Unpredictability of some systems e.g. weather comes from the fact that
measurement tools can't be 100% accurate and small differences in initial
values of the system result in very large differences in the subsequent states of
the system as time goes on.

#### Chapter 2

• Prevailing view until the 1960s was that chaotic systems would ultimately settle into a stable, predictable state. Now we know that chaotic systems are systems that are unpredictable and stable.

### Chapter 3 - Application of Chaos Theory to Biology/Ecology

- Ecologists were divided into two camps when it came to population ecology.
  - The Determistics Believed that ecological systems had deterministic steady state behaviour which hard mathematics could be applied to
  - The Erratics Believed that noise in ecological systems overcome any deterministic behaviour making these systems random.
- Ultimately, it was shown that ecological systems have deterministic behaviour that is indistinguishable from random noise.

#### Chapter 4

- Fractal dimensions is the idea of describing irregular shapes by dimensions that can be in between two dimensions.
  - For example, the "Koch curve" has a fractal dimension of 1.26 which means its dimension is in between one and two dimensions
  - The "Koch curve" is very close to the structure of a coastal boundaries
  - Hence, the mathematical properties of a Koch curve are more applicable to studying coastal boundaries when compared to an one dimensional line

- Similarly, fractal shapes and their dimensions have a wide range of real world applications in fields such as physics, chemistry, seismology, metallurgy, probability theory, physiology etc
- Fractal geometry provides the visual representation of nonlinear systems just as euclidean geometry does for linear systems.

#### Chapter 5

- There are two "attractors"
  - o "Fixed point" Motion eventually comes to a rest
  - "Periodic" Motion oscillates in a closed loop
  - "Strange" Motion never comes to a rest but never goes into a closed loop either.

### Chapter 6

- As some parameter in the system is varied, the attractor bifurcates into two attractors. As the parameter is increased further, each of these two attractors bifurcate again creating four attractors. Then eight... sixteen and so on until they are so many attractors that the system becomes chaotic.
- Theorists were able to predict these "period doublings" in many one dimensional mathematical systems.

#### Chapter 7

 Experiments demonstrate "period doublings" in real world fluids such as liquid helium and then other things such as electronic oscillators, lasers and chemical reactions

#### Chapter 8

- Mandelbrot set is a two dimensional set
  - It can be visualized as a fractal with infinite magnification that contains no repeating structures

- o It's defined as the set of complex numbers c (numbers like 3 + 2i) for which the function  $f(z) = z^2 + c$  does not diverge to infinity, starting with z=0
- It's computed iteratively using a computer with its "magnification" being the number of iterations run on the function before it concludes it isn't diverging to infinity.
- At the boundary are points where an increasing number of iterations are needed as points do not seem to fall into or out of the set
- The boundary of the Mandelbrot set finds application in real world chaotic systems which have two attractors. This area of study is called fractal basin boundaries
  - o Lorenz attractor is a chaotic system with just one chaotic attractor
  - There are nonchaotic systems with two steady state attractors

# Chapter 9

• Chaos researchers manage to find a way to reconstruct phase space of an attractor from real world observational data of chaotic systems

# Chapter 10

• Hearts, circadian rhythms are most likely chaotic systems as well as many other biological processes and pathologies

#### Chapter 11