity to self-correct their behavior without being called out in front of the group.

Specific examples:

- Give a gentle non-verbal cue, such as eye contact or a smile, to help them realize the need to refocus.
- If a participant says something inappropriate, pause and stare at them with a neutral expression for a few seconds, and then continue on with the discussion as if nothing has happened.

☆☆ **Set boundaries clearly and calmly**

Why is this important?

- Boundaries define the behaviors that are acceptable and unacceptable.
- Setting boundaries with gentle positivity ensures that participants see them as a natural part of the learning process rather than as punitive rules.

Specific examples:

- Briefly explain why the boundary exists to give it purpose beyond authority. For example, "We raise our hands so everyone has a chance to share their thoughts," helps participants understand that the rule is about creating a fair environment, not about control.
- Set a norm for the behavior you want to see, rather than focusing on the things that you don't want to see: "Right now is an opportunity to listen to others' ideas. You'll have your chance to share soon."

☆☆ **If a participant is consistently being disruptive, address the situation firmly and respectfully**

Why is this important?

- If a facilitator sees behaviors that are unacceptable, it is their responsibility to communicate that and to check and oppose the behavior as a way of setting the expectation in the Math Circle.

Specific examples:

- Pull the participant aside for a private, respectful conversation. Ask them how they feel about the group and whether anything is making it hard for them to participate. Express that you want them to succeed, but that certain behaviors are getting in the way.
- It's important to publicly address threats to someone else's dignity. If a participant puts another down for their gender, race, identity, level of intelligence, or mistake, don't pull that kid aside. State something like, "Telling Karen that her intelligence is related to her gender is inappropriate and incorrect. There is a lot of evidence that girls and boys have the same levels of intelligence. It's also not helpful. If you see someone making a calculation mistake, you can help them by gently correcting them."
- If the disruptive behavior continues despite all efforts, consider reaching out to the participant's guardians or mentors for insight and support. They may be able to provide helpful context or strategies.

Conclusion

Facilitating a Math Circle is both an art and a science, requiring empathy, adaptability, and a deep appreciation for the unique ways individuals engage with mathematics and problem-solving. By focusing on building trust, ensuring equitable participation, promoting agency, providing balanced guidance, and avoiding and addressing disengagement and disruption, facilitators can foster an environment where curiosity thrives and participants feel empowered to explore and learn.

As you apply the principles in this guide, remember that every group and session is different. Be flexible, open to feedback, and willing to experiment with new strategies to meet the needs of your participants. By embracing this dynamic process, you can help participants not only grow as problem solvers but also develop a lifelong love and appreciation for the beauty of mathematics.

Thank you for committing to this meaningful work. Your role as a facilitator has the potential to make a profound and lasting impact on how your participants view and engage with math.

Appendix: Annotated Resources for Math Student Circles

This appendix to *Leading a Math Student Circle: Guide for Facilitators* provides a selection of sources of math circle problems, puzzles, and resources. All of these resources are free to use. There is also a reading list of books at the end.

Free Sources of Puzzles and Problems

Source of Problems/Puzzles	Ages	Description
The Infinite Pickle	8+	A delightful book of puzzles that start out fairly simple, but get really deep really quickly. There are many variations on each puzzle, and you and your participants can get lost for hours in them!
MathCircle.org Activity Database	6+	A gently-curated source of problems added to this database by mathematicians and math circle leaders.
JRMF Puzzles Database	5+	A highly-curated mix of activities and puzzles, both for in-person and online circles.
Nrich Maths	4+	Problems aligned to the British Mathematics curriculum that involve different levels of problem-solving, and are divided by topic, interest, and problem-solving skills.
Math for Love	6+	Lessons and games designed to promote thinking skills along with math calculation skills.
San Francisco Math Circle Activities	8+	Descriptions / overviews of activities for in-person Math Circles. Many activities have links to handouts, but there are not full lesson plans.

Tech Resources

Resource	Description
PolyPad	Lots of different manipulatives for any math you can think of: base-10 blocks, shape blocks, algebra tiles, and lots more!
Virtual Graph Paper	An infinite virtual grid paper plus drawing tools.
Geogebra	A graphing calculator on steroids. This tool is very powerful, but easy to learn. There are free tutorials online.
Desmos Graphing Calculator	A variety of graphing and calculation resources that are user-friendly.

Books and Printed Sources

Book	Description
Algorithmic Puzzles by Anany Levintin and Maria Levintin	A source of puzzles and games that can be easily adapted to math circles.
Out of the Labyrinth: Setting Mathematics Free by Robert and Ellen Kaplan	An early account of math circles taught on the Harvard campus in the 1990's. This book includes math circle topics and experiences the authors had in running early math circles.
MSRI Math Circle Library from American Mathematical Society	A collection of books published by the American Mathematical Society of different math problems used in various math circles.
Art of the Infinite: Pleasures of Mathematics by Robert and Ellen Kaplan	This book explores topics in mathematics by delving into the concept of infinity, revealing its profound implications and its presence in our understanding of the world.
Mathematics for Human Flourishing by Francis Su	An inspiring perspective on how engaging with mathematics can enrich our lives, cultivate virtues, and promote a more fulfilling and compassionate society.
Books from Natural Math	A collection of books that can easily be used as inspiration for math circles topics.