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B. Tech. (Mech. Engg.) (Semester – 1st)

MATHEMATICS – I (CALCULUS & LINEAR ALGEBRA)

Subject Code: BMATH 2101

Paper ID: [18112302]

Time: 03 Hours **Maximum Marks: 60**

Instruction for candidates:

1. Section A is compulsory. It consists of 10 parts of two marks each.
2. Section B consist of 5 questions of 5 marks each. The student has to attempt any 4 questions out of it.
3. Section C consist of 3 questions of 10 marks each. The student has to attempt any 2 questions.

Section – A **(2 marks each)**

Q1. Attempt the following:

a) Evaluate $\frac{x^3-a^3}{x^2-a^2}$.

b) Verify Rolle's theorem for $f(x) = (x - 1)^2 (x - 2)^2$ in the interval $[1, 2]$.

c) Check the convergence of the sequence whose n^{th} term is $a_n = \frac{n+1}{2}$.

d) Find radius of convergence of the series $f(x) = \sum_{n=1}^{\infty} \frac{x^n}{n}$

e) Find the limit $\frac{2n+1}{n+3}$.

f) Check the continuity of the function $f(x) = x^2$ at x=0.

g) Find curl of the function $(x) = xy + x^2$.

h) Find the eigen values of the matrix [1 2 0 1] .

i) Find the determinant of the matrix [2 4 − 1 2] .

j) Check whether the matrix [2 3 3 5] is symmetric or not?

Section – B **(5 marks each)**

Q2. Solve I = $\int_0^{\frac{\pi}{2}} sin^7\theta . cos^7\theta d\theta$ by using Beta function.

Q3. Test the convergence of the series $\frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \ldots$.

Q4. Evaluate the improper integral $\int_1^2 \frac{x}{\sqrt{x-1}}$ dx.

Q5. Solve the system of linear equation using matrix method:

4x + 3y + 2z = −7, 2x + y − 4z = −1, x + 2y + z = 1

Q6. Examine the function for maxima and minima: $f(x, y) = x^3 + 3x^2 - y^2$.

Section – C **(10 marks each)**

Q7. (a) Evaluate $(x \tan\frac{1}{x})$.

(b) Find Taylor's series expansion of $f(x) = \sin x$ about origin.

Q8. (a) Test the convergence of the series by D'Alembert 's ratio test $\frac{1!}{5} + \frac{2!}{5^2} + \frac{3!}{5^3} + \ldots$.

(b) If $\frac{x}{y}$) , then verify $\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial y \partial x}$.

Q9. (a) Verify Caley - Hamilton theorem for the matrix: $A = [1\ 0\ 2\ 0\ -1\ 1\ 0\ 1\ 0]$.

(b) If $A = [1\ 1\ 3\ 1\ 3\ -3\ -2\ -4\ -4]$, then find A^{-1}.