LUMEN LEARNING

College Algebra Corequisite Instructor Guide

Module 1: Algebra Essentials

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Learning Outcomes

Detailed Course Learning Outcome Spreadsheet is linked here.

Topic	Student Learning Goals
Introduction to Real Numbers	• Categorize real numbers as counting numbers, whole numbers, rational numbers, irrational numbers, or integers • Recognize and use the properties of real numbers • Simplify and evaluate an algebraic equation
Exponents and Scientific Notation	• Understand and use the rules for exponents • Change numbers between scientific notation and standard notation • Solve calculations using scientific notation
Roots and Rational Exponents	• Calculate square roots and apply them in basic operations like addition, subtraction, and rationalizing the denominators. • Use the product and quotient rules to simplify expressions that include square roots. • Understand how to use rational exponents in expressions and recognize their connection to roots for simplifying calculations.

Summary of Module

Background You'll Need

The assumed prerequisite skills:

- Use prime factorization techniques to break down numbers into their prime factors.
- Add, subtract, multiple and divide fractions
- Learn to spot the difference between letters and numbers in math expressions and combine similar terms to simplify them.

Introduction to Real Numbers

In this section, students explore the foundations of real numbers and algebraic expressions. They begin by categorizing different types of real numbers, visualizing their relationships on a number line. Students then dive into the properties of real numbers, practicing how to apply commutative, associative, distributive, identity, and inverse properties to simplify calculations. They learn to navigate complex expressions using the order of operations (PEMDAS), solving problems step-by-step. The section then transitions to algebraic concepts, introducing students to constants, variables, and algebraic expressions. Students gain hands-on experience evaluating expressions by substituting values for variables and simplifying the results. They practice combining like terms, using the distributive property, and simplifying fractions within algebraic contexts. Throughout the section, students apply these skills to real-world scenarios, using formulas to solve practical problems involving areas, volumes, and other measurable quantities.

Exponents and Scientific Notation

In this section, students explore exponential notation, rules for exponents, and scientific notation. They begin by understanding exponential notation as a compact way to represent repeated multiplication. Students then dive into various exponent rules, including the product rule, quotient rule, power rule, zero exponent rule, and negative exponent rule. They practice applying these rules to simplify and evaluate expressions with exponents. The section transitions to scientific notation, teaching students how to convert between standard and scientific notation for very large and very small numbers. Students learn to perform calculations using scientific notation, including multiplication and division. Throughout the section, students apply these skills to real-world scenarios, such as calculating atomic quantities or dealing with astronomical distances.

Radicals and Rational Exponents

In this section, students explore square roots, higher-order roots, and rational exponents. They begin by understanding the concept of square roots and their properties, including the product and quotient rules for radicals. Students learn to simplify radical expressions, combine like radicals, and rationalize denominators. They practice adding, subtracting, and multiplying expressions involving radicals. The section then introduces higher-order roots (cube roots, fourth roots, etc.) and the concept of principal nth roots. Students learn to work with rational exponents as an alternative notation for radicals, understanding the relationship between fractional exponents and roots. Throughout the section, students apply these concepts to simplify complex expressions, solve equations involving radicals, and rationalize denominators in fractions containing radicals. They also learn to convert between radical notation and exponential notation with rational exponents.

Module Resources

Cheat Sheet

Algebra Essentials: Cheat Sheet

Worksheets/Handouts

Classifying Connections: The Real Number Game Handout

Classifying Connections: The Real Number Game Answer Key

Real Number Operations Discussion

Scientific Notation and Radical Expressions Writing Task

Activity One: Classifying Connections: The Real Number Game

Evidence-Based Teaching Practice

Engagement

Educators engage students in activities that require them to do more than passively listen to a live or recorded lecture by having students actively participate in evaluating expressions, classifying numbers, and discussing their findings in small groups and as a class.

Higher Order Thinking

Educators engage students in activities specifically designed to cultivate level-appropriate critical thinking, complex problem solving, analytical reasoning, abstract reasoning, and creative thinking by requiring students to evaluate mathematical expressions, classify results, and analyze patterns and relationships between different types of numbers.

Collaboration

Educators design group projects that require students to work together in order to help them master both core content and critical skills related to collaboration and teamwork by having students work in groups to evaluate expressions, classify results, and create groupings of number types.

Background

Students have learned about classifying real numbers as natural, whole, integers, rational or irrational. They have studied the properties of real numbers and the order of operations, and they have practiced simplifying and evaluating expressions involving fractions, variables, exponents, scientific notation, square roots and nth roots. This activity gives students the opportunity to bring together the essential components of algebra while thinking about how different classifications of numbers relate to one another.

Instructions

Time Estimate: 45-60 minutes

1. **Conversation starter**
   Ask students if they've played the game called Connections by the New York Times. Briefly explain how to play the game, emphasizing the concept of grouping related items.

   https://www.nytimes.com/games/connections

2. **Review**
   You may wish to review the classification of real numbers, ensuring students remember the definitions and relationships between natural, whole, integer, rational, and irrational numbers.

3. **Split the class into groups of 3-5 students**
   Split the class into groups of 3-4 students. Ensure each group has access to paper or a digital platform for calculations and note-taking.

4. **Distribute Materials**
   Provide each group with the handout containing 16 mathematical expressions and scissors. If working digitally, share a document where students can easily move the expressions around.

5. **Expression Evaluation**
   Instruct groups to evaluate each of the 16 expressions without using calculators. They should write their answers below each expression.

6. **Classification Discussion**
   Have groups discuss and classify each result as natural, whole, integer, rational, or irrational. Encourage them to consider how expressions with one type of number can result in a different classification. Challenge groups to create four groups (integers, irrational, rational, and whole numbers) of four expressions. Remind them that a number can fit multiple classifications, but there's only one way to group them into four distinct categories.

7. **Class Discussion**
   Facilitate a class discussion about the groupings and any surprising results or challenges faced.

Discussion Prompts

- **Consider the expressions that resulted in irrational numbers. What common operations or number types do you see in these expressions? Can you create a new expression that you're certain will result in an irrational number?**

  Misconception: Students often believe that any expression involving irrational numbers (like π or √2) will always result in an irrational number. However, this isn't always the case. For example, π - π = 0, which is rational.

- **Look at the expressions that resulted in rational numbers. What patterns do you notice in these expressions? Are there any operations that seem to "generate" rational numbers more often?**

  Goal: This question aims to deepen students' understanding of rational numbers and the operations that produce them. Students should recognize that arithmetic operations between rational numbers always yield rational results, reinforcing the concept of closure. They should also note that division often generates rational numbers, even with integers, connecting this to the formal definition of rational numbers.

- **In real-world applications, we often use rational approximations for irrational numbers. How might this affect calculations and decision-making in fields like engineering or finance?**

  Goal: This question encourages students to connect mathematical concepts with practical applications. Students should consider how approximations can lead to accumulated errors in complex calculations, potentially causing significant issues in precise engineering or financial models. However, they should also recognize that these approximations often provide sufficient accuracy for many practical purposes and allow for easier computation.

Reflection

After the activity, we recommend that students complete exit cards. Have each student write on a piece of paper one key concept they learned from the activity and one concept they have questions about. Below are some suggestions for students:

- Which type of expression did you find most challenging to simplify or evaluate, and why?
- How does the simplification process affect the classification of the resulting value? Give an example.
- Describe a real-world scenario where understanding the classification of numbers would be important.

Online Variation

For an online classroom, create a Google Slides document with the 16 expressions as movable text boxes. Share this document with students, allowing them to edit. Have students work in virtual breakout rooms to discuss, evaluate, and group the expressions. Use a shared document or chat function for students to submit their exit card responses. To facilitate discussion, you can use online polling tools to gauge initial responses to discussion questions, then use video conferencing features like "raise hand" or chat to call on students for more detailed explanations. Encourage students to use the digital whiteboard feature to show their work when evaluating complex expressions.

Assignments

Real Number Operations Discussion

In this discussion, students will explore practical applications of real number operations, exponents, and radicals across various fields. Students will identify a real-world scenario where these mathematical concepts are used to solve problems or model relationships, analyzing how specific operations help address challenges in areas such as architecture, finance, science, technology, sports, or healthcare. Students will create detailed posts demonstrating at least three different mathematical operations from the module with specific numerical examples, explaining how understanding these operations aids in problem-solving within their chosen field. Additionally, students will engage with their peers by responding to at least two classmates' posts.

Real Number Operations Discussion

We ask that you make your own copy to edit and adjust to fit the needs of your classroom

Scientific Notation and Radical Expressions Writing Task

In this assignment, students will apply scientific notation, exponents, and radical expressions to solve practical problems in a real-world scenario involving solar energy design. Working as part of a hypothetical team designing a solar power installation for a small community, students will calculate energy requirements using scientific notation, determine panel quantities using rational exponents, design panel arrays using properties of exponents, calculate support post heights using the Pythagorean theorem with simplified radicals, analyze energy production decay rates with rational exponents, and perform cost analyses.

Scientific Notation and Radical Expressions Writing Task

We ask that you make your own copy to edit and adjust to fit the needs of your classroom