Analytical method of finding velocity and acceleration in slider crank mechanism

Formulae for Analytical method of finding velocity and acceleration in slider crank mechanism

Ratio \( n = \frac{\text{connecting rod length}}{\text{crank radius}} = \frac{l}{r} \)

<table>
<thead>
<tr>
<th>Part</th>
<th>Velocity</th>
<th>Acceleration</th>
</tr>
</thead>
<tbody>
<tr>
<td>Crank</td>
<td>( \omega = \frac{2\pi N}{60} ) rad/sec</td>
<td>0 {Because crank has constant angular velocity}</td>
</tr>
<tr>
<td>Piston /slider</td>
<td>( V_p = \omega r [\sin \theta + \frac{\sin^2 \theta}{2n}] ) m/sec</td>
<td>( a_p = \omega^2 r [\cos \theta + \frac{\cos 2\theta}{n}] ) m/s^2</td>
</tr>
<tr>
<td>Connecting rod</td>
<td>( \omega_{cr} = \frac{\omega \cos \theta}{(r^2 - \sin^2 \theta)^{1/2}} ) rad/sec</td>
<td>( \alpha_{cr} = - \omega^2 \left[ \frac{\sin \theta (r^2 - 1)}{(r^2 - \sin^2 \theta)^{3/2}} \right] ) rad/sec^2</td>
</tr>
</tbody>
</table>
Numerical Problems On finding velocity and acceleration in slider crank mechanism

1. The crank of a petrol engine is 4 cm and length of connecting rod is 16 cm. Find the velocity and acceleration of the piston at the instant when the crank is at $30^\circ$ from IDC and rotating at 2500 rpm. Also, find the inertia force due to reciprocating parts having a mass of 0.3 kg.

$\{\text{vel}=6.37 \text{ m/s, acc}=2716.965 \text{ m/s}^2, \text{force}=815.09 \text{ N}\}$

2. The stroke of a steam engine is 15 cm and the connecting rod is 30 cm in length. Determine the velocity and acceleration of the piston when the crank has made $45^\circ$ measured from IDC position and rotates at 600 rpm. Also determine angular velocity and angular acceleration of the connecting rod.

$\{\text{Vel piston}=0.39 \text{ m/s, acc piston}=209.36 \text{ m/s}, \text{ang vel of CR}=11.107 \text{ rad/sec, ang acc of CR}=-697.89 \text{ rad/sec}^2\}$

3. A reciprocating engine mechanism has its crank 10 cm long and the length of the connecting rod is 40 cm. Find the velocity and acceleration of the piston by analytical method when the crank rotates uniformly at angular speed of 10 rad/sec for the following positions 1) at IDC 2) at $\theta = 30^\circ$ 3) at $\theta = 90^\circ$

$\{\text{Ans (1) vel}=0, \text{acc}=12.5 \text{ m/s}^2 \ (2) \text{ vel}=0.608 \text{ m/s, acc}=9.285 \text{ m/s}^2 \ (3) \text{ vel}=1.125 \text{ m/s, acc}=-1.25 \text{ m/s}^2\}$

4. The crank and connecting rod of a steam engine are 0.3m and 1.5m in length. The crank rotates at 180 rpm clockwise. Determine the velocity and acceleration of the piston when the crank is at 40 degrees from the inner dead centre.

$\{\text{Ans (1) vel}=4.19 \text{ m/s, acc}=85.35 \}$
5. In a slider crank mechanism the length of the crank and connecting rod are 150mm and 600mm respectively. The crank position is 60° from inner dead centre. The crank shaft speed is 450 r.p.m. Using analytical method fine
   1. Velocity and acceleration of the slider
   2. Velocity and acceleration of the connecting rod.

   \( \{ \text{Vel piston}=6.9 \text{m/s}, \text{acc piston}=124.94 \text{ m/s}^2, \text{ang vel of CR}=5.9 \text{ rad/sec}, \text{ang acc of CR}=-481 \text{ rad/sec}^2 \} \)

6. In a slider crank mechanism the length of connecting rod is 80 cm, ratio of length of connecting rod to length of crank is 4. Determine (i) Velocity of slider (ii) Velocity of connecting rod, if crank makes an angle of 120° with IDC.

Klein’s Construction Method

Q.1.Explain the Klein’s construction method of finding the velocity and acceleration of various links/points in slider-crank Mechanism.

Ans: The Klein’s construction is graphical method to obtain the velocity, acceleration of links or important points on the links. This construction is drawn directly on the configuration diagram. hence it does not involve two or three separate figures. However, it is applicable only for the slider-crank mechanism.

Steps to find the velocity and acceleration of various links
step 1) Draw the basic configuration diagram by measuring the angle made by crank and also other dimension of crank and connecting rod.
Step 2) Extend the connecting rod upto the vertical centre line of the crank circle and mark point M, the triangle created OAM is the velocity triangle.

Step 3) Locate the midpoint of the connecting rod as point G.
Step 4: With Centre as “A” and radius equal to AM draw the circle.

Step 5: With Centre as “G” and radius equal to GA or GB draw the circle.
Step 6: Both circles will intersect each other at two points, join these two points.

Step 7: This line will intersect the connecting rod at point “Q” and line of stroke at point “N”. Name these two points.
Step 8: Now OAM is the velocity triangle and the OAQN is the acceleration diagram. Which can be used to find the required velocity of acceleration of the links of various points on the links.

**Numerical problems**

1. The crank and connecting rod of a reciprocating engine are 200 mm and 700 mm respectively. The crank is rotating in clockwise direction at 120 rad/s. Find with the help of Klein’s construction:
   1. Velocity and acceleration of the piston,
   2. Velocity and acceleration of the mid point of the connecting rod, and 3. Angular velocity and angular acceleration of the connecting rod, at the instant when the crank is at 30° to I.D.C. (inner dead centre).

2. In a slider crank mechanism, the length of the crank and connecting rod are 150 mm and 600 mm respectively. The crank position is 60° from inner dead centre. The crank shaft speed is 450 r.p.m. clockwise. Using Klein’s construction, determine
   1. Velocity and acceleration of the slider,
   2. Velocity and acceleration of point D on the connecting rod which is 150 mm from crank pin C, and 3. angular velocity and angular acceleration of the connecting rod

3. The crank of a reciprocating engine is rotating in clockwise direction with a constant angular velocity of 30 rad/sec. The lengths of crank and connecting rod are 200 mm and 750 mm respectively. Using Klein’s construction find
   1. Velocity of piston
   2. Velocity of midpoint of connecting rod
   3. Acceleration of piston
   4. Angular acceleration of connecting rod when the crank has turned through 30 degrees from inner dead center.

{Ans : 3.8 m/s, 4.2 m/s, 180 m/s², 119.57 rad/sec²}
Videos On Kleins Construction:
1. https://www.youtube.com/watch?v=jgwpribxRc
2. https://www.youtube.com/watch?v=k202Yisjc5g

Relative Velocity Method

Relative Velocity Method important points

Important points to remember:
1. Relative velocity of a link is always perpendicular to link.
2. Angular velocity of crank is found by formula
   \[ \omega = 2 \times N \times 60 \text{ rad/sec}. \]
   This angular velocity is converted to linear velocity by formula
   \[ \text{AB} = l(AB) \text{ m/s} \]
3) All fixed points in the mechanism are represented by a single point on velocity diagram.
4) For finding linear or angular velocity of any link use formula
   \[ \text{Linear velocity} = \text{Angular velocity} \times \text{Length of Link} \]

A. Problems on Four Bar chain

1. In a four bar chain ABCD, AD is fixed and is 150 mm long. The crank AB is 40mm long and rotates at 120 r.p.m, while the link CD=80 mm oscillates about D. BC and AD are equal length. Find the angular velocity of link CD when angle isBAD=600

   \{ \text{Ans } cd=0.385 \text{ m/s and } cd=4.8 \text{ rad/sec} \}
2) PQRS is a four bar chain with PS fixed length of links are PQ = 62.5 mm, QR =175 mm, RS = 112.5 mm, PS = 200 mm. The crank PQ rotate at 10 rad/sec. in clockwise direction. Determine the angular velocity of point R, graphically by using relative velocity method.

\[ \text{\{rs}=3.82 \text{ rad/sec } \]

3) In a four bar chain mechanism, the lengths of links AB,BC,CD and DA are 60 mm,160mm,100mm and 200mm respectively. The link AB makes an angle of 600 with the fixed link AD and rotates at an angular velocity of 30 rad/sec. Find the angular velocity of link BC.

\[ \text{\{Ans :BC}=10.5 \text{ rad/sec } \]

4) In a four-link mechanism, the crank AB rotates at 36 rad/sec. The lengths of a link are AB = 200 mm, BC = 400 mm, CD = 450 mm and AD = 600 mm. AD is the fixed link. At the instant when AB is at right angles to AD determine the velocity of: (i) The midpoint of link BC (ii) A point on the link CD, 100 mm from the pin connecting the link CD & AD.

\[ \text{\{vel of midpoint }= 2.2 \text{ m/s }, \text{velocity of another point on link CD}=1.4 \text{ m/sec}\]

A. Problems on Slider crank chain.

6) The crank and connecting rod of a steam engine are 0.5m and 2m long respectively. The crank makes 180 r.p.m in the clockwise direction. When it has turned 450 from inner dead centre position. Determine (1) Velocity of piston (2) Angular velocity of connecting rod (3) Velocity of point E on the connecting rod 1.5 m from the gudgeon pin.

\[ \text{\{ p}=8.15 \text{ m/sec, pb}=3.4 \text{ rad/sec, E}=8.5 \text{ m/s } \]

7) In a slider crank mechanism, the length of crank OB and connecting rod AB are 125 mm and 500 mm respectively. The centre of gravity G of the connecting rod is 275 mm from the slider. The crank speed is 600 rpm clockwise. When the crank has turned 45º from the inner dead centre position, determine (i) Velocity of slider ‘A’ (ii) Velocity of the point ‘G’ graphically. Draw the
configuration diagram also.

\{Velocity of slider Vector \text{oa} = V_{oa} = 6.79 \text{ m/s}, \ Velocity of conn. Rod Vector \text{ab} = V_{ab} = 5.66 \text{ m/s} \ Velocity of point \text{‘G’ Vector \text{og} = V_{g} = 7.2 \text{ m/s} } \}

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8) In a single slider crank mechanism OAB, the crank OA(30 cm long) revolves at 200 rpm. The connecting rod AB is 75 cm long. Determine the velocity of Slider B and the velocity of midpoint of the connecting rod, when crank makes an angle of 35 degrees with the IDC.

\{Velocity of slider = 5.68 \text{ m/sec}, \ velocity of midpoint 5.38 \text{ m/sec}\}

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9) In a single slider crank mechanism the crank OA is 7.5 cm long and rotates with uniform speed 60 rpm. The connecting rod AB is 30 cm long. Determine the linear velocity of the piston B, When crank makes an angle of 45 degrees with idc.

\{ \text{Velocity of piston :0.39 m/sec}\}

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