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FINALTERM EXAMINATION
MTH301- Calculus II

Question No: 1 (Marks: 1) - Please choose one

Q 1. Intersection of two straight lines is ----------

► Surface

► Curve

a. ► Plane

b. ► **Point**

Question No: 2 (Marks: 1) - Please choose one

Plane is a ---------- surface.

► One-dimensional

► **Two-dimensional**

► Three-dimensional

► Dimensionless

Question No: 3 (Marks: 1) - Please choose one

Let w = f(x, y, z) and x = g(r, s), y = h(r, s), z = t(r, s) then by chain rule

$\frac{\partial w}{\partial r} =$

- $\frac{\partial w}{\partial x}\frac{\partial x}{\partial r}+\frac{\partial w}{\partial y}\frac{\partial y}{\partial r}+\frac{\partial w}{\partial z}\frac{\partial z}{\partial r}$
- $\frac{\partial w}{\partial r}\frac{\partial x}{\partial r}+\frac{\partial w}{\partial r}\frac{\partial y}{\partial r}+\frac{\partial w}{\partial r}\frac{\partial z}{\partial r}$
- $\frac{\partial w}{\partial x}\frac{\partial x}{\partial r}\frac{\partial x}{\partial s}+\frac{\partial w}{\partial y}\frac{\partial y}{\partial r}\frac{\partial y}{\partial s}+\frac{\partial w}{\partial z}\frac{\partial z}{\partial r}\frac{\partial z}{\partial s}$
- $\frac{\partial w}{\partial r}\frac{\partial r}{\partial x}+\frac{\partial w}{\partial r}\frac{\partial r}{\partial y}+\frac{\partial w}{\partial r}\frac{\partial r}{\partial z}$

Question No: 4 (Marks: 1) - Please choose one

What are the parametric equations that correspond to the following vector equation?

$\vec{r}(t)=\sin^2 t\,\hat{i}+(1-\cos 2t)\,\hat{j}$

- $x=\sin^2 t \quad , \quad y=1-\cos 2t \quad , \quad z=0$
- $y=\sin^2 t \quad , \quad x=1-\cos 2t \quad , \quad z=0$
- $x=\sin^2 t \quad , \quad y=1-\cos 2t \quad , \quad z=1$
- $x=\sin^2 t \quad , \quad y=\cos 2t \quad , \quad z=1$

Question No: 5 (Marks: 1) - Please choose one

What are the parametric equations that correspond to the following vector equation?

$r(t)=(2t-1)\hat{i}-3\sqrt{t}\,\hat{j}+\sin 3t\,\hat{k}$

- $z=2t-1 \quad , \quad x=-3\sqrt{t} \quad , \quad y=\sin 3t$

- $y = 2t - 1 \quad , \quad x = -3\sqrt{t} \quad , \quad z = \sin 3t$
- $x = 2t - 1 \quad , \quad z = -3\sqrt{t} \quad , \quad y = \sin 3t$
- $x = 2t - 1 \quad , \quad y = -3\sqrt{t} \quad , \quad z = \sin 3t$

Question No: 6 (Marks: 1) - Please choose one

What is the derivative of following vector-valued function?

$\vec{r}(t) = (\cos 5t, \tan t \ , 6\sin t)$

- $\vec{r}'(t) = \left(\frac{\sin 5t}{5}, \sec t \ , 6\cos t\right)$
- $\vec{r}'(t) = (\frac{-\sin 5t}{5}, \sec t \ , 6\cos t)$
- $\vec{r}'(t) = (-5\sin 5t, \sec^2 t \ , 6\cos t)$
- $\vec{r}'(t) = (\sin 5t, \sec^2 t \ , -6\cos t)$

Question No: 7 (Marks: 1) - Please choose one

What is the derivative of following vector-valued function?

$\vec{r}(t) = \left(t^4, \sqrt{t+1} \ , \frac{3}{t^2}\right)$

- $\vec{r}'(t) = \left(4t^3, \frac{1}{\sqrt{t+1}} \ , \frac{-6}{t^3}\right)$

- ▶ $\vec{r'}(t) = \left(4t^3, \frac{1}{2\sqrt{t+1}}, \frac{6}{t^3}\right)$
- ▶ $\vec{r'}(t) = \left(4t^4, \frac{1}{2\sqrt{t+1}}, \frac{-6}{t^3}\right)$
- ▶ $\vec{r'}(t) = \left(4t^3, \frac{1}{2\sqrt{t+1}}, \frac{-6}{t^3}\right)$

Question No: 8 (Marks: 1) - Please choose one

The following differential is exact
$dz = (x^2 y + y)\ dx - x\ dy$

- ▶ True
- ▶ **False**

Question No: 9 (Marks: 1) - Please choose one

Which one of the following is correct Wallis Sine formula when n is even and $n \geq 2$?

- ▶ $\int_0^{\frac{\pi}{2}} \sin^n x\ dx = \frac{\pi}{2} \frac{(n-1)}{n} \frac{(n-3)}{(n-2)} \frac{(n-5)}{(n-4)} \cdots\cdots\cdots \frac{5}{6} \frac{3}{4} \frac{1}{2}$
- ▶ $\int_0^{\frac{\pi}{2}} \sin^n x\ dx = \frac{(n-1)}{n} \frac{(n-3)}{(n-2)} \frac{(n-5)}{(n-4)} \cdots\cdots\cdots \frac{6}{7} \frac{4}{5} \frac{2}{3}$
- ▶ $\int_0^{\frac{\pi}{2}} \sin^n x\ dx = \frac{\pi}{2} \frac{(n)}{(n-1)} \frac{(n-2)}{(n-3)} \frac{(n-4)}{(n-5)} \cdots\cdots\cdots \frac{6}{5} \frac{4}{3} \frac{2}{1}$

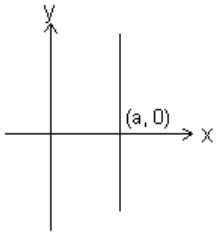
▶ $\int_0^{\frac{\pi}{2}} \sin^n x\, dx = \frac{(n)}{(n-1)} \frac{(n-2)}{(n-3)} \frac{(n-4)}{(n-5)} \text{--------} \frac{6}{5} \frac{4}{3} \frac{2}{1}$

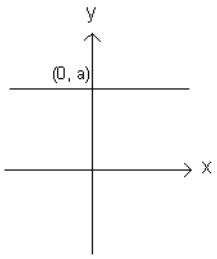
Question No: 10 (Marks: 1) - Please choose one

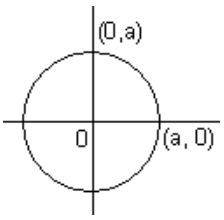
Match the following equation in polar co-ordinates with its graph.

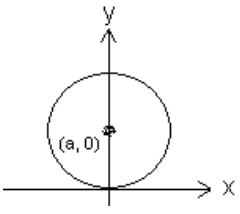
$r \cos\theta = a$

where a is an arbitrary cons tan *t*

▶ 

▶ 

▶ 

▶ 

Question No: 11 (Marks: 1) - Please choose one

If the equation of a curve, in polar co-ordinates, remains unchanged after replacing (r, θ) by $(r, \pi - \theta)$ then the curve is said to be symmetric about which of the following?

► Initial line

► y-axis

► Pole

Question No: 12 (Marks: 1) - Please choose one

If the equation of a curve, in polar co-ordinates, remains unchanged after replacing (r, θ) by $(-r, \theta)$ then the curve is said to be symmetric about which of the following?

► Initial line

► y-axis

► **Pole**

Question No: 13 (Marks: 1) - Please choose one

What is the amplitude of a periodic function defined by $f(x) = \sin\frac{x}{3}$?

► 0

► 1

► $\frac{1}{3}$

► Does not exist

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Question No: 14 (Marks: 1) - Please choose one

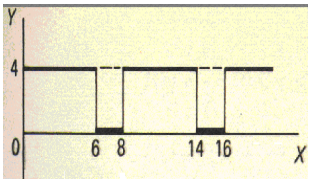
What is the period of a periodic function defined by $f(x) = 4\cos 3x$?

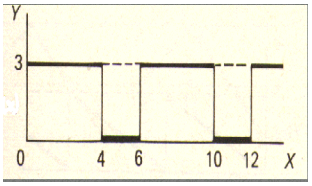
- ▶ $\frac{\pi}{4}$
- ▶ $\frac{\pi}{3}$
- ▶ $\frac{2\pi}{3}$
- ▶ π

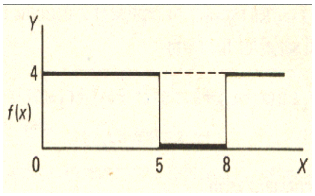
Question No: 15 (Marks: 1) - Please choose one

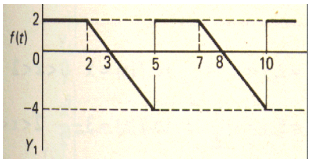
Match the following periodic function with its graph.

$$f(x) = \begin{cases} 3 & 0 < x < 4 \\ 0 & 4 < x < 6 \end{cases}$$

- ▶ 

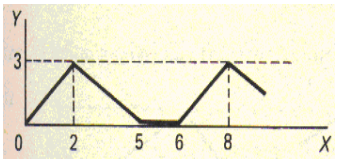
- ▶ 

- ▶ 

► 

Question No: 16 (Marks: 1) - Please choose one

What is the period of periodic function whose graph is as below?



► 2

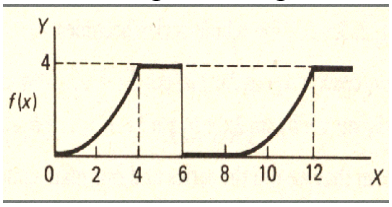
► 5

► 6

► 8

Question No: 17 (Marks: 1) - Please choose one

What is the period of periodic function whose graph is as below?



► 0

► 4

► 6

► 8

Question No: 18 (Marks: 1) - Please choose one

Let L denotes the Laplace Transform.

If $L\{F(t)\} = f(s)$ where s is a constant.and $\lim_{t\to 0}\left(\frac{F(t)}{t}\right)$ exists then which of the following equation holds?

- ▶ $L\left(\frac{F(t)}{t}\right) = f(s+a)$
- ▶ $L\left(\frac{F(t)}{t}\right) = f(s-a)$
- ▶ $L\left(\frac{F(t)}{t}\right) = \int_s^\infty f(s)\,ds$
- ▶ $L\left(\frac{F(t)}{t}\right) = -\frac{d}{ds}\{f(s)\}$

Question No: 19 (Marks: 1) - Please choose one

Which of the following is Laplace inverse transform of the function $f(s)$ defined by $f(s) = \frac{3}{s-2} - \frac{2}{s}$?

- ▶ $3te^{2t} - 2$
- ▶ $3e^{2t} - 2t$
- ▶ $3e^{2t} - 2$
- ▶ None of these.

Question No: 20 (Marks: 1) - Please choose one

Let (x_1, y_1, z_1) and (x_2, y_2, z_2) be any two points in three dimensional space. What does the formula $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$ calculates?

► Distance between these two points

► Midpoint of the line joining these two points

► Ratio between these two points

Question No: 21 (Marks: 1) - Please choose one

Let the functions $P(x, y)$ and $Q(x, y)$ are finite and continuous inside and at the boundary of a closed curve C in the xy-plane. If $(P\,dx + Q\,dy)$ is an exact differential then

$\oint_C (P\,dx + Q\,dy) =$

► Zero

► One

► Infinite

Question No: 22 (Marks: 1) - Please choose one

What is Laplace transform of the function $F(t)$ if $F(t) = t$?

- ▶ $L\{t\} = \frac{1}{s}$
- ▶ $L\{t\} = \frac{1}{s^2}$
- ▶ $L\{t\} = e^{-s}$
- ▶ $L\{t\} = s$

Question No: 23 (Marks: 1) - Please choose one

What is the value of $L\{e^{5t}\}$ if L denotes laplace transform?

- ▶ $L\{e^{5t}\} = \frac{1}{s-5}$
- ▶ $L\{e^{5t}\} = \frac{s}{s^2+25}$
- ▶ $L\{e^{5t}\} = \frac{5}{s^2+25}$
- ▶ $L\{e^{5t}\} = \frac{5!}{s^6}$

Question No: 24 (Marks: 1) - Please choose one

Evaluate the line integral $\int_C (3x+2y)\,dx + (2x-y)\,dy$ where C is the line segment from (0, 0) to (0, 2).

- ▶ 1

► 0

► 2

► -2

Question No: 25 (Marks: 1) - Please choose one

Evaluate the line integral $\int_C (2x+y)\,dx+(x^2-y)\,dy$ where C is the line segment from (0, 0) to (2, 0).

► 0

► -4

► 4

► Do not exist

Question No: 26 (Marks: 1) - Please choose one

Which of the following are direction ratios for the line joining the points $(1, 3, 5)$ and $(2, -1, 4)$?

► 3, 2 and 9

► 1, -4 and -1

► 2, -3 and 20

► 0.5, -3 and 5/4

Question No: 27 (Marks: 1) - Please choose one

$If\ R = \{(x,y)/0 \le x \le 2\ and\ 1 \le y \le 4\}, then$

$\iint_R (6x^2 + 4xy^3)dA =$

► $\int_1^4 \int_0^2 (6x^2 + 4xy^3)dydx$

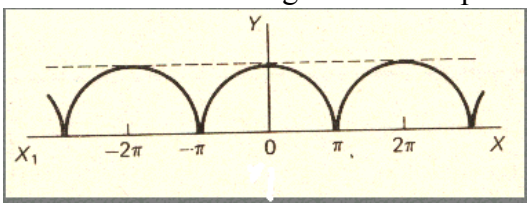
► $\int_0^2 \int_1^4 (6x^2 + 4xy^3)dxdy$

► $\int_1^4 \int_0^2 (6x^2 + 4xy^3)dxdy$

► $\int_2^4 \int_0^1 (6x^2 + 4xy^3)dxdy$

Question No: 28 (Marks: 1) - Please choose one

Which of the following is true for a periodic function whose graph is as below?

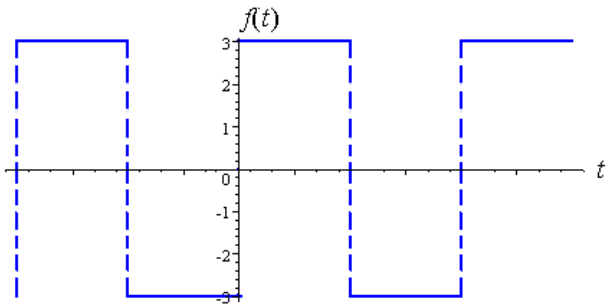


► Even function

► Odd function

► Neither even nor odd function

Question No: 29 (Marks: 1) - Please choose one



Which of the following is true for a function whose graph is given above

► An odd function

► An even function

► Neither even nor odd

Question No: 30 (Marks: 1) - Please choose one

At each point of domain, the function ----------

► Is defined

► Is continuous

► Is infinite

► Has a limit

Question No: 31 (Marks: 2)

Determine whether the following differential is exact or not.

$dz = 4x^3y^3\ dx + 3x^4y^2\ dy$

Solution:

$dz = 4x^3y^3\ dx + 3x^4y^2\ dy$

$\frac{\partial p}{\partial y} = 12x^3y^2$

$\frac{\partial Q}{\partial X} = 12x^3y^2$

$\frac{\partial p}{\partial y} = \frac{\partial Q}{\partial X}$

yes

Question No: 32 (Marks: 2)

Evaluate

$$\int_{-\pi}^{\pi} \sin\ nx\ dx$$

where n is an integer other than zero.

Solution:

$$\int_{-\pi}^{\pi} \sin nx\, dx$$
$$= \left[\frac{-\cos nx}{n}\right]_{-\pi}^{\pi}$$
$$= \left[\frac{-\cos n\pi}{n} + \frac{\cos n\pi}{n}\right]$$
$$= \frac{1}{n}(-\cos n\pi + \cos n\pi)$$
$$= 0$$

Question No: 33 (Marks: 2)

Find Laplce transform of the function $F(t)$ if $F(t) = e^{3t}$

Solution:

$$L(e^{3t}) = \int_0^{\infty} e^{3t} - e^{-st}$$
$$= \int_0^{\infty} e^{-(s-3)t}.dt$$
$$= \{\frac{e^{-(s-3)t}}{-(s-3)}\} \lim 0 - \infty$$
$$= \frac{-1}{s-3}(\frac{1}{e^{(s-3)t}})$$
$$= \frac{-1}{s-3}(0-1)$$
$$= \frac{1}{s-3} \text{.....Ans}$$

Question No: 34 (Marks: 3)

Determine the Fourier co-efficient a_0 of the periodic function defined below:

$f(x) = 2x+1 \qquad 0 < x < 2$

Solution:

$$a_{\mathbb{N}} = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)dx$$
$$f(x) = (2x+1)$$
$$(0,2)$$
$$= \int_0^2 (2x+1)dx$$
$$= \left[x^2 + x\right]_0^2$$
$$= 6$$

Question No: 35 (Marks: 3)

Determine whether the following differential is exact or not.

$$dz = (3x^2e^{2y} - 2y^2e^{3x})\ dx + (2x^3e^{2y} - 2ye^{3x})\,dy$$

Solution:

dz = Pdx + Qdy

Therefore,

For dz to be an exact differential it must satisfy $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$

But this test fails becuase $\frac{\partial P}{\partial y} \neq \frac{\partial Q}{\partial x}$

Not Exact

Question No: 36 (Marks: 3)

Use Wallis sine formula to evaluate $\int_0^{\frac{\pi}{2}} \left(\sin^3 x + \sin^5 x\right)dx$

Solution:

$$\int_0^{\frac{\pi}{2}} \sin^3 x dx$$
$$= \frac{n-1}{n}.$$
$$= \frac{3-1}{3}$$
$$= \frac{2}{3}$$
$$\int_0^{\frac{\pi}{2}} \sin^5 x dx$$
$$= \frac{n-1}{n}.\frac{n-3}{n-2}$$
$$= \frac{5-1}{5}.\frac{5-3}{5-2}$$
$$= \frac{4}{5}.\frac{2}{3}$$
$$\int_0^{\frac{\pi}{2}} \left(\sin^3 x + \sin^5 x\right) dx$$
$$= \frac{2}{3} + \frac{4}{5}.\frac{2}{3}$$

Question No: 37 (Marks: 5)

Evaluate the following line integral which is independent of path.

$$\int_{(0,0)}^{(3,2)} (2xe^y)\, dx + (x^2e^y)\, dy$$

Solution:

$$p = \frac{\partial z}{\partial x} = 2e^y \qquad \int 2e^y dx$$

$$Q = \frac{\partial z}{\partial y} = x^2 e^y \qquad \int x^2 e^y dy$$

$$z = \int_{(0,0)}^{(3,2)} 2xe^y + x^2 ye^y$$

$$z = 6e^2 + 18e^2$$

$$z = 24e^2$$

Question No: 38 (Marks: 5)

Determine the Fourier coefficients b_n for a periodic function $f(t)$ of period 2 defined by

$$f(t) = \begin{cases} 4\ (1 + t) & -1 < t < 0 \\ 0 & 0 < t < 1 \end{cases}$$

Solution:

$$bn = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx dx$$

$$= \frac{1}{\pi}\int_{-1}^{1} 4(1+t)\sin nx dx$$

$$= \frac{1}{\pi}\left[\frac{-4(1+t)\cos nx}{n}\right]_{-1}^{1}$$

$$= \frac{-4(1+t)}{\pi n}\left[\cos n(1) - \cos n(-1)\right]$$

$$= \frac{-4(1+t)}{\pi n}(\cos n + \cos n)$$

Question No: 39 (Marks: 5)

Determine whether the following vector field $\vec{F}$ is conservative or not.

$$\vec{F}(x,\ y,\ z) = (4x - z)\hat{i} + (3y + z)\hat{j} + (y - x)\hat{k}$$

……..