

Day 3.6: ARC LENGTH

The formula giving the arc length of the curve $y = f(x)$ from $x = a$ to $x = b$ is given by

$$L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$$

For most functions, this integral expression is too difficult (or impossible) to evaluate by hand, and in that case, you should just set up the integral and then evaluate it with your calculator.

Similarly, the formula giving the arc length of the curve $x = g(y)$ from $y = c$ to $y = d$ is given by

$$L = \int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2} dy$$

The formula giving the arc length of a parametric curve from $t = e$ to $t = f$ is given by

$$L = \int_e^f \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$$

Remarkably, the formula giving the arc length of the polar curve $r = h(\theta)$ from $\theta = \alpha$ to $\theta = \beta$ is given by

$$L = \int_\alpha^\beta \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} d\theta$$

1) Let $f(x) = \frac{4}{3}x + 1$ from $x = 0$ to $x = 3$.

a) Find the arc length of f without using integrals.

b) Find the arc length of f using integrals **without** using your calculator.

2) Let $f(x) = \sqrt{9-x^2}$ from $x=0$ to $x=3$.

a) Find the arc length of f without using integrals.

b) Find the arc length of f using integrals using your calculator.

3) Find the arc length of $f(x) = \sin x$ from $x=0$ to $x=\pi$ (one arc of the sine curve).

4) Find the arc length of the cardioid $r = 1 + \cos\theta$. Show a set-up step.

5) Find the full arc length (both loops) of the limaçon $r = 1 + 2\cos\theta$. Show a set-up step.

- 6) The length of the arc of $y = \ln \cos x$ from $x = \frac{\pi}{4}$ to $x = \frac{\pi}{3}$ equals

(A) $\ln \frac{\sqrt{3} + 2}{\sqrt{2} + 1}$ (B) 2 (C) $\ln(1 + \sqrt{3} - \sqrt{2})$
(D) $\sqrt{3} - 2$ (E) $\frac{\ln(\sqrt{3} + 2)}{\ln(\sqrt{2} + 1)}$

- 7) The length of the arc of the curve $y^2 = x^3$ cut off by the line $x = 4$ is

(A) $\frac{4}{3}(10\sqrt{10} - 1)$ (B) $\frac{8}{27}(10^{3/2} - 1)$ (C) $\frac{16}{27}(10^{3/2} - 1)$
(D) $\frac{16}{27}10\sqrt{10}$ (E) none of these