Self-inverse and periodic functions

Some functions, if repeatedly used (i.e. iterated) return to a start value and it can be interesting to explore these - by operating with fractions preferably, or using a calculator.

It can be demanding to firmly establish how many steps it takes to return to a start value (the period) algebraically.

The 'usual' self-inverse functions are of a form similar to:
$f(x) = 10 - x$ and
$f(x) = 12/x$

What happens if you work out 10 - 3 and then subtract the result from 10?

This leads to an appreciation that 10 - (10 - n) is n

However, there are many more examples of self-inverse rational functions
Here are a few:

explore these functions

$$f(x) = \frac{x + 5}{2x - 1}$$

$$f(x) = \frac{3x - 1}{2x - 3}$$

$$f(x) = \frac{2x + 1}{x - 2}$$

$$f(x) = \frac{3x + 5}{4x - 3}$$

$$f(x) = \frac{4x - 1}{2x - 4}$$

rational functions

f(7) means substitute 7 in place of x
$f^2(7)$ means work out f(7) and then use this value as a new input value
(i.e. do ‘f’ then do ‘f’ again)

(1)

$$f(x) = \frac{2x + 1}{x - 2}$$

(i) work out:
(a) f(7) (b) f(1) (c) f(12) (d) f(3) (e) f(– 3) (f) f(2.5)

(ii) work out and simplify:
(a) f(27) (b) f(52) (c) f(17) (d) f(2.2) (e) f(2.1) (f) f(2⅓)

(2)

$$f(x) = \frac{3x + 1}{x - 3}$$

(i) work out:
(a) f(4) (b) f(5) (c) f(1) (d) f(13) (e) f(8) (f) f(– 2)

(ii) work out and simplify:
(a) f(23) (b) f(28) (c) f(53) (d) f(103) (e) f(3.5) (f) f(3.4)

(3)

$$f(x) = \frac{2x + 2}{x - 2}$$

(i) work out:
(a) f(3) (b) f(4) (c) f(1) (d) f(8) (e) f(5) (f) f(– 4)

(ii) work out and simplify:
(a) f(6) (b) f(7) (c) f(12) (d) f(14) (e) f(17) (f) f(22) (g) $f^2(32)$ (h) $f^2(62)$

(4)

$$f(x) = \frac{4x + 2}{x - 4}$$

(i) work out:
(a) f(5) (b) f(6) (c) f(7) (d) f(10) (e) f(13) (f) f(22)

(ii) work out and simplify:
(a) f(14) (b) f(16) (c) f(19) (d) f(24) (e) f(34) (f) f(40) (g) $f^2(49)$ (h) $f^2(64)$
(i) f(?) = 4.2 (j) f(?) = 4.1 (k) f(?) = 4

For practice in dividing fractions and working with directed numbers, self-inverse functions provide an interesting context:

- put a number ('seed' value) into one of the functions (above)
- feed the output (y-value) back into the function

What happens?
Try this with several input (start) numbers...

Why is the process problematic if the denominator is = 0?
Other than for a problematic x value, does the number always seem to return to the start?
What if x is a fraction, or a negative number, or a negative fraction?

What happens with these functions?

explore these functions

$$f(x) = \frac{1 - x}{1 + x}$$

$$f(x) = \frac{x - 1}{x + 1}$$

$$f(x) = \frac{1 + x}{1 - x}$$

$$f(x) = \frac{2x - 4}{x}$$

$$f(x) = 1 - \frac{1}{x}$$