Homework #1 (Discrete Linear Regulator Problem)

Problem 1. Example 3.10 (p84)

1) Reproduce the results in the Example 3.10 and plot the curves in p85 by using Matlab. Provide the code with the plots. (You need to produce all curves for the variables, f_1, f_2, x_1, x_2, and u.)
2) Replay the problem when N=50, x(0) = $[3, 5]^T$.

Example 3.10 The linear discrete system

$$x(k+1) = [0.9974\ 0.0539\ \ -0.1078\ 1.1591\ ]\, x(k) + \ [0.0013\ 0.0539\ ]u(k) \qquad (3.10-22)$$

is to be controlled to minimize the performance measure

$$J = \frac{1}{2}\sum_{k=0}^{N-1}\left(0.25\, x_1^2(k) + \ 0.05\, x_2^2(k) + \ 0.05\, u^2(k)\right) \qquad (3.10-23)$$

Determine the optimal control law.

Equations (3.10-19), (3.10-21), and (3.10-20a) are most easily solved by using a digital computer with A and B as specified in Eq. (3.10-22),

$$F(N-K) = -\left[R(N-K) + \ B^T(N-K)P(K-1)B(N-K)\right]^{-1} \times B^T(N-K)P(K-1)A(N-K)$$

$$(Equaitions\ 3.10-19)$$

$$V(N-K) \triangleq A(N-K) + \ B(N-K)F(N-K) \qquad (Equations\ 3.10-21)$$

$$P(K) = V^T(N-K)P(K-1)V(N-K) + \ F^T(N-K)R(N-K)F(N-K) + \ Q(N-K)$$

$$(Equations\ 3.10-21a)$$

The discrete system of Eq. (3.10-1) with A and B constant matrices is completely controllable if and only if the n × mn matrix is of rank n.

$$\left[B \ \vdots\ AB\ \vdots\ \ \dots\ \vdots A^{n-1}B\right]$$

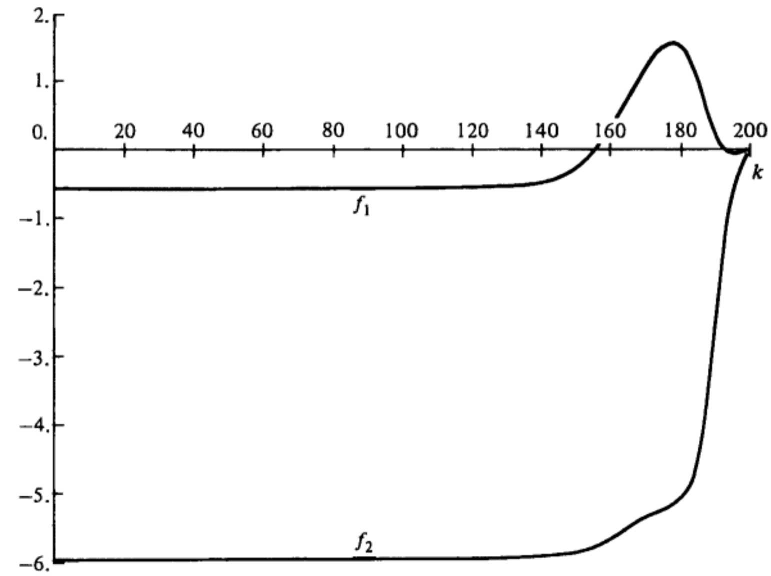

Figure 3-9(a) Feedback gain coefficients for optimal control of a second-order discrete linear regulator

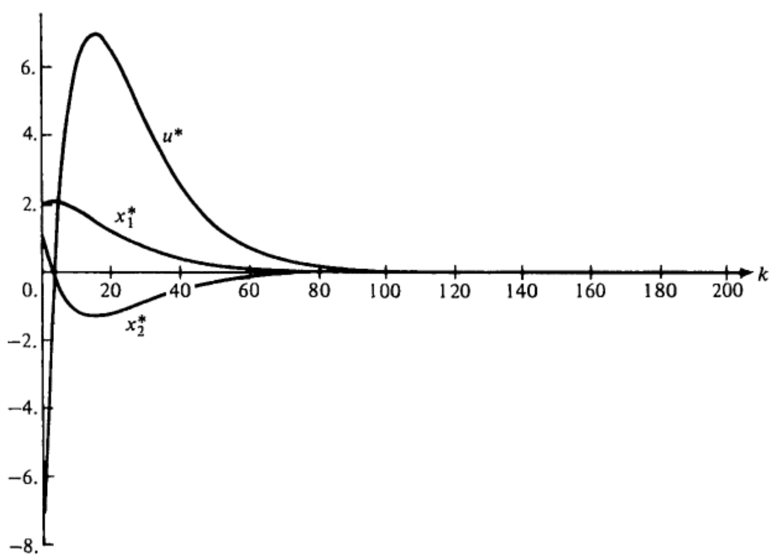

Figure 3-9(b) Optimal control and trajectory for a second-order discrete linear regulator

$$H = 0,\ Q = [0.25\ 0.00\ 0.00\ 0.05],\ \text{and}\ R = 0.05$$

The optimal feedback gain matrix $F(k)$ is shown in Fig. 3-9(a) for $N = 200$. Looking backward from $k = 199$, we observe that at $k \approx 130$ the $F(k)$ matrix has reached the steady-state value.

$$F(k) = [-\ 0.5522 \quad -\ 5.9668],\ 0 \le k \lesssim 130 \qquad (3.10 - 24)$$

The optimal control history and the optimal trajectory for $x(0) = [2 \quad 1]^T$ are shown in Fig. 3-9(b). Notice that the optimal trajectory has essentially reached **0** at $k = 100$. Thus, we would expect that insignificant performance degradation would be caused by simply using the steady-state value of **F** given in (3.10-24) rather than F(k) as specified in Fig. 3-9(a).

Problem 2. 3-3 in p96. The first-order discrete system

$$x(k + 1) = 0.5x(k) + u(k)$$

is to be transferred to the origin in two stages ($x(2) = 0$) while the performance measure

$$J = \sum_{k=0}^{1} [|x(k)| + 5|u(k)|]$$

is minimized

(a) Use the method of dynamic programming to determine the optimal control law for each of the heavily dotted points in Fig. 3-P3. Assume that the admissible control values are quantized into the levels 1, 0.5, 0, −0.5, −1.

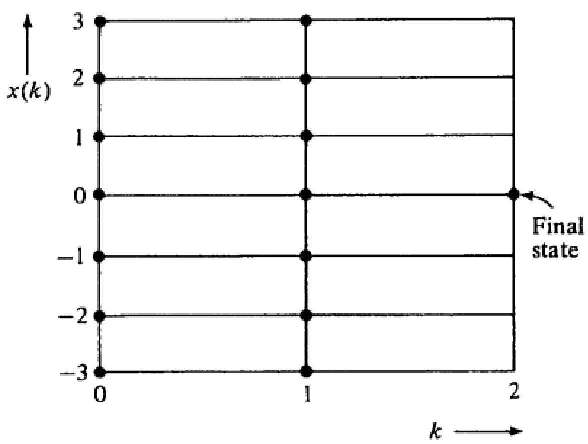

Figure 3-P3

(b) Find the optimal control sequence $\{u^*(0),\ u^*(1)\}$ that corresponds to the initial state $x(0) = -2.$

Problem 3. 3-6 in p98 (Do not use computer programming.)

A discrete system described by the difference equation

$$x(k + 1) = x(k) + u(k)$$

is to be controlled to minimize the performance measure

$$J = \sum_{k=0}^{2} \left[2\left|x(k) - 0.1k^2\right| + |u(k - 1)|\right]$$

The state and control values must satisfy the constraints

$$0.0 \le x(k) \le 0.4,\ \ k = 0,\ 1,\ 2$$

$$-0.2 \le u(k) \le 0.2, \quad k = 0, 1$$

(a) Use the dynamic programming algorithm to determine the optimal control law $u^*(x(k), k)$. Quantize the state into the values $x(k) = 0, \ 0.1, 0.2, 0.3, 0.4$ (k = 0, 1, 2) and the control into the values $u(k) = \ -0.2, \ -0.1, \ 0, \ 0.1, \ 0.2$ (k=0, 1).

(b) Determine the optimal control sequence $\{u^*(0), u^*(1)\}$ if the initial state values is $x(0) = 0.2$

Problem 4. There is a 2-state, single-input, and single-output system. (Use Matlab)

$$x_{k+1} = [1\ 1\ 0\ 1]x_k + [0\ 1]u_k$$

with initial state $x_1 = [1\ 0]$, horizon $N = 20$. If the cost is defined as:

$$J = \sum_{k=0}^{N-1} \left\{ x^T(k)Qx(k) + u^T(k)Ru(k) \right\}$$

and, the coefficients are given, $Q = [1\ 0\ 0\ 1]$, $R = \rho$.

1) Obtain the trajectories of the optimal control and the states when $\rho = 0.3$ and $\rho = 10$.
2) Plot the optimal trade-off curve of J_{out} vs. J_{in} according ρ (0-100), where the functions are expressed as:

$$J_{in} = \sum_{k=0}^{N-1} \|u_k\|^2$$

$$J_{out} = \sum_{k=0}^{N-1} \|x_k\|^2$$