Prompt: If $f(x) = \sin sin\, x\ \sinh sinh\, x$, then $f^{4n}(x) = (-\ 4)^{n} f(x)$

1. To show the prompt is true

$f(x) = \sin sin\, x \sinh sinh\, x$

$f'(x) = \sin sin\, x \cosh cosh\, x + \cos cos\, x \sinh sinh\, x$

$f''(x) = \sin sin\, x \sinh sinh\, x + \cos cos\, x \cosh cosh\, x - \sin sin\, x \sinh sinh\, x + \cos cos\, x \cosh cosh\, x \quad = 2$ c

$f'''(x) = 2x \sinh sinh\, x \ - \sin sin\, x \cosh cosh\, x)$

$f^{iv}(x) = 2x \cosh cosh\, x \ - \sin sin\, x \sinh sinh\, x \ - \sin sin\, x \sinh sinh\, x - \cos cos\, x \cosh cosh\, x\ )$

$= 2x \sinh sinh\, x\ ) = x \sinh sinh\, x \ = (-\ 4)^{1} f(x)$

$f^{v}(x) = (-\ 4)^{1} f'(x)$

$f^{vi}(x) = (-\ 4)^{1} f''(x)$

$f^{vii}(x) = (-\ 4)^{1} f'''(x)$

$f^{viii}(x) = (-\ 4)^{1} f^{iv}(x) = (-\ 4)^{2} f(x)$

$f^{ix}(x) = (-\ 4)^{2} f'(x)$

$f^{x}(x) = (-\ 4)^{2} f''(x)$

$f^{xi}(x) = (-\ 4)^{2} f'''(x)$

$f^{xii}(x) = (-\ 4)^{2} f^{iv}(x) = (-\ 4)^{3} f(x)$

The derivatives will continue in the same way and, in general, $f^{4n}(x) = (-\ 4)^{n} f(x)$

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2. Using $\sinh sinh\, x = \frac{e^{x} - e^{-x}}{2}$ **to differentiate**

$f(x) = \sin sin\, x \sinh sinh\, x \ = \sin sin\, x \left(\frac{e^{x} - e^{-x}}{2}\right) = \frac{1}{2}\left(e^{x} \sin sin\, x \ - e^{-x} \sin sin\, x\right)$

$f'(x) = \frac{1}{2}\left(e^{x} \cos cos\, x \ + e^{x} \sin sin\, x \ - e^{-x} \cos cos\, x \ + e^{-x} \sin sin\, x\right)$

$= \frac{1}{2}\left[(e^{x} - e^{-x}) \cos cos\, x \ + (e^{x} + e^{-x} x\right]$

$f''(x) = \frac{1}{2}\left[\left(e^{x} - e^{-x}\right) x) \ + \left(e^{x} + e^{-x}\right) \cos cos\, x + \ (e^{x} + e^{-x} x \ + \left(e^{x} - e^{-x}\right) \sin sin\, x\right]$

$= \left(e^{x} + e^{-x}\right) \cos cos\, x$

$f'''(x) = \left(e^{x} + e^{-x}\right)(-\sin sin\, x) + \left(e^{x} - e^{-x}\right)\cos cos\, x$

$f^{iv}(x) = \left(e^{x} + e^{-x}\right)(-\cos cos\, x) + \left(e^{x} - e^{-x}\right)(-\sin sin\, x) + \left(e^{x} - e^{-x}\right)(-\sin sin\, x) + \left(e^{x} + e^{-x}\right)\cos cos$

$=-2\sin\left(e^{x} - e^{-x}\right) sin\, x =-4\left(\frac{e^{x}-e^{-x}}{2}\right)\sin sin\, x =-4\sin sin\, x \sinh sinh\, x = (-4)^{1} f(x)$

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3. An additional generalisation

$f''(x) = 2\cos cos\, x \cosh cosh\, x$

$f^{vi}(x) = (-4)^{1} f''(x)$

$f^{x}(x) = (-4)^{2} f''(x)$

In general, $f^{4n-2}(x) = 2(-4)^{n-1}(\cos cos\, x \cosh cosh\, x)$