ELL Mathematical Language Routines

Understanding Language/SCALE Stanford | GRADUATE SCHOOL OF EDUCATION

The eight mathematical language development routines included here were selected because they are the most effective and practical for simultaneously learning mathematical practices, content, and language. These routines also can be used in most lessons and across grade levels. A “math language routine” refers to a structured but adaptable format for amplifying, assessing, and developing students’ language. The routines emphasize the use of language that is meaningful and purposeful, not inauthentic or simply answer-based. These routines can be adapted and incorporated across lessons in each unit to fit the mathematical work wherever there are productive opportunities to support students in using and improving their English and disciplinary language.

These routines facilitate attention to student language in ways that support in-the-moment teacher-, peer-, and self-assessment. The feedback enabled by these routines will help students revise and refine not only the way they organize and communicate their own ideas, but also ask questions to clarify their understandings of others’ ideas.

MLR1: Stronger and Clearer Each Time

Purpose: To provide a structured and interactive opportunity for students to revise and refine both their ideas and their verbal and written output. This routine provides a purpose for student conversation as well as fortifies output. The main idea is to have students think or write individually about a response, use a structured pairing strategy to have multiple opportunities to refine and clarify the response through conversation, and then finally revise their original written response. Throughout this process, students should be pressed for details, and encouraged to press each other for details.

Example 1. Successive Pair Shares

1. PRE-WRITE: Have students, individually, look at a problem and write down their idea/reasoning for solving the problem a certain way, or any thoughts or questions about it, in complete sentences if possible. This is the pre-write sample; there will be a post-write to see if the sharing with others makes a difference.
2. THINK TIME: Then give a minute for students to think about what they will say to the first partner to explain what they are doing, or did, to solve it. (They can't look at what they wrote while talking).
3. STRUCTURED PAIRING: Use a successive pairing structure. (For example: Have students get into groups of 6 or 8, with inner circles of 3 or 4 facing outer circles of 3 or 4). Remind students that oral clarity and explaining reasoning are important. Even if they have the right answer or they both agree, the goal is either (1) to be able to clearly explain it to others as a mathematician would or (2) for the other person to truly understand the speaker's ideas. Goal (1) is appropriate when students are further along in the development of a concept; goal (2) is appropriate closer to when students are first introduced to a concept.
4. IN PAIRS: When one partner is listening, he or she can ask clarifying questions, especially related to justifying (Why did you do that?…). The other person then also shares and the listener also asks clarifying questions to draw more language and ideas out of quiet partners, if needed.
5. SWITCH: Partners switch one, two, or three more times, strengthening and clarifying their idea each time they talk to a new partner. Optionally, turns can emphasize strength (focus on math concepts and skills) or clarity (how to describe the math to others).
6. POST-WRITE: Have students return to seats and write down their final explanations, in sentences (they can use drawings, too, explained by sentences). Turn in.

MLR2: Collect and Display

Purpose: To capture students' oral words and phrases into a stable, collective reference. The intent of this routine is to stabilize the fleeting language that students use during partner, small-group, or whole-class activities in order for student's own output to be used as a reference in developing their mathematical language. The teacher listens for, and scribes, the student output using written words, diagrams and pictures; this collected output can be organized, revoiced, or explicitly connected to other language in a display for all students to use. This routine provides feedback for students in a way that increases accessibility while simultaneously supporting meta-awareness of language.

Example 1. Gather and Show Student Discourse

During pair/group work, circulate and listen to student talk during pair work or group work, and jot notes about common or important words and phrases, together with helpful sketches or diagrams. Scribe students' words and sketches on visual display to refer back to during whole class discussions throughout the unit. Refer back to these words, phrases, and diagrams by asking students to explain how they are useful, asking students to clarify their meaning, and asking students to reflect on which words and visuals help to communicate ideas more precisely.

Example 2. Oral to Written Math Explanation

1. POSE: Pose a substantive math question, one that is discussion-worthy.
2. JOT: Allow students a few minutes to jot down notes quietly
3. COMPARE: Allow students to compare responses (teacher listens closely)
4. ELICIT: Ask for a volunteer or select a student to give oral explanation (several students might contribute), or provides a mock-up explanation with a common error
5. SCRIBE: Teacher scribes the student explanation – asking for confirmation (often students will revise and improve)
6. STUDENT FEEDBACK: Do you agree with the explanation? What is helpful? What questions do you have? What suggestions do you have to improve?
7. TEACHER FEEDBACK: Teacher highlights a point that was not mentioned (strengths and areas for improvement), points out vocab, forms
8. STUDENTS REVISE: Students revise their initial jottings in their journals, using the information that was provided in the class discussion.

MLR3: Critique, Correct, and Clarify

Purpose: To give students a piece of mathematical writing that is not their own to analyze, reflect on, and develop. The intent is to prompt student reflection with an incorrect, incomplete, or ambiguous written argument or explanation, and for students to improve upon the written work by correcting errors and clarifying meaning. This routine fortifies output and engages students in meta-awareness. Teachers can demonstrate with meta-think-alouds and press for details when necessary.

Example 1. Critique a Partial or Flawed Response

1. PRESENT: Present a partial/broken argument, explanation, or solution method. Teacher can play the role of the student who produced the response, and ask for help in fixing it.
    - Given response could include a common error.
    - Given response should include an ambiguous term or phrase, or an informal way of expressing a mathematical idea.
2. PROMPT: Prompt students to identify the error(s) or ambiguity, analyze the response in light of their own understanding of the problem, and work both individually and in pairs to propose an improved response.
3. SHARE: Pairs share out draft improved response.
4. REFINE: Students refine their own draft response.

Example 2. Always-Sometimes-Never

Use a structure or graphic organizer to evaluate or critique whether mathematical statements are always, sometimes, or never true. (Examples: “A rectangle is a parallelogram” or “A negative integer minus another negative integer equals a positive integer.”) Use the graphic organizer to frame and assess the reasoning process as students work toward evaluating and improving a response.

MLR4: Information Gap

Purpose: To create a need for students to communicate. This routine allows teachers to facilitate meaningful interactions by giving partners or team members different pieces of necessary information that must be used together to solve a problem or play a game. With an information gap, students need to orally (and/or visually) share their ideas and information in order to bridge the gap.

Example 1. Info Gap Cards

In one version of this activity, Partner A has the general problem on a card, and Partner B has the information needed to solve it on the “data card.” Data cards can also contain diagrams, tables, graphs, etc. Partner A needs to realize what is needed and ask for information that is provided on Partner B’s data card. Partner B should not share information unless Partner A specifically asks for it. Neither partner should read their cards to one another nor show their cards to their partners. As they work the problem, they justify their responses using clear and connected language.

1. READ, then THINK-ALOUD: The problem card partner (Partner A) reads his or her card silently and thinks aloud about what information is needed. Partner B reads the data card silently.
2. QUESTION 1: Partner B asks, “What specific information do you need?” Partner A needs to ask for specific information from Partner B.
3. QUESTION 2: When partner A asks, Partner B should ask for justification: “Why do you need that information?” before telling it to Partner A.
4. EXPLANATIONS: Partner A then explains how he or she is using the information to solve the problem. Partner B helps and asks for explanations, even if he or she understands what Partner A is doing.
5. FOLLOW-UP: As a follow-up step, have both students use blank cards to write their own similar problem card and data card for other pairs to use.

Example 2. Info Gap Games

Students play a guessing game or matching game in which they have a real reason to talk (e.g., students need to work together to develop a strategy to win a game; each student is provided with different information; one student has something in mind and other students use their understanding of a mathematical concept to guess what it is).

Example: Guess my ratio. One student identifies a ratio between two distinct features/objects in a given set and keeps it a secret; other students try to figure out which features/objects are in the identified ratio.

MLR5: Co-Craft Questions and Problems

Purpose: To allow students to get inside of a context before feeling pressure to produce answers, and to create space for students to produce the language of mathematical questions themselves. Through this routine, students are able to use conversation skills as well as develop meta-awareness of the language used in mathematical questions and problems. Teachers should push for clarity and revoice oral responses as necessary.

Example 1. Co-Craft Questions

1. PRESENT SITUATION: Teacher presents a situation – a context or a stem for a problem, with or without values included. (Example: A bird is flying at 30 mph)
2. STUDENTS WRITE: Students write down possible mathematical questions that might be asked about the situation. These should be questions that they think are answerable by doing math. They can also be questions about the situation, information that might be missing, and even about assumptions that they think are important. (1-2 minutes)
3. PAIRS COMPARE: In pairs, students compare their questions. (1-2 minutes)
4. STUDENTS SHARE: Students are invited to share their questions, with some brief discussion. (2-3 minutes)
5. REVEAL QUESTIONS: The actual questions students are expected to work on are revealed, and students are set to work.

Example 2. Co-Craft Problems

1. PAIRS CREATE NEW PROBLEMS: Students get into pairs and co-create problems similar to a given task.
2. STUDENTS SOLVE THEIR OWN PROBLEMS: Students solve their own problems before trading them with other pairs.
3. EXCHANGE PROBLEMS: Students solve other pairs' problems, and check solutions and methods with the pair who created each problem.
4. TOPIC SUPPORT: Teacher can provide possible topics of interest to students, or brainstorm as a whole class for 2 minutes before pairing up.

MLR6: Three Reads

Purpose: To ensure that students know what they are being asked to do, and to create an opportunity for students to reflect on the ways mathematical questions are presented. This routine supports reading comprehension of problems and meta-awareness of mathematical language. It also supports negotiating information in a text with a partner in mathematical conversation.

Example 1.

Students are supported in reading a situation/problem three times, each time with a particular focus:

1. Students read the situation with the goal of comprehending the text (describe the situation without using numbers),

2. Students read the situation with the goal of analyzing the language used to present the mathematical structure.
3. Students read the situation in order to brainstorm possible mathematical solution methods.

This routine works well in conjunction with Mathematical Language Routine 5, in which the question stem is tentatively withheld in order to focus on the comprehension of what is happening in the text.

MLR7: Compare and Connect

Purpose: To foster students' meta-awareness as they identify, compare, and contrast different mathematical approaches, representations, and language. Teachers should demonstrate thinking out loud (e.g., exploring why we one might do/say it this way, questioning an idea, wondering how an idea compares or connects to other ideas and/or language), and students should be prompted to reflect and respond. This routine supports meta-cognitive and meta-linguistic awareness, and also supports mathematical conversation.

Example 1. Compare and Connect Solution Strategies

Instructional time is dedicated to students understanding one another's solution strategies by relating and connecting other students' approaches to their own approach.

1. SET-UP: Ways to set this up so that multiple strategies are likely to be generated by each pair of students:
    - I solve it one way, you solve it another.
    - Divide and conquer: you do one and I do another.
    - I have a piece of info, you have a piece of info.
2. WHAT IS SIMILAR, WHAT IS DIFFERENT: Students first identify what is similar and what is different about the approaches. This can also be an initial discussion about what worked well in this or that approach, and what might make this or that approach more complete or easy to understand.
3. MATHEMATICAL FOCUS: Students are asked to focus on specific mathematical relationships, operations, quantities and values. For example:
    - Why does this approach include multiplication, and this one does not?
    - Where is the 10 in each approach?
    - Which unit rate was used in this approach?

MLR8: Discussion Supports

Purpose: To support rich discussions about mathematical ideas, representations, contexts, and strategies. The examples provided can be combined and used together with any of the other routines. They include multi-modal strategies for helping students comprehend complex language and ideas, and can be used to make classroom communication accessible, to foster meta-awareness of language, and to demonstrate strategies students can use to enhance their own communication and construction of ideas.

Examples

- Revoice student ideas to demonstrate mathematical language use by restating a statement as a question in order to clarify, apply appropriate language, and involve more students.
- Press for details in students' explanations by requesting for students to challenge an idea, elaborate on an idea, or give an example. This is intended to get more participation from students, deepen student understanding, and provide extensions.
- Show central concepts multi-modally by utilizing different types of sensory inputs: acting out scenarios or inviting students to do so, showing videos or images, using gesture, and talking about the context of what is happening.
- Practice phrases or words through choral response.
- Think-Aloud Demonstrating

Teacher uses a think-aloud to raise students' awareness about their mathematical language and thinking. Teacher talks through his thinking about a new mathematical concept as he solves a related problem or does a task. While doing so she demonstrates detailing her steps, describing and justifying her reasoning, and questioning her strategies. Students' attention is brought to the importance of describing their thinking to others. They are then allowed opportunities (possibly in conjunction with other routines) to talk through their own thinking.

A few discussion starters include:

- How can you remember that word or term?
- What was most confusing about the math concepts we explored today?
- What was most/least challenging for me to learn? Why?
- Is this always true? Sometimes true?
- How does *stronger, clearer each time* help you to develop your math and language skills?
- What things did we do in class today that help you think about how to solve the problems?

General Instructional Routines

Algebra Talk

What: One expression is displayed at a time. Students are given a few minutes to quietly think and give a signal when they have an answer and a strategy. The teacher selects students to share different strategies for each one, "Who thought about it a different way?" Their explanations are recorded for all to see. Students might be pressed to provide more details about why they decided to approach a problem a certain way. It may not be possible to share every possible strategy for the given limited time; the teacher may only gather two or three distinctive strategies per problem. Problems are purposefully chosen to elicit different approaches.

Where: Warm-ups

Why: Algebra Talks build algebraic thinking by encouraging students to think about the numbers and variables in an expression and rely on what they know about structure, patterns, and properties of operations to mentally solve a problem. Algebra Talks promote seeing structure in expressions and thinking about how changing one number affects others in an equation. While participating in these activities, students need to be precise in their word choice and use of language (MP6).

Anticipate, Monitor, Select, Sequence, Connect

What: Fans of *5 Practices for Orchestrating Productive Mathematical Discussions* (Smith and Stein, 2011) will recognize these as the 5 Practices. In this curriculum, much of the work of anticipating, sequencing, and connecting is handled by the materials in the activity narrative, launch, and synthesis sections. But teachers will need to take this ball and run with it by developing the capacity to prepare for and conduct whole-class discussions. The book itself would make excellent fodder for a teacher PLC or study group.

Where: Many classroom activities lend themselves to this structure.

Why: In a problem-based curriculum, many activities can be described as “do math and talk about it,” but the 5 Practices lend more structure to these activities so that they more reliably result in students making connections and learning new mathematics.

Group Presentations

Some activities instruct students to work in small groups to solve a problem with mathematical modeling, invent a new problem, design something, or organize and display data, and then create a visual display of their work. Teachers need to help groups organize their work so that others can follow it, and then facilitate different groups’ presentation of work to the class. Teachers can develop specific questioning skills to help more students make connections and walk away from these experiences with desired mathematical learning. For example, instead of asking if anyone has any questions for the group, it is often more productive to ask a member of the class to restate their understanding of the group’s findings in their own words.

Notice and Wonder

What: Students are shown some media or a mathematical representation. The prompt to students is "What do you notice? What do you wonder?" Students are given a few minutes to write down things they notice and things they wonder. After students have had a chance to write down their responses, the teacher asks several students to share things they noticed and things they wondered; these are recorded by the teacher for all to see. Usually, the teacher steers the conversation to wondering about something mathematical that the class is about to focus on.

Where: Appears frequently in warm-ups but also appears in launches to classroom activities.

Why: The purpose is to lower the bar for entry into a mathematical task for all students with these two low-stakes questions; by thinking about them and responding, students gain entry into the context and might get their curiosity piqued. Taking steps to become familiar with a context and the mathematics that might be involved is making sense of problems (MP1). Note: *Notice and Wonder* and *I Notice/I Wonder* are trademarks of NCTM and the Math Forum and used in these materials with permission.

Number Talk

What: One problem is displayed at a time. Students are given a few minutes to quietly think and give a signal when they have an answer and a strategy. The teacher selects students to share different strategies for each problem, "Who thought about it a different way?" Their explanations are recorded for all to see. Students might be pressed to provide more details about why they decided to approach a problem a certain way. It may not be possible to share every possible strategy for the given limited time; the teacher may only gather two or three distinctive strategies per problem. Problems are purposefully chosen to elicit different approaches, often in a way that builds from one problem to the next.

Where: Warm-ups

Why: Number talks build computational fluency by encouraging students to think about the numbers in a computation problem and rely on what they know about structure, patterns, and properties of operations to mentally solve a problem. Dot images are similar to number talks, except the image used is an arrangement of dots that students might count in different ways. While participating in these activities, students need to be precise in their word choice and use of language (MP6).

Poll the Class

What: Every student in class reports a response to a prompt. Teachers need to develop a mechanism by which poll results are collected and displayed so that this frequent form of classroom interaction is seamless. Smaller classes might be able to conduct a roll call by voice. For larger classes, students might be given mini-whiteboards or a set of colored index cards to hold up. Free and paid commercial tools are also readily available.

Where: Used to register an initial response or an estimate, most often in activity launches or to kick off a discussion. Can also be used when data needs to be collected from each student in class, for example, "What is the length of your ear in centimeters?"

Why: Going on record with an estimate or a gut reaction makes people want to know if they were right and increases investment in the outcome. If coming up with an estimate is too daunting, ask students for a guess that they are sure is too low or too high. Putting some boundaries on possible outcomes of a problem is an important skill for mathematical modeling (MP4). Collecting data from the class to use in an activity makes the outcome of the activity more interesting.

Take Turns

What: In Take Turns, students work with a partner or small group. They take turns in the work of the activity, whether it be spotting matches, explaining, justifying, agreeing or disagreeing, or asking clarifying questions. If they disagree, they are expected to support their case and listen to their partner's arguments. The first few times students engage in these activities, the teacher should demonstrate, with a partner, how the discussion is expected to go. Once students are familiar with these structures, less set-up will be necessary. While students are working, the teacher can ask students to restate their question more clearly or paraphrase what their partner said.

Where: Classroom Activities

Why: Building in an expectation, through the routine, that students explain the rationale for their choices and listen to another's rationale deepens the understanding that can be achieved through these activities. Specifying that students take turns deciding, explaining, and listening limits the phenomenon where one student takes over and the other does not participate. Taking turns can also give students more opportunities to construct logical arguments and critique others' reasoning (MP3).

Think Pair Share

What: Students have quiet time to think about a problem and work on it individually, and then time to share their response or their progress with a partner. Once these partner conversations have taken place, some partnerships are selected to share their thoughts with the class.

Where: Classroom activities

Why: This is a teaching routine useful in many contexts whose purpose is to give all students enough time to think about a prompt and form a response before they are expected to try to verbalize their thinking. First they have an opportunity to share their thinking in a low-stakes way with one partner, so that when they share with the class they can feel calm and confident, as well as say something meaningful that might advance everyone's understanding. Additionally, the teacher has an opportunity to eavesdrop on the partner conversations so that she can purposefully select students to share with the class.

True or False

What: One statement is displayed at a time. Students are given a few minutes to quietly think and give a signal when they have an answer. The teacher selects students to share different ways of reasoning for each statement. "Who thought about it a different way?" While students may evaluate each side of the equation to determine if it is true or false, encourage students to think about ways to reason that do not require complete computations. It may not be possible to share every possible reasoning approach for the given limited time; the teacher may only gather two or three distinctive strategies per problem. Statements are purposefully chosen to elicit different approaches, often in a way that builds from one statement to the next.

Where: Warm-ups

Why: Depending on the purpose of the task, the true or false structure encourages students to reason about numeric and algebraic expressions using base-ten structure, the meaning of fractions, meaning and properties of operations, and the meaning of comparison symbols. While the structure of a true or false is similar to that of a number talk, number talks are often focused on computational strategies, while true or false tasks are more likely to focus on more structural aspects of the expressions. Often students can determine whether an equation, an inequality, or a statement is true or false without doing any direct computation. While participating in these activities, students need to be precise in their word choice and use of language (MP6).

Which One Doesn't Belong?

What: Students are presented with four figures, diagrams, graphs, or expressions with the prompt "Which one doesn't belong?" Typically, each of the four options "doesn't belong" for a different reason, and the similarities and differences are mathematically significant. Students are prompted to explain their rationale for deciding that one option doesn't belong and given opportunities to make their rationale more precise.

Where: Warm-ups

Why: Which One Doesn't Belong fosters a need to define terms carefully and use words precisely in order to compare and contrast a group of geometric figures or other mathematical representations.