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**B. Tech. (Mech. Engg.) (Semester – 1<sup>st</sup>)**  
**MATHEMATICS – I (CALCULUS & LINEAR ALGEBRA)**

**Subject Code: BMATH 2101**

**Paper ID: [18112302]**

**Time: 03 Hours**

**Maximum Marks: 60**

**Instruction for candidates:**

1. Section A is compulsory. It consists of 10 parts of two marks each.
2. Section B consist of 5 questions of 5 marks each. The student has to attempt any 4 questions out of it.
3. Section C consist of 3 questions of 10 marks each. The student has to attempt any 2 questions.

**Section – A**

**(2 marks each)**

**Q1.** Attempt the following:

- a) Evaluate  $\frac{x^3 - a^3}{x^2 - a^2}$ .
- b) Verify Rolle's theorem for  $f(x) = (x - 1)^2 (x - 2)^2$  in the interval  $[1, 2]$ .
- c) Check the convergence of the sequence whose  $n^{th}$  term is  $a_n = \frac{n+1}{2}$ .
- d) Find radius of convergence of the series  $f(x) = \sum_{n=1}^{\infty} \frac{x^n}{n}$ .
- e) Find the limit  $\frac{2n+1}{n+3}$ .
- f) Check the continuity of the function  $f(x) = x^2$  at  $x=0$ .
- g) Find curl of the function  $(x) = xy + x^2$ .
- h) Find the eigen values of the matrix  $\begin{bmatrix} 1 & 2 & 0 & 1 \end{bmatrix}$ .
- i) Find the determinant of the matrix  $\begin{bmatrix} 2 & 4 & -1 & 2 \end{bmatrix}$ .
- j) Check whether the matrix  $\begin{bmatrix} 2 & 3 & 3 & 5 \end{bmatrix}$  is symmetric or not?

**Section – B**

**(5 marks each)**

**Q2.** Solve  $I = \int_0^{\frac{\pi}{2}} \sin^7 \theta \cdot \cos^7 \theta d\theta$  by using Beta function.

**Q3.** Test the convergence of the series  $\frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \dots$ .

**Q4.** Evaluate the improper integral  $\int_1^2 \frac{x}{\sqrt{x-1}} dx$ .

**Q5.** Solve the system of linear equation using matrix method:

$$4x + 3y + 2z = -7, \quad 2x + y - 4z = -1, \quad x + 2y + z = 1$$

Q6. Examine the function for maxima and minima:  $f(x, y) = x^3 + 3x^2 - y^2$ .

**Section – C**

**(10 marks each)**

Q7. (a) Evaluate  $(x \tan \frac{1}{x})$ .

(b) Find Taylor's series expansion of  $f(x) = \sin x$  about origin.

Q8. (a) Test the convergence of the series by D'Alembert's ratio test  $\frac{1!}{5} + \frac{2!}{5^2} + \frac{3!}{5^3} + \dots$ .

(b) If  $\frac{x}{y}$ , then verify  $\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial y \partial x}$ .

Q9. (a) Verify Caley - Hamilton theorem for the matrix:  $A = \begin{bmatrix} 1 & 0 & 2 & 0 \\ -1 & 1 & 0 & 1 & 0 \end{bmatrix}$ .

(b) If  $A = \begin{bmatrix} 1 & 1 & 3 & 1 & 3 \\ -3 & -2 & -4 & -4 \end{bmatrix}$ , then find  $A^{-1}$ .