Roll No…………………….. **Total No. of Printed Pages: [02]**

Total No. of Questions: [09]

B. Sc. (Hons.) Maths (Semester – 6th)
LINEAR PROGRAMMING AND OPTIMIZATION
Subject Code: BMATS1-601
Paper ID: [19131226]

Time: 03 Hours **Maximum Marks: 60**

Instruction for candidates:

1. Section A is compulsory. It consists of 10 parts of two marks each.
2. Section B consist of 5 questions of 5 marks each. The student has to attempt any 4 questions out of it.
3. Section C consist of 3 questions of 10 marks each. The student has to attempt any 2 questions.

Section – A **(2 marks each)**

Q1. Attempt the following:

a. Define linear programming problem.

b. Explain basic feasible solution.

c. Prove that the set of all feasible solutions of a LPP forms a convex set.

d. Write the standard and canonical forms of a linear programming problem.

e. Define slack and surplus variables?

f. Write the dual of the LPP:

Minimize $z = 4x_1 + 6x_2 + 18x_3$ subject to constraints

$x_1 + 3x_2 \geq 3$, $x_2 + 2x_3 \geq 5$, $x_1, x_2, x_3 \geq 0$.

g. State complementary slackness theorem.

h. What is the two-person zero-sum game?

i. Solve the following game:

$$\begin{array}{cc} & \textit{Player B} \\ \textit{Player A} & \begin{bmatrix} 1 & 3 & 1 \\ 0 & -4 & -3 \\ 1 & 5 & -1 \end{bmatrix} \end{array}.$$

j. Two players A and B match coins. If the coins match, then A wins 2 units of the value, If the coins do not match, then B wins 2 units of the value. Determine the optimum strategies for the players and the value of the game.

Section – B **(5 marks each)**

Q2. A firm manufactures headache pills in two sizes A and B. Size A contains 2 grains of aspirin, 5 grains of bicarbonate and 1 grain of codeine. Size B contains 1 grain of aspirin, 8 grains of bicarbonate and 6 grains of codeine. It is found by users that it requires at least 12 grains of aspirin, 74 grains of bicarbonate and 24 grains of codeine for providing immediate effect. It is required to estimate the least number of pills a patient should take to get immediate relief. Formulate the problem as a standard LPP.

Q3. Use graphical method to solve the following LPP:

Maximize $z = 4x_1 + 3x_2$ subject to constraints

$2x_1 + x_2 \leq 1000, \quad x_1 + x_2 \leq 800, \quad x_1 \leq 400, \quad x_2 \leq 700, \quad x_1, x_2 \geq 0$.

Q4. Solve the following LPP by using Big-M method:

Maximize $z = -2x_1 - x_2$ subject to constraints

$3x_1 + x_2 = 3, \; 4x_1 + 3x_2 \geq 6, \; x_1 + 2x_2 \leq 4$ and $x_1, x_2 \geq 0$.

Q5. Prove that the dual of the dual is the primal.

Q6. Solve the following game graphically:

$$\begin{array}{cc} & PlayerB \\ PlayerA & \begin{bmatrix} 2 & 1 & 0 & -2 \\ 1 & 0 & 3 & 2 \end{bmatrix} \end{array}.$$

Section – C **(10 marks each)**

Q7. Use two-phase simplex method to solve the LPP:

Minimize $z = x_1 - 2x_2 - 3x_3$ subject to constraints

$-2x_1 + x_2 + 3x_3 = 2, \; 2x_1 + 3x_2 + 4x_3 = 1$, $x_1, x_2, x_3 \geq 0$

Q8. Solve the following LPP by using dual simplex method:

Minimize $z = 2x_1 + x_2$ subject to constraints

$3x_1 + x_2 \geq 3$, $4x_1 + 3x_2 \geq 6$, $x_1 + 2x_2 \geq 3$ and $x_1, x_2 \geq 0$.

Q9. Solve the following game by linear programming method:

$$\begin{array}{cc} & \textit{Player B} \\ \textit{Player A} & \begin{bmatrix} 1 & -1 & 3 \\ 3 & 5 & -3 \\ 6 & 2 & -2 \end{bmatrix} \end{array}.$$