Mark Scheme

Q1.

Question	Scheme	Marks	AOs
	Integrate a w.r.t. time	M1	1.1a
	$\mathbf{v} = \frac{5t^2}{2}\mathbf{i} - 10t^{\frac{3}{2}}\mathbf{j} + \mathbf{C} \text{ (allow omission of } \mathbf{C}\text{)}$	A1	1.1b
	$\mathbf{v} = \frac{5t^2}{2}\mathbf{i} - 10t^{\frac{3}{2}}\mathbf{j} + 20\mathbf{i}$	A1	1.1b
	When $t = 4$, $v = 60i - 80 j$	M1	1.1b
	Attempt to find magnitude: $\sqrt{(60^2 + 80^2)}$	M1	3.1a
	Speed = 100 m s ⁻¹	A1ft	1.1b
	- Marie		6 mark

Notes:

1st M1: for integrating a w.r.t. time (powers of t increasing by 1)

1st A1: for a correct v expression without C

 2^{nd} A1: for a correct v expression including C

 2^{nd} M1: for putting t = 4 into their v expression

3rd M1: for finding magnitude of their v

3rd A1: ft for 100 m s⁻¹, follow through on an incorrect v

Question	Scheme	Marks	AOs
(a)	$\mathbf{v}_B = (-16\mathbf{i} - 3\mathbf{j}) + 5(2.4\mathbf{i} + \mathbf{j})$	M1	3.4
	$\mathbf{v}_{B} = (-4\mathbf{i} + 2\mathbf{j})$	A1	1.1b
	$\sqrt{(-4)^2+2^2}$	M1	3.1a
	$\sqrt{20} = 2\sqrt{5}$, 4.5 or better (m s ⁻¹)	A1	1.1b
		(4)	
(b)	Using A as the initial position: $\mathbf{r}_C = \mathbf{v}_A t + \frac{1}{2} \mathbf{a} t^2 + \mathbf{r}_A \qquad \text{where } t = T$ $(4\mathbf{i} + c\mathbf{j}) = (-16\mathbf{i} - 3\mathbf{j})T + \frac{1}{2}(2.4\mathbf{i} + \mathbf{j})T^2 + (44\mathbf{i} - 10\mathbf{j})$ $\mathbf{OR} \qquad \begin{pmatrix} 4 \\ c \end{pmatrix} = \begin{pmatrix} -16 \\ -3 \end{pmatrix} T + \frac{1}{2}\begin{pmatrix} 2.4 \\ 1 \end{pmatrix} T^2 + \begin{pmatrix} 44 \\ -10 \end{pmatrix}$ Equating i-components, to give a quadratic equation in T only. Allow t instead of T. N.B. Allow omission of 44 for this M mark. Also allow ± 4 but M0 if 4 is not used at all	M1	3.1a
	i.e. $4 = -16T + \frac{1}{2} \times 2.4T^2$ scores M1A0A0 $4 = -16T + \frac{1}{2} \times 2.4T^2 + 44$	A1	1.1b
	(T =) 10	A1	1.1b

ALTERNATIVE <u>using B</u> as the initial position:		
(The position vector of B, $\mathbf{r}_{_{\!B}}$, should be $-6\mathbf{i}-12.5\mathbf{j}$ but no credit for		
finding this)		
$\mathbf{r}_C = \mathbf{v}_B t + \frac{1}{2} \mathbf{a} t^2 + \mathbf{r}_B$ using their \mathbf{v}_B from (a) and their \mathbf{r}_B		
$(4\mathbf{i} + c\mathbf{j}) = (-4\mathbf{i} + 2\mathbf{j})t + \frac{1}{2}(2.4\mathbf{i} + \mathbf{j})t^2 + (-6\mathbf{i} - 12.5\mathbf{j})$		
$\binom{4}{c} = \binom{-4}{2}t + \frac{1}{2}\binom{2.4}{1}t^2 + \binom{-6}{-12.5}$		
	M1	3.1a
Equating i-components, to give a quadratic equation in t only. Allow if		
they have T instead of t.		

2	N.B. Allow omission of their -6 or if they use 44 for this M mark. Also		
	allow ± 4 but M0 if 4 is not used at all. e.g. $4 = -4t + \frac{1}{2} \times 2.4t^2$ scores M1A0A0		
	$4 = -4t + \frac{1}{2} \times 2.4t^2 - 6$	A1	1.1b
	t = 5 so (T =) 10	A1	1.1b
		(3)	
(c)	Equating j-components, with their value of T or t substituted, to give an equation, which must have a square term, in c only. N.B. Allow $\pm c$ in their equation.	M1	
	(N.B. Allow omission of -10 or their -12.5 for this M mark		
	i.e. if using A as initial position $c = (-3 \times 10) + \frac{1}{2} \times 1 \times 10^2 \text{ scores M1M0A0}$		2.1
	OR if using B as initial position		
	$c = (2 \times 5) + \frac{1}{2} \times 1 \times 5^2 \qquad \text{scores M1M0A0)}$		
	if using A as initial position	M1	
	$c = (-3 \times 10) + \frac{1}{2} \times 1 \times 10^2 + (-10)$		
	N.B. Allow $\pm c$ and/or $\pm (-10)$ in their equation		
	OR		1.1b
	if using B as initial position		
	$c = (2 \times 5) + \frac{1}{2} \times 1 \times 5^2 + (-12.5)$		
	N.B. Allow $\pm c$ and/or $\pm (-12.5)$ in their equation		
	c = 10	A1	1.1b
		(3)	

			(10 marks)
Notes:	Accept	column vectors throughout	
а	M1	Use of $\mathbf{v} = \mathbf{u} + \mathbf{a}t$ with $t = 5$ to give an unsimplified \mathbf{v}_B M0 if $\mathbf{u} = 0$	

		N.B. If using integration, they must get to the same stage i.e. have found the constant and put t = 5
	A1	M0 if they omit the constant altogether
	A1	Correct v_B with i's and j's collected
	M1	Use of Pythagoras on their V_B to give a magnitude (need the root)
	A1	Must be positive
b	M1	Equating components of i to give an equation in <i>T</i> or <i>t</i> only. N.B. (they could use integration to get to the same stage) for this M mark, they only need to be equating the i -components, and receive no credit until they do so. M0 if u = 0
	A1	A correct equation in T or t only (could be in $(T-5)$ if using B as initial position)
	A1	T = 10
c	M1	Equating components of j to give an equation in <i>c</i> only but allow omission of their initial position
	M1	With their value of T or t and must include $t = 0$ position (should be -10 if using A OR their -12.5 if using B)
	A1	cao

Question	Scheme	Marks	AOs
(i)(a)	Integrate a wrt t to obtain velocity	M1	3.4
	$\mathbf{v} = (t - 2t^2)\mathbf{i} + \left(3t - \frac{1}{3}t^3\right)\mathbf{j} \ (+\mathbf{C})$	Al	1.1b
	$8i - \frac{28}{3}j \ (m \ s^{-1})$	A1	1.1b
		(3)	
(i)(b)	Equate i component of v to zero	M1	3.1a
	$t - 2t^2 + 36 = 0$	A1ft	1.1b
	t = 4.5 (ignore an incorrect second solution)	A1	1.1b
		(3)	
(ii)	Differentiate r wrt to t to obtain velocity	M1	3.4
	$\mathbf{v} = (2t - 1)\mathbf{i} + 3\mathbf{j}$	A1	1.1b
	Use magnitude to give an equation in t only	M1	2.1
	$(2t-1)^2 + 3^2 = 5^2$	A1	1.1b
	Solve problem by solving this equation for t	M1	3.1a
	t = 2.5	A1	1.1b
		(6)	
		(12 n	narks

Notes: A	ccept	column vectors throughout
(i)(a)	M1	At least 3 terms with powers increasing by 1 (but M0 if clearly just multiplying by t)
	A1	Correct expression
	A1	Accept 8i-9.3j or better. Isw if speed found.
(i)(b)	M1	Must have an equation in t only (Must have integrated to find a velocity vector)
	A1 ft	Correct equation follow through on their v but must be a 3 term quadratic
	A1	cao
(ii)	M1	At least 2 terms with powers decreasing by 1 (but M0 if clearly just dividing by t)
	A1	Correct expression
	M1	Use magnitude to give an equation in t only, must have differentiated to find a velocity (M0 if they use $\sqrt{x^2 - y^2}$)
	A1	Correct equation $\sqrt{(2t-1)^2+3^2}=5$
	M1	Solve a 3 term quadratic for t which has come from differentiating and using a magnitude. This M mark can be implied by a correct answer with no working.
	A1	2.5

Question Number	Scheme	Marks	Notes
(a)	Integrate: $\mathbf{v} = (t^3 - 2t^2)\mathbf{i} + (3t^2 - 5t)\mathbf{j} + \mathbf{C}$	M1	At least 3 powers going up. Condone errors in constants. Must be two separate component equations if not in vector form. Could be in column vector form. Allow with no "+ C" -1 each integration error. i.e. All correct A1A1
		A2	1 error A1A0, 2 or more errors A0A0 Allow with no "+ C"
	$t = 3 : \mathbf{v} = 9\mathbf{i} + 12\mathbf{j} + \mathbf{C} = 11\mathbf{i} + 10\mathbf{j}$ $\mathbf{C} = 2\mathbf{i} - 2\mathbf{j}$	DM1	Substitute given values to find C. Dependent on the previous M mark
	$\mathbf{v} = (t^3 - 2t^2 + 2)\mathbf{i} + (3t^2 - 5t - 2)\mathbf{j}$	A1 (5)	Correct velocity (any equivalent form)
(b)	Parallel to $\mathbf{i} \implies 3t^2 - 5t - 2 = 0$	M1	Set j component of their v equal to zero and solve for t Correct answers imply method, but incorrect answers need to show method clearly.
	(3t+1)(t-2)=0, $t=2$	A1	Correct only. Ignore $-\frac{1}{3}$ if present.
	00 000 G-	DM1	Substitute their t to find v. Dependent on the previous M mark.
	$ \mathbf{v} = 8 - 8 + 2 = 2 \text{ (m s}^{-1})$	A1 (4)	The answer must be a scalar – the Q asks for speed. Results from negative t must be rejected.
		[9]	90

Q		Scheme Marks		Marks
			B1	
(a)	$t=\frac{5}{4}$		M1	1.25
(b)	$\mathbf{r} = \left(2t^2 - 5t\right)\mathbf{i} + 3t$	j(+c)		Integrate the velocity vector
			A1 DM1	NB Also correct to use suvat with $a = 4i$ and $u = -5i + 3j$. Correct
	$t = 0 2\mathbf{i} + 5\mathbf{j} = \mathbf{c}$	HOLD COMPANY TO A STATE OF THE	A1	Use ro to find C
	$\mathbf{r} = \left(2t^2 - 5t\right)\mathbf{i} + 3t$		27-141	oe
	$(2t^2 - 5t + 2)\mathbf{i} + (3t^2 - 5t)\mathbf{i}$		B1	
(c)	$\mathbf{r}_{Q} = 11\mathbf{i} + 2\mathbf{j} - 2t\mathbf{i} - 2ti$			Correct j component of r _Q Do not actually require the whole thing - can answer the Q by considering only the j component.
	$\mathbf{r}_{Q} = \mathbf{r}_{P} = d\mathbf{i} + 14\mathbf{j}$. (3, 13)	$2t^2 - 5t$	
	3t + 5 = 14	$2t^{2} - 3t - 9$ $(2t + 3)(t - 3) = 0$ $t = 3$	M1	Form an equation in t only
	t = 3	A1 ft	A1	
	$2+ct=14 \Rightarrow c=6$		A1 ft	Their t
	$d = 11 - 2 \times 3 = 5$ $d = 2 \times 3^2 - 5 \times 3 + 2$	or $2 \Rightarrow d = 5$		Their t
	Alt: $2t^2 - 5t + 2 =$	$11 - 2t = d \Rightarrow t = \frac{11 - d}{2}$		
	$2\left(\frac{11-d}{2}\right)^2 - 5\left(\frac{11}{2}\right)^2$	= · /		
	$d^2 - 19d + 70 = 0$	=(d-5)(d-14)		

Question Number	Scheme	Marks	Notes
(a)	$\mathbf{a} = \frac{\mathbf{d}\mathbf{v}}{\mathbf{d}t} = 6t\mathbf{i} + (4 - 2t)\mathbf{j}$ When $t = 1$, $\mathbf{a} = 6\mathbf{i} + 2\mathbf{j}$	M1 A1 DM1	Differentiate v to obtain a. Accept column vector or i and j components dealt with separately. Substitute $t = 1$ into their a. Dependent on 1^{12} M1
	$ \mathbf{a} = \sqrt{6^2 + 2^2} = \sqrt{40} = 6.32 \text{ (m s}^2\text{)}$	DM1 A1 (5)	Use of Pythagoras to find the magnitude of their a. Allow with their t. Dependent on 1 st M1 Accept awrt 6.32, 6.3 or exact equivalents.
(b)	$\mathbf{r} = \int (3t^2 - 1)\mathbf{i} + (4t - t^2)\mathbf{j} dt$ $= (t^3 - t + C)\mathbf{i} + (2t^2 - \frac{1}{3}t^3 + D)\mathbf{j}$ $t = 0, \mathbf{r} = \mathbf{i} \Rightarrow C = 1, D = 0$	MI A1 DM1	Integrate \mathbf{v} to obtain \mathbf{r} Condone C , D missing Use $t = 0$, $\mathbf{r} = \mathbf{i}$ to find $C \& D$ Substitute $t = 3$ with their $C \& D$ to find \mathbf{r} . Dependent on both
	When $t = 3$, $r = 25i + 9j$ (m)	DM1 A1 (5)	previous Ms. cao. Must be a vector.

Q7.

(a)	Speed = $\sqrt{8^2 + 48^2} = \sqrt{2368} = 48.7 \text{(ms}^{-1}\text{)}$	M1 A1
	And the second s	(2
(b)	$\mathbf{a} = 2\mathbf{i} - 6t\mathbf{j}$	M1 A1
	When $t = 4$, $a = 2i - 24j$ (ms ⁻²)	A1
		(3
(c)	$\mathbf{r} = t^2 \mathbf{i} - t^3 \mathbf{j} + \mathbf{C}$	M1 A1
*****	t = 1, -4i + j = i - j + C, C = -5i + 2j	DM1
	$\mathbf{r} = (t^2 - 5)\mathbf{i} + (-t^3 + 2)\mathbf{j}$	
	When $t = 4$, $\mathbf{r} = (16-5)\mathbf{i} + (-64+2)\mathbf{j} = 11\mathbf{i} - 62\mathbf{j}$	DM1 A1
	When t = 4,1 = (10-5)1 + (-04 + 2)j = 111 - 02j	DWI AI
		1

Question Number	Scheme	Marks
(a)	$\mathbf{a} = \frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = 6t\mathbf{i} - 4\mathbf{j}$	M1 A1
(b)	Using $\mathbf{F} = \frac{1}{2}\mathbf{a}$, sub $t = 2$, finding modulus	M1, M1, M1
	e.g. at $t = 2$, $a = 12i - 4j$	
	$\mathbf{F} = 6\mathbf{i} - 2\mathbf{j}$	
	$ \mathbf{F} = \sqrt{(6^2 + 2^2)} \approx \underline{6.32 \mathbf{N}}$	A1(CSO)
	M1 Clear attempt to differentiate. Condone i or j missing. A1 both terms correct (column vectors are OK)	
	The 3 method marks can be tackled in any order, but for consistency on epen grid please enter as:	
	M1 F=ma (their a, (correct a or following from (a)), not v. $F=\frac{1}{2}$ a). Condone a not a vector for this mark	
	M1 subst t = 2 into candidate's vector F or a (a correct or following from (a), not v) M1 Modulus of candidate's F or a (not v)	
	A1 CSO All correct (beware fortuitous answers e.g. from 6ti+4j)) Accept 6.3, awrt 6.32, any exact equivalent e.g. $2\sqrt{10}$, $\sqrt{40}$, $\frac{\sqrt{160}}{2}$	

Question	Scheme	Marks	AOs
(a)	$(4i - j) + (\lambda i + \mu j) = (4 + \lambda)i + (-1 + \mu)j$	M1	3.4
	Use ratios to obtain an equation in λ and μ only	M1	2.1
	$\frac{(4+\lambda)}{(-1+\mu)} = \frac{3}{1}$ or $\frac{\frac{1}{4}(4+\lambda)}{\frac{1}{4}(-1+\mu)} = \frac{3}{1}$	A1	1.1b
	$\lambda - 3\mu + 7 = 0$ * Allow $0 = \lambda - 3\mu + 7$ but nothing else.	A1*	1.1b
		(4)	

(1	b)	$\lambda = 2 \Rightarrow \mu = 3$; Resultant force = $(6\mathbf{i} + 2\mathbf{j})$ (N)	M1	3.1a
		(6i + 2j) = 4a OR $ (6i + 2j) = 4a$	M1	1.1b
		Use of $\mathbf{r} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$ with $\mathbf{u} = 0$, their \mathbf{a} and $t = 4$: Or they may integrate their \mathbf{a} twice with $\mathbf{u} = 0$ and put $t = 4$:	DM1	2.1
		$\mathbf{r} = \frac{1}{2} \times \frac{(6\mathbf{i} + 2\mathbf{j})}{4} 4^2 = (12\mathbf{i} + 4\mathbf{j})$ $\sqrt{12^2 + 4^2}$ ALTERNATIVE 1 for last two M marks:	M1	1.1b
		Use of $s = ut + \frac{1}{2}at^2$, with $u = 0$, their a and $t = 4$: $S = \frac{1}{2} \times \sqrt{1.5^2 + 0.5^2} \times 4^2$		
		Use of Pythagoras to find mag of a: $a = \sqrt{1.5^2 + 0.5^2}$ M1		
		ALTERNATIVE 2 for last two M marks: Use of $s = ut + \frac{1}{2}at^2$, with $u = 0$, their a and $t = 4$: $DM1$ $s = \frac{1}{2} \times \left(\frac{\sqrt{6^2 + 2^2}}{4}\right) \times 4^2$		
		Use of Pythagoras to find $ (6\mathbf{i} + 2\mathbf{j}) $: $= \sqrt{6^2 + 2^2}$ M1		
		$\sqrt{160}$, $2\sqrt{40}$, $4\sqrt{10}$ oe or 13 or better (m)	A1	1.18
			(5)	
			(9	marks
lotes:	Accept	column vectors throughout		
a	M1	Adding the two forces, i's and j's must be collected (or must be a s vector) seen or implied	ingle col	umn
	M1	Must be using ratios; Ignore an equation e.g. $(4 + \lambda)\mathbf{i} + (-1 + \mu)\mathbf{j} =$	3i + j if ti	hey go

on to use ratios.

		III 20.1 20.1 20.1 20.1 20.1 20.1 20.1 20.1
		However, if they write $4 + \lambda = 3$ and $-1 + \mu = 1$ then $3(-1 + \mu) = 3$ so
		$4 + \lambda = 3(-1 + \mu)$ with no use of a constant, it's M0
		They may use the acceleration, with a factor of $\frac{1}{4}$ top and bottom, see alternative
		Allow one side of the equation to be inverted
	A1	Correct equation
	A1*	Given answer correctly obtained. Must see at least one line of working, with the LH fraction 'removed'.
b	M1	Adding \mathbf{F}_1 and \mathbf{F}_2 to find the resultant force, λ and μ must be substituted N.B. M0 if they use $\mu = 2$ coming from $-1 + \mu = 1$ in part (a).
	M1	Use of $\mathbf{F} = 4\mathbf{a}$ Or $ \mathbf{F} = 4a$, where \mathbf{F} is their resultant. (including $3\mathbf{i} + \mathbf{j}$) This is an independent mark, so could be earned, for example, if they have subtracted the forces to find the 'resultant' N.B. M0 if only using \mathbf{F}_1 or \mathbf{F}_2
	DM 1	Dependent on previous M mark for Either: use of $\mathbf{r} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$ with $\mathbf{u} = 0$, their a and $t = 4$ to produce a displacement vector Or: integrate twice, with $\mathbf{u} = 0$, their a and $t = 4$ to produce a displacement Vector Or: use of $s = ut + \frac{1}{2}at^2$ with $u = 0$, their a and $t = 4$ to produce a length
	M1	Use of Pythagoras, with square root, to find the magnitude of their displacement vector, \mathbf{a} or \mathbf{F} (M0 if only using \mathbf{F}_1 or \mathbf{F}_2) depending on which method they have used.

Question	Scheme	Marks	AO
(a)	Differentiate v	M1	1.1a
	$(\mathbf{a} =) 6\mathbf{i} - \frac{15}{2} t^{\frac{1}{2}} \mathbf{j}$	A1	1.1b
	$=6i-15j \text{ (m s}^{-2})$	A1	1.1b
		(3)	
(b)	Integrate v	M1	1.1a
	$(\mathbf{r} =) (\mathbf{r}_0) + 3t^2 \mathbf{i} - 2t^{\frac{5}{2}} \mathbf{j}$	A1	1.1b
	= $(-20i + 20j) + (48i - 64j) = 28i - 44j$ (m)	A1	2.2a
		(3)	
		(6)	

Ma	rks	Notes
		N.B. Accept column vectors throughout and condone missing brackets in working but they must be there in final answers
a	M1	Use of $\mathbf{a} = \frac{\mathbf{d}\mathbf{v}}{\mathbf{d}t}$ with attempt to differentiate (both powers decreasing by 1) M0 if \mathbf{i} 's and \mathbf{j} 's omitted and they don't recover
	A1	Correct differentiation in any form
	A1	Correct and simplified. Ignore subsequent working (ISW) if they go on and find the magnitude.
b	M1	Use of $\mathbf{r} = \int \mathbf{v} dt$ with attempt to integrate (both powers increasing by 1) M0 if \mathbf{i} 's and \mathbf{j} 's omitted and they don't recover
	A1	Correct integration in any form. Condone ro not present
	A1	Correct and simplified.

Question	Scheme	Marks	AOs
	Allow column vectors throughout this question		
(a)	Differentiate v wrt t	M1	3.1a
	$\frac{3}{2}t^{-\frac{1}{2}}\mathbf{i} - 2\mathbf{j} \text{ isw}$	A1	1.1b
		(2)	Ö
(b)	$3t^{\frac{1}{2}} = 2t$	M1	2.1
	Solve for t	DM1	1.1b
	$t = \frac{9}{4}$	A1	1.1b
		(3)	
(c)	Integrate v wrt t	M1	3.1a
	$\mathbf{r} = 2t^{\frac{3}{2}}\mathbf{i} - t^2\mathbf{j}(+\mathbf{C})$	A1	1.1b
	$t = 1$, $r = -j \implies C = -2i$ so $r = 2t^{\frac{3}{2}}i - t^2j - 2i$	A1	2.2a
		(3)	
(d)	$\sqrt{(3t^{\frac{1}{2}})^2 + (2t)^2} = 10$ or $(3t^{\frac{1}{2}})^2 + (2t)^2 = 10^2$	M1	2.1
	$9t + 4t^2 = 100$	M(A)1	1.1b
	t = 4	A1	1.1b
	r = 14i - 16j	M1	1.1b
	$\sqrt{14^2 + (-16)^2}$	M1	3.1a
	$\sqrt{452} (2\sqrt{113}) (m)$	A1	1.1b
		(6)	
		(14 n	narks)

Not	es:	
а	M1	Both powers decreasing by 1 (M0 if vector(s) disappear but allow recovery)
ĺ	A1	cao
b	M1	Complete method, using v, to obtain an equation in t only, allow a sign error
	DM1	Dependent on M1, solve for t
	A1	cao
c	M1	Both powers increasing by 1 (M0 if vectors disappear but allow recovery)
	A1	Correct expression without C
	A1	cao
d	M1	Use of Pythagoras on v and 10 to set up equation in t
	M(A)1	Correct 3 term quadratic in t
	A1	cao
	M1	Substitute their numerical t value into their r
	M1	Use of Pythagoras to find the magnitude of their r
	A1	cso

uestion	Scheme	Marks	AOs
(a)	7i – 3j seen or implied by Pythagoras	B1	1.1b
	Use Pythagoras: $\sqrt{7^2 + (-3)^2}$	M1	3.1a
	$\sqrt{58}$, 7.6 or better (m s ⁻¹)	A1	1.1b
		(3)	
(b)	$t^2 - 3t + 7 = 2t^2 - 3$ OR $\frac{t^2 - 3t + 7}{2t^2 - 3} = \frac{1}{1} = 1$	M1	2.1
	t = 2 only	A1	1.1b
		(2)	
(c)	Differentiate v wrt t to give a vector.	M1	3.1a
(c)	Differentiate \mathbf{v} wrt \mathbf{t} to give a vector. $(2t-3)\mathbf{i} + 4t\mathbf{j}$	M1 A1	
(c)	2 SYPER		
(c)	2 SYPER	A1	1.1b
19005	$(2t-3)\mathbf{i} + 4t\mathbf{j}$	A1 (2)	1.1b
19005	$(2t-3)\mathbf{i} + 4t\mathbf{j}$ $2t-3 = 0$	A1 (2) M1	3.1a 1.1b 3.1a 1.1b

Note	es: All	ow column vectors throughout.
a	B1	сао
	M1	Use of Pythagoras, including the square root, on a velocity vector at t = 0
	A1	cao. Must come from a <u>correct</u> v.
b	M1	Equating \mathbf{i} and \mathbf{j} components of \mathbf{v} or a ratio of 1:1 to obtain a quadratic in t only. If they use a constant, e.g. $t^2-3t+7=k$ and $2t^2-3=k$, k must be eliminated to earn this mark. N.B. M0 (since wrong working seen) if they write down $\mathbf{i}+\mathbf{j}=\left(t^2-3t+7\right)\mathbf{i}+\left(2t^2-3\right)\mathbf{j}$ OR $\begin{pmatrix}1\\1\end{pmatrix}=\begin{pmatrix}t^2-3t+7\\2t^2-3\end{pmatrix}$ OR $t^2-3t+7=1$ and $t^2-3=1$

		and then $t^2 - 3t + 7 = 2t^2 - 3$
	A1	t = 2
		N.B. Allow M1A1 for a correct trial and error method where they obtain $\mathbf{v} = 5\mathbf{i} + 5\mathbf{j}$ when $t = 2$ but M0 if they don't get $t = 2$
С	M1	At least one power decreasing by 1 in each component in their v (M0 if clearly dividing by t) Both i and j needed in their answer or a column vector Allow recovery if the i and j disappear and then reappear.
	A1	cao (must be a vector) isw e.g. if they find the magnitude or put $t=0$ or differentiate again i's and j's do not need to be collected. N.B. Allow M1A0 for $2t-3\mathbf{i}+4t\mathbf{j}$
d	М1	2t-3=0 or (their derivative of the i-component of v) = 0 N.B. M0 if they equate the derivative of both components of v to zero.
	A1	N.B. Correct answer, with no working, can score both marks.

Question	Scheme	Marks	AOs
(a)	Use of $\mathbf{v} = \mathbf{u} + \mathbf{a}t$ or integrate to give: $\mathbf{v} = (-2\mathbf{i} + 2\mathbf{j}) + 2(4\mathbf{i} - 5\mathbf{j})$	M1	3.1a
	(6i-8j) (m s ⁻¹)	A1	1.1b
		(2)	
(b)	Solve problem through use of $\mathbf{r} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$ or integration (M0 if $\mathbf{u} = 0$) Or any other complete method e.g use $\mathbf{v} = \mathbf{u} + \mathbf{a}T$ and $\mathbf{r} = \frac{(\mathbf{u} + \mathbf{v})T}{2}$:	M1	3.1a
	$-4.5\mathbf{j} = 2t\mathbf{j} - \frac{1}{2}t^25\mathbf{j} \qquad \text{(j terms only)}$	A1	1.1b
	The first two marks could be implied if they go straight to an algebraic equation.		
	Attempt to equate j components to give equation in T only $(-4.5 = 2T - \frac{5}{2}T^2)$	M1	2.1
	T = 1.8	A1	1.1b
		(4)	
(c)	Solve problem by substituting their T value (M0 if $T < 0$) into the i component equation to give an equation in λ only: $\lambda = -2T + \frac{1}{2}T^2 \times 4$	M1	3.1a
	$\lambda = 2.9 \text{ or } 2.88 \text{ or } \frac{72}{25} \text{ oe}$	A1	1.1b
		(2)	1

Not	es: Acc	ept column vectors throughout (8 marks)
2a	M1	For any complete method to give a v expression with correct no. of terms with $t = 2$ used, so if integrating, must see the initial velocity as the constant. Allow sign errors.
	A1	Cao isw if they go on to find the speed.
2b M1 in t (or T) only, using $a = (4i - 5j)$, so if integrating, RHS correct structure.		For any complete method to give a vector expression for j component of displacement in t (or T) only, using $a = (4i - 5j)$, so if integrating, RHS of equation must have the correct structure. Allow sign errors.
	A1	Correct j vector equation in t or T . Ignore i terms.
	M1	Must have earned 1 st M mark. Equate j components to give equation in T (allow t) only (no j's) which has come from a displacement. Equation must be a 3 term quadratic in T.
	A1	cao
2c	M1	Must have earned 1 st M mark in (b) Complete method - must have an equation in λ only (no i's) which has come from an appropriate displacement. (e.g M0 if $a = 0$ has been used) Expression for λ must be a quadratic in T
	A1	cao

Q14.

Question	Scheme	Marks	AOs
(a)	Use of $\mathbf{v} = \mathbf{u} + \mathbf{a}t$: $(10.5\mathbf{i} - 0.9\mathbf{j}) = 0.6\mathbf{j} + 15\mathbf{a}$	M1	3.1b
	$\mathbf{a} = (0.7\mathbf{i} - 0.1\mathbf{j}) \text{ m s}^{-2}$ Given answer	A1	1.1b
		(2)	
(b)	Use of $\mathbf{r} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$	M1	3.1b
	$\mathbf{r} = 0.6\mathbf{j} \ t + \frac{1}{2} (0.7\mathbf{i} - 0.1\mathbf{j}) \ t^2$	A1	1.1b
		(2)	
(c)	Equating the i and j components of r	M1	3.1b
	$\frac{1}{2} \leftarrow 0.7 \ t^2 = 0.6 \ t - \frac{1}{2} \leftarrow 0.1 \ t^2$	A1ft	1.1b
	t = 1.5	A1	1.1b
		(3)	
(d)	Use of $\mathbf{v} = \mathbf{u} + \mathbf{a}t$: $\mathbf{v} = 0.6\mathbf{j} + (0.7\mathbf{i} - 0.1\mathbf{j}) t$	M1	3.1b
	Equating the i and j components of v	M1	3.1b
	t = 0.75	A1 ft	1.10
		(3)	

Notes:

(a)

M1: for use of v = u + at

A1: for given answer correctly obtained

(b)

M1: for use of $\mathbf{r} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$

A1: for a correct expression for \mathbf{r} in terms of t

(c)

M1: for equating the i and j components of their r

A1ft: for a correct equation following their r

A1: for t = 1.5

(d)

M1: for use of v = u + at for a general t

M1: for equating the i and j components of their v

A1ft: for t = 0.75, or a correct follow through answer from an incorrect equation

Question Number	Scheme	Marks
(a)	$\tan \theta = \frac{2}{9} \theta = 12.5^{\circ}$ bearing 103°	M1 A1 A1 (3)
(b)	224 175011 1222 1273	
(i) (ii)	$\mathbf{p} = (9\mathbf{i} + 10\mathbf{j}) + t(9\mathbf{i} - 2\mathbf{j})$ $\mathbf{q} = (\mathbf{i} + 4\mathbf{j}) + t(4\mathbf{i} + 8\mathbf{j})$	M1 A1 A1 (3)
(c)	$\overrightarrow{QP} = (8+5t)\mathbf{i} + (6-10t)\mathbf{j}$	M1 A1 (2)
(d)	$D^{2} = (8+5t)^{2} + (6-10t)^{2}$ $= 125t^{2} - 40t + 100$	M1 A1
	$100 = 125t^2 - 40t + 100$	M1 M1
	0 = 5t(25t - 8) t = 0 or 0.32	A1 A1 (6)
	Notes	
(a)	M1 for $\tan \theta = \pm \frac{2}{9}$ or $\pm \frac{9}{2}$ or use $\sin \theta$ or $\cos \theta$	
	First A1 for θ - $\pm 13^{\circ}$ or $\pm 77^{\circ}$ or $\pm 12.5^{\circ}$ or $\pm 77.5^{\circ}$ or better Second A1 for 103°	
(b)	M1 for clear attempt at $\mathbf{p} = (9\mathbf{i} + 10\mathbf{j}) + t(9\mathbf{i} - 2\mathbf{j})$ or $\mathbf{q} = (\mathbf{i} + 4\mathbf{j}) + t(4\mathbf{i} + 8\mathbf{j})$ (Allow slips but must be a '+' sign and $\mathbf{r} + t\mathbf{v}$)	
(i)	slips but must be a '+' sign and $\mathbf{r} + t\mathbf{v}$) First A1 for $\mathbf{p} = (9\mathbf{i} + 10\mathbf{j}) + t(9\mathbf{i} - 2\mathbf{j})$ oe	
(ii)	Second A1 for $\mathbf{q} = (\mathbf{i} + 4\mathbf{j}) + t(4\mathbf{i} + 8\mathbf{j})$ oe	
(c)	M1 for $\mathbf{p} - \mathbf{q}$ or $\mathbf{q} - \mathbf{p}$ with their \mathbf{p} and \mathbf{q} substituted A1 for correct answer $\overrightarrow{QP} = (8+5t)\mathbf{i} + (6-10t)\mathbf{j}$ (don't need \overrightarrow{QP} but on R.H.S must be identical coefficients of \mathbf{i} and \mathbf{j} but allow column vectors)	
(d)	First M1 for attempt to find QP or QP^2 in terms of t only, using correct formula First A1 for a correct expression (with or without $$) $125t^2 - 40t + 100$ Second M1 for $$ (3 term quadratic) = 10 or (3 term quadratic) = 100. Third M1 for quadratic expression = 0 and attempt to solve (e.g. factorising or using formula) Second A1 for $t = 0$ (if they divide by t and lose this value but get 0.32, M1A0A1) Third A1 for $t = 0.32$ oe	

Question Number	Scheme	Mai	rks
(a)	$\tan \theta = \frac{5}{20}$	Ml	
	$\theta = 14.036^{\circ}$ $\theta = 104^{\circ}$ nearest degree	A1 A1	(3)
(b)	YOR		
(6)	$\mathbf{p} = 400\mathbf{i} + t(15\mathbf{i} + 20\mathbf{j})$ $\mathbf{q} = 800\mathbf{j} + t(20\mathbf{i} - 5\mathbf{j})$	Ml Al	(3)
(c)	Equate their j components: $20t(\mathbf{j}) = (800 - 5t)(\mathbf{j})$ t = 32 $\mathbf{s} = 800 \mathbf{j} + 32(20 \mathbf{i} - 5 \mathbf{j})$ $= 640 \mathbf{i} + 640 \mathbf{j}$	M1 A1 M1 A1	(4) 10
(a)	Notes Allow column vectors throughout M1 for $\tan \theta = \pm \frac{5}{20}$ or $\pm \frac{20}{5}$ (or any other complete method) First A1 for $\pm 14.04^{\circ}$ or $\pm 75.96^{\circ}$ Second A1 for $\pm 104^{\circ}$		
(b) (i) (ii)	M1 for clear attempt at either p or q (allow slip but t must be attached to the velocity vector and position vector and velocity vector must be paired up correctly) First A1 $400\mathbf{i} + t(15\mathbf{i} + 20\mathbf{j})$ " \mathbf{p} =" not needed but must be clear it's P Second A1 $800\mathbf{j} + t(20\mathbf{i} - 5\mathbf{j})$ " \mathbf{q} =" not needed but must be clear it's Q	5	
(c)	First M1 for equating their \mathbf{j} components; allow \mathbf{j} 's on both sides First A1 for $t = 32$ Second M1 <u>independent</u> for substituting their t value into their \mathbf{q} from (b) Second A1 for $640\mathbf{i} + 640\mathbf{j}$	8	

Question Number	Scheme	Marks
(a)	$\mathbf{F}_{2} = k\mathbf{i} + k\mathbf{j}$	BI
	$(-1+a)\mathbf{i} + (2+b)\mathbf{j}$	05.0500 I
		M1
	$\frac{-1+a}{2+b} = \frac{1}{3}$	DM1 A1
	$a = b = k = 2.5$; $\mathbf{F}_2 = 2.5\mathbf{i} + 2.5\mathbf{j}$	DM1 A1; A1
	ALTERNATIVE:	(7)
	$\mathbf{F}_{2} = k\mathbf{i} + k\mathbf{j}$	Bl
	$(-1+a)\mathbf{i} + (2+b)\mathbf{j} = p(\mathbf{i} + 3\mathbf{j})$	M1 for LHS
	-1+a=p	DAGE AT
	2+b=3p	DM1 A1
	$a = b = k = 2.5$; $\mathbf{F}_2 = 2.5\mathbf{i} + 2.5\mathbf{j}$	DM1 A1; A1 (7)
(b)	$\mathbf{v} = 3\mathbf{i} - 22\mathbf{j} + 3(3\mathbf{i} + 9\mathbf{j})$	Ml
	= 12 i + 5 j	Al
	$ \mathbf{v} = \sqrt{12^2 + 5^2} = 13 \text{ ms}^{-1}$	Ml Al cso (4)
	\$650E	11
3	Notes	G.
(a)	B1 for $\mathbf{F}_2 = k\mathbf{i} + k\mathbf{j}$ $(k \neq 1)$ seen or implied in working, including for an	
	incorrect final answer, with the wrong k value.	
	First M1 for adding the 2 forces (for this M mark we only need	
	$\mathbf{F}_2 = a\mathbf{i} + b\mathbf{j}$), with \mathbf{i} 's and \mathbf{j} 's collected (which can be implied by later	
	working) but allow a slip.	
	(M0 if a and b both assumed to be 1) Second M1 dependent on first M1 for ratio of their ents = $1/3$ or $3/1$	
	Second M1, dependent on first M1, for ratio of their cpts = 1/3 or 3/1 (Must be correct way up for the M mark)	
	First A1 for a correct equation which may involve two unknowns	
	Third M1, dependent on first and second M1, for solving for k oe	
	Second A1 for a correct k value	
	Third A1 for 2.5i+2.5j	

ALTERNATIVE: Using two simultaneous equations

B1 for $\mathbf{F}_2 = k\mathbf{i} + k\mathbf{j}$ $(k \neq 1)$ seen or implied in working.

First M1 for adding the 2 forces (for this M mark we only need

 $\mathbf{F}_2 = a\mathbf{i} + b\mathbf{j}$), with \mathbf{i} 's and \mathbf{j} 's collected (LHS of equation) (M0 if <u>a</u> and

b both assumed to be 1) but allow a slip

Second M1, dependent on first M1, for equating coeffs to produce *two* equations in 2 or 3 unknowns. Must have p and 3p (M0 if p is assumed to be 1 or k)

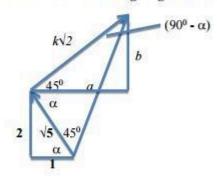
First A1 for two correct equations

Third M1, dependent on first and second M1, for solving for k oe

Second A1 for a correct k value

Third A1 for 2.5i+2.5i

ALTERNATIVE: Using magnitudes and directions



 $\mathbf{F}_2 = k\mathbf{i} + k\mathbf{j}$, seen or implied

Correct vector triangle

$$\frac{k\sqrt{2}}{\sin 45^{\circ}} = \frac{\sqrt{5}}{\sin(90^{\circ} - \alpha)}, \quad \alpha = \arctan 2$$

$$2k = 5$$

$$k = 2.5; \quad F = 2.5; \quad k = 2.5;$$

$$k = 2.5$$
; $\mathbf{F}_2 = 2.5\mathbf{i} + 2.5\mathbf{j}$

B1 M1

DM1 A1

DM1 A1; A1 (7)

ALTERNATIVE: Using magnitudes and directions

B1 for $\mathbf{F}_2 = k\mathbf{i} + k\mathbf{j}$ seen or implied in working.

First M1 for a correct vector triangle (for this M mark we only need $\mathbf{F}_2 = a\mathbf{i} + b\mathbf{j}$). (M0 if <u>a and b both</u> assumed to be 1 and/or longest side is assumed to be $\sqrt{10}$)

Second M1, dependent on first M1, for using sine rule on vector triangle

First A1 for a correct equation. 450 may not appear exactly.

Third M1, dependent on first and second M1, for solving for k oe Second A1 for a correct k value

Third A1 for 2.5i+2.5j

(b)	First M1 for use of $\mathbf{v} = \mathbf{u} + \mathbf{a}t$ with $t = 3$ First A1 for $12\mathbf{i} + 5\mathbf{j}$ seen or implied. However, if a wrong \mathbf{v} is seen A0 Second M1 for finding magnitude of their \mathbf{v} Second A1 for 13	

Question Number	Scheme	Marks
a	F = ma : 3i - 2j = 0.5a	M1
	$a = 6\mathbf{i} - 4\mathbf{j}$	A1
	$ a = \sqrt{6^2 + (-4)^2} = 2\sqrt{13} \text{ (m s}^{-2}) ***$	M1A1 (4)
b	v = u + at: $v = (i + 3j) + 2(6i - 4j)$	M1A1 ft
	$= 13i - 5j \text{ m s}^{-1}$	A1 (3)
c	Distance = $2 \mathbf{v} = 2\sqrt{4+1} = 2\sqrt{5} = 4.47$ (m)	M1A1
		(2)
d	When $t = 3.5$, velocity of P is $(i + 3j) + 3.5(6i - 4j) = 22i - 11j$	M1A1 ft
	Given conclusion reached correctly. E.g. $22i - 11j = 11(2i - j)$	A1 (3)
		[12]
	Notes for Question	70 73 78

Question (a)

Either:

First M1 for use of F = m a

First A1 for a = 6i - 4jSecond M1 for $a = \sqrt{(6^2 + (-4)^2)}$ (Allow $\sqrt{(6^2 + 4^2)}$)

Second A1 for $a = 2\sqrt{13}$ (ms⁻²) Given answer

Or:

First M1 for $F = \sqrt{(3^2 + (-2)^2)}$ (Allow $\sqrt{(3^2 + 2^2)}$)

First A1 $F = \sqrt{13}$

Second M1 for $\sqrt{13} = 0.5 a$

Second A1 for $a = 2\sqrt{13}$ (ms⁻²) Given answer

Question (b)

M1 for (i + 3j) + (2 x their a)

First A1 ft for a correct expression

Second A1 for 13i - 5j; isw if they go on to find the speed

Question (c)

M1 for $2\sqrt{(2^2 + (-1)^2)}$ or $\sqrt{(4^2 + (-2)^2)}$

A1 for $2\sqrt{5}$ or $\sqrt{20}$ or 4.5 or 4.47 or better

Question (d)

M1 for (i + 3j) + (3.5 x their a), or possibly, their (b) + (1.5 x their a)

First A1 ft for a correct expression of form ai + bj

Second A1 for given conclusion reached correctly e.g. 22i-11j=11(2i-j) oe Given answer

Q19.

Question Number	Scheme	Marks
(a)	$(4\mathbf{i} - 2\mathbf{j}) + (2\mathbf{i} + q\mathbf{j}) = (6\mathbf{i} + (q - 2)\mathbf{j})$ 6 = 2(q - 2) ratio 2:1 q = 5	M1A1 DM1 A1 (4
(b)	6i + 3j = 1.5a a = (4i + 2j) m s ⁻² v = u + at = (-2i + 4j) + 2(4i + 2j)	M1 A1 M1
	$= 6\mathbf{i} + 8\mathbf{j}$ $speed = \sqrt{6^2 + 8^2}$ $= 10 \text{ m s}^{-1}$	A1ft M1
	= 10 m s ⁻¹	A1 (6)

Notes for Question

Question (a)

First M1 for (4i - 2j) + (2i + qj)

First A1 for $(6\mathbf{i} + (q-2)\mathbf{j})$ (seen or implied)

Second M1, dependent on first M1, for using 'parallel to (2i + j)' to obtain an equation in q only. Second A1 for q = 5

Question (b)

First M1 for their resultant force = 1.5a

First A1 for a = 4i + 2j

Second M1 for (-2i + 4j) + 2x (their a)

(M0 if force is used instead of a)

Second A1 ft for their velocity at t = 2

Third M1 for finding the magnitude of their velocity at t = 2

Third A1 for 10 (ms⁻¹)

N.B. In (b), if they use scalars throughout, M0A0M0A0M0A0

Q20.

Question Number	Scheme	Marks
	$-6\mathbf{i} + \mathbf{j} = \mathbf{u} + 3(2\mathbf{i} - 5\mathbf{j})$ $\Rightarrow \mathbf{u} = -12\mathbf{i} + 16\mathbf{j}$ $\Rightarrow u = \sqrt{(-12)^2 + 16^2} = 20$	M1 A1 A1 cso M1 A1 [5]

Question Number	Scheme	Marks
(a)	$\tan \theta = \frac{2}{1} \Rightarrow \theta = 63.4^{\circ}$ angle is 153.4°	M1 A1 A1 (3)
(b)	$(4 + p)\mathbf{i} + (q - 5)\mathbf{j}$ (q - 5) = -2(4 + p) 2p + q + 3 = 0*	B1 M1 A1 A1 (4)
(c)	$q = 1 \Rightarrow p = -2$ $\Rightarrow \mathbf{R} = 2\mathbf{i} - 4\mathbf{j}$ $\Rightarrow \mathbf{R} = \sqrt{2^2 + (-4)^2} = \sqrt{20}$ $\sqrt{20} = m8\sqrt{5}$ $\Rightarrow m = \frac{1}{4}$	B1 M1 M1 A1 f.t. M1 A1 f.t. A1 cao
	4	[14]

Question Number	Scheme	Marks
(a)	$\tan \theta = \frac{8}{6}$	M1
	θ≈53°	A1 (2)
(b)	$\mathbf{F} = 0.4(6\mathbf{i} + 8\mathbf{j}) (= 2.4\mathbf{i} + 3.2\mathbf{j})$	M1
	$ \mathbf{F} = \sqrt{(2.4^2 + 3.2^2)} = 4$	M1 A1 (3)
(c)	$\mathbf{v} = 9\mathbf{i} - 10\mathbf{j} + 5(6\mathbf{i} + 8\mathbf{j})$	M1 A1
	$=39\mathbf{i}+30\mathbf{j} \left(\mathrm{ms}^{-1}\right)$	A1 (3)
	111000	(8 marks)

Q23.

Question Number	Scheme	Marks
(a)	speed = $\sqrt{2^2 + (-5)^2}$	M1
15,000)	$=\sqrt{29} = 5.4$ or better	A1 (2)
(b)	((7i+10j)-(2i-5j))/5	M1 A1
	=(5i+15j)/5=i+3j	A1
	$\mathbf{F} = m\mathbf{a} = 2(\mathbf{i} + 3\mathbf{j}) = 2\mathbf{i} + 6\mathbf{j}$	DM1 A1ft (5)
(c)	$\mathbf{v} = \mathbf{u} + \mathbf{a}t = (2\mathbf{i} - 5\mathbf{j}) + (\mathbf{i} + 3\mathbf{j})t$	M1
	(-5+3t)j	A1
	Parallel to $i \Rightarrow -5 + 3t = 0$	M1
	t = 5/3	A1
		(4) [11]

Question	Scheme	Marks	AOs
(a)	Put $t = 2$ in v and use Pythagoras: $\sqrt{12^2 + (-6\sqrt{2})^2}$	M1	3.1a
	$\sqrt{216}, 6\sqrt{6}$ or 15 or better (m s ⁻¹)	A1	1.1b
		(2)	
(b)	Differentiate v wrt t to obtain a	M1	3.4
	$6ti - 3t^{\frac{1}{2}}j$ oe (m s ⁻²) isw	A1	1.1b
		(2)	
(c)	Integrate v wrt t to obtain r	M1	3.4
	$\mathbf{r} = t^3 \mathbf{i} - 4t^{\frac{3}{2}} \mathbf{j} \ (+\mathbf{C})$	A1	1.1b
	$(i-4j) = 4^3i - 4 \times 4^{\frac{3}{2}}j + C$	M1	3.1a
	(-62i+24j) (m) isw e.g. if they go on to find the distance.	A1	1.1b
		(4)	

	A1	N.B. C does not need to be found and this is a method mark, so allow slips.
	M1	Putting $\mathbf{r} = (\mathbf{i} - 4\mathbf{j})$ and $t = 4$ into their displacement vector expression which must have C (allow C) to give an equation in C only, seen or implied. Must have attempted to integrate v for this mark to be available.
	A1	(r =) not required
c	M1	Both powers increasing by 1 M0 if i or j is missing but allow recovery.
	A1	cao. Do not accept a column vector.
b	M1	Both powers decreasing by 1. Allow a column vector. M0 if i or j is missing but allow recovery in (b).
	A1	N.B. Correct answer with no working can score 2 marks.
a	M1	Need square root but -ve sign not required. Allow i's and/or j's to go missing from their v at $t = 2$, provided they have applied Pythagoras correctly.