Set Theory and Logic

Guided Notes

This resource is designed to help you actively engage with key mathematical concepts. By completing these notes, you'll establish connections between concepts, identify patterns, and develop a deeper understanding of the material.

Fill in the blanks, solve examples, and note questions as you go. Writing in your own words strengthens your memory and helps identify areas needing more attention. Bring your questions to class discussions where your instructor can address specific concerns. Don't focus on perfection—you'll learn math best through active engagement!

Set Theory Basics

Key Concept: Sets and Set Notation

A **set** is _____

Elements of a set are _____

A set can be defined by:

- _____
- _____

Important characteristics of sets:

- Repeated elements are only listed _____
- Order of elements is _____

The symbol $\in$ means _____

The empty set is denoted by _____

Key Concept: Subsets

A subset of a set *A* is _____

If *B* is a subset of *A*, the notation is: _____

A proper subset of a set *A* is _____

If *B* is a proper subset of *A*, the notation is: _____

Key Concept: Number of Subsets

When a set has *n* elements:

- Number of subsets: _____
- Number of proper subsets: _____

Practice with Sets

Write each of the following as a set using set notation:

1. The set of odd numbers less than 10: _____
2. The set of primary colors: _____

Determine whether each element belongs to the given set:

Set $A = \{2, 4, 6, 8, 10\}$

1. Is $4 \in A$? _____
2. Is $5 \in A$? _____
3. Is $10 \in A$? _____

Practice with Subsets

For the set $C = \{a, b, c\}$:

1. List all possible subsets: _____
2. How many subsets does *C* have? _____
3. How many proper subsets does *C* have? _____

Practice with Number of Subsets

Find the number of subsets and proper subsets for:

1. A set with 4 elements:
    - Number of subsets: _____
    - Number of proper subsets: _____
2. A set with 5 elements:
    - Number of subsets: _____
    - Number of proper subsets: _____

Think About It

If you double the number of elements in a set, what happens to the number of possible subsets? Why does this relationship exist?

_____

_____

Set Operations

Key Concept: Basic Set Operations

Union ($\cup$): _____

- Notation: _____
- More formally: _____

Intersection ($\cap$): _____

- Notation: _____
- More formally: _____

Complement (A^c or A'): _____

- Notation: _____
- A complement is relative to the _____

Difference ($\setminus$): _____

- Notation: _____
- More formally: _____

Key Concept: Universal Set

A universal set is _____

Practice with Set Operations

Given sets:

$$A = \{1, 3, 5, 7\} \text{ and } B = \{1, 2, 3, 4\}$$

Find:

1. $A \cup B =$ _____
2. $A \cap B =$ _____
3. If Universal Set $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, find $A^c =$ _____
4. $A \setminus B =$ _____
5. $B \setminus A =$ _____

Think About It

In everyday language, how do the words "and" and "or" relate to the intersection and union of sets? Can you give a real-world example?

Venn Diagrams

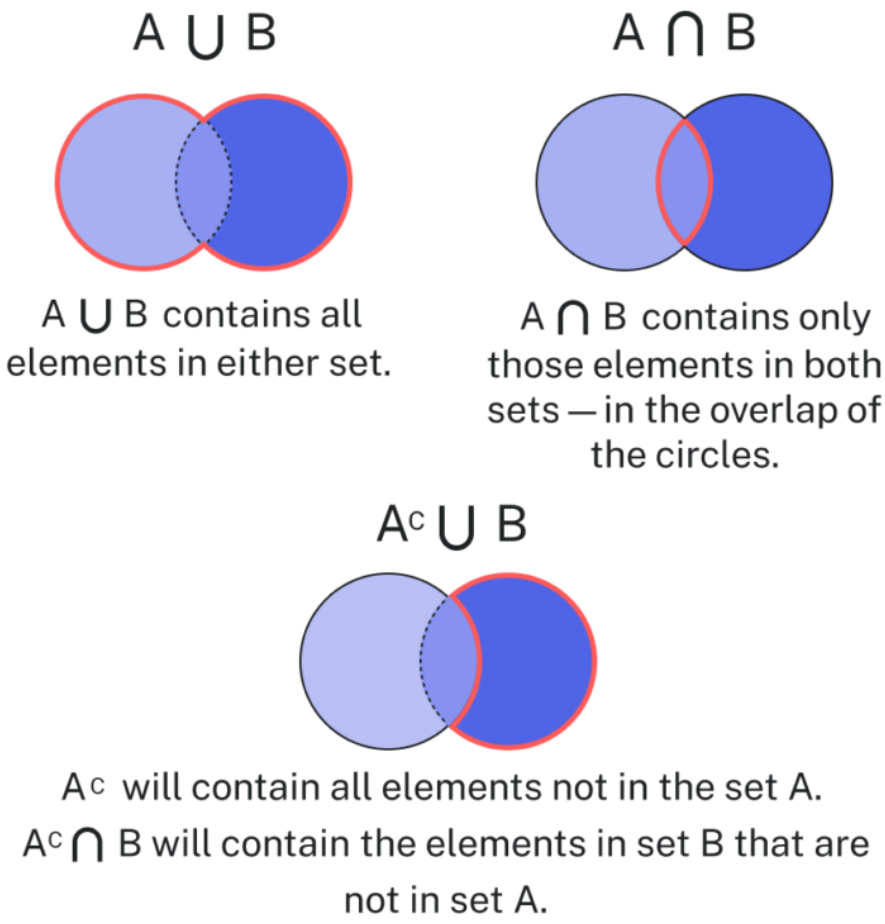
Key Concept: Venn Diagrams

A **Venn diagram** is _____

Key components of a Venn diagram:

- _____
- _____
- _____

Venn diagrams can be used to illustrate the union, intersection, and complements of sets.



Practice with Venn Diagrams

Draw a Venn diagram for the following sets: $A = \{1, 2, 3\}$ and $B = \{2, 3, 4, 5\}$

Shade the region representing $A \cap B$ in your diagram.

Draw a new Venn diagram and shade the region representing $A \cup B^c$.

Think About It

How can Venn diagrams help solve complex problems involving multiple sets? What are their limitations?

Cardinality

Key Concept: Cardinality

Cardinality is _____

Notation for the cardinality of set A: _____

Cardinality Properties

1. $n(A \cup B) =$ _____

2. $n(A^c) =$ _____
3. $n(A \cap B) =$ _____

Practice with Cardinality

1. If $A = \{1, 3, 5, 7, 9\}$, find $n(A)$: _____
2. If $n(A) = 20$, $n(B)=15$, and $n(A \cap B) = 5$, find:
    - $n(A \cup B) =$ _____
    - $n(A \backslash B) =$ _____
3. In a survey of 50 students - 28 like math, 32 like science, 15 like both subjects.

    Find:

    - Students who like at least one subject: _____
    - Students who like neither subject: _____
    - Students who like exactly one subject: _____

Think About It

How can understanding cardinality help you make informed decisions with data in everyday life? Give a specific example.

Logic

Key Concept: Boolean Logic

Boolean logic is _____

In connection to sets, a boolean search is true if _____

Connection to Set Operations:

Boolean Logic	Set Operations
A and *B*	
A or *B*	
Not *A*	

Practice with Boolean Logic

Translate the following searches using Boolean logic:

1. Movies that are comedies and were released after 2010: _____
2. Historical novels that are not biographies: _____
3. Jobs that require a college degree or 5 years of experience: _____

Which of these Boolean searches would find MORE results?

- "cats AND dogs"
- "cats OR dogs"

Explain why: _____

Think About It

In everyday conversation, how does the meaning of "or" differ from how it's used in Boolean logic? Give an example where this difference might cause confusion.

Quantified Statements

Key Concept: Quantifiers

A **universal quantifier** states that _____

- Notation: _____

An **existential quantifier** states that _____

- Notation: _____

Key Concept: Negating Quantified Statements

Complete the following:

1. The negation of "all *A* are *B*" is _____
2. The negation of "no *A* are *B*" is _____
3. The negation of "at least one *A* is *B*" is _____
4. The negation of "at least one *A* is not *B*" is _____

Practice with Quantifiers

Translate each statement using quantifiers:

1. Every student passed the test: _____
2. Some students forgot their homework: _____
3. No fish can breathe out of water: _____

Practice with Negating Statements

Write the negation of each statement:

1. All cars have four wheels: _____
2. Some birds cannot fly: _____
3. Every student likes math: _____

Think About It

Why is it significant that the negation of a universal statement is an existential statement, and vice versa? How might this help with proving or disproving claims?

Conditional Statements

Key Concept: Statements and Conditionals

A **statement** is _____

A **conditional** is _____

Practice with Conditionals

Write the following as conditional statements:

1. If a student scores at least 70%, they pass the course.
    - Condition: _____
    - Result: _____
2. You can apply for a loan if your credit score is above 650.
    - Condition: _____
    - Result: _____

Practice with Excel Conditionals

For the Excel expression: `=IF(A1>100, A1*0.9, A1*0.8)`

1. If A1 = 90, the result will be: _____
2. If A1 = 110, the result will be: _____
3. Write an Excel expression that gives a 5% discount if the total is over $200, and no discount otherwise: _____

Truth Tables

Key Concept: Truth Tables

A **truth table** is _____

Logical Symbols

- The symbol $\wedge$ is used for _____ and means _____
- The symbol $\vee$ is used for _____ and means _____
- The symbol ~ is used for _____ and means _____

Basic Truth Tables

Complete the following truth tables:

A	***B***	***A*** $\vee$ ***B***
T	T	
T	F	
F	T	
F	F	

A	***B***	***A*** $\wedge$ ***B***
T	T	
T	F	
F	T	
F	F	

A	~***A***
T	
F	

Practice with Truth Tables

Construct a truth table for $A \wedge \sim B$:

A	***B***	~***B***	**A** $\wedge$ **~B**
T	T		
T	F		
F	T		
F	F		

Think About It

Why are truth tables useful tools for analyzing logical expressions? How might they help with debugging computer programs or electronic circuits?

Implications

Key Concept: Implications

Implications are _____

- Antecedent: _____
- Consequence: _____
- Notation: _____

Key Concept: Truth Values for Implications

Complete the truth table for $p \rightarrow q$:

p	q	$p \rightarrow q$
T	T	
T	F	
F	T	
F	F	

Key Concept: Related Statements

For the implication "if p then q" ($p \rightarrow q$):

- The **converse** is _____
- The **inverse** is _____
- The **contrapositive** is _____

Practice with Implications

Consider the statement: "If it's snowing, then it's cold outside."

1. Write the converse: _____
2. Write the inverse: _____
3. Write the contrapositive: _____
4. Which of these statements are logically equivalent to the original? _____

Think About It

Why does an implication remain true when its antecedent is false? How does this relate to making valid arguments in mathematics?

Equivalence and DeMorgan's Laws

Key Concept: Logical Equivalence

Statements *p* and *q* are logically equivalent if _____

Equivalent statements from what we've learned:

- A conditional and its _____ are logically equivalent
- The converse and _____ are logically equivalent

Key Concept: DeMorgan's Laws

- $\sim (P \wedge Q) =$ _____

  In words: To negate an "and" statement, _____

- $\sim (P \vee Q) =$ _____

  In words: To negate an "or" statement, _____

Practice with DeMorgan's Laws

Apply DeMorgan's laws to negate these statements:

1. "It is sunny and warm."
   Negation: _____
2. "The car is red or blue."
   Negation: _____
3. "The book is interesting or the movie is exciting."
   Negation: _____

Think About It

How might DeMorgan's laws be useful when designing computer systems or solving complex logical problems? Give a specific example.

Argument Types

Key Concept: Logical Arguments

A logical argument is _____

An inductive argument is _____

A deductive argument is _____

Practice with Argument Types

For each argument below, identify whether it is inductive or deductive:

1. All mammals have hair. Humans are mammals. Therefore, humans have hair.
   Type: _____
   Reason: _____

2. Every time I've worn this shirt, my team has won. Therefore, this shirt is lucky.
   Type: _____
   Reason: _____
3. Water has boiled at 100°C in every experiment we've conducted. Therefore, water boils at 100°C at sea level.
   Type: _____
   Reason: _____

Think About It

What is the relationship between inductive reasoning and the scientific method? Why can scientific theories never be absolutely "proven"?

Evaluating Arguments

Key Concept: Evaluating Inductive Arguments

An inductive argument is evaluated based on: _____

A **strong inductive argument** is _____

A **weak inductive argument** is _____

Key Concept: Evaluating Deductive Arguments

A deductive argument is considered **valid** when _____

A deductive argument is considered **sound** when _____

Practice with Evaluating Arguments

Evaluate each argument:

1. Premise: All students at this university pay tuition. Premise: Tanya pays tuition. Conclusion: Tanya is a student at this university.
    - Valid or invalid? _____ Why? _____
    - Sound or unsound? _____ Why? _____
2. Premise: If it rains, the ground gets wet. Premise: The ground is wet. Conclusion: It rained.
    - Valid or invalid? _____ Why? _____
    - Sound or unsound? _____ Why? _____

Think About It

Why is it important to distinguish between the validity of an argument (its logical structure) and its soundness (the truth of its premises)? How might this distinction help you evaluate claims you encounter in everyday life?

Analyzing Arguments with Venn/Euler Diagrams

Key Concept: Using Venn Diagrams for Arguments

Steps to analyze an argument with Venn/Euler diagrams:

1. _____
2. _____
3. _____

Practice with Venn/Euler Diagrams

Analyze this argument using a Venn/Euler diagram:

- Premise: All doctors have medical degrees.
- Premise: Sarah has a medical degree.

- Conclusion: Sarah is a doctor.

Is this argument valid? _____ Explain: _____

Think About It

How do Venn diagrams help visualize the logical relationships between sets in an argument? What types of arguments are best suited for analysis with Venn diagrams?

Analyzing Arguments with Truth Tables

Key Concept: Using Truth Tables for Arguments

Steps to analyze an argument with truth tables:

1. _____
2. _____
3. _____

Key Concept: Syllogism

A **syllogism** is _____

General form:

- Premise: _____
- Premise: _____

- Conclusion: _____

This is sometimes called the _____ property for implication.

Practice with Truth Tables

Analyze this argument using a truth table:

- Premise: If I study, I will pass the test.
- Premise: If I pass the test, I will graduate.
- Conclusion: If I study, I will graduate.

Step 1: Define variables

- Let *P* = _____
- Let *Q* = _____
- Let *R* = _____

Step 2: Write premises and conclusion symbolically

- Premise 1: _____
- Premise 2: _____
- Conclusion: _____

Step 3: Complete partial truth table

P	**Q**	**R**	**P→Q**	**Q→R**	**P→R**	**[(P→Q) ∧ (Q→R)]→(P→R)**
T	T	T				
T	T	F				
T	F	T				

Is this argument valid? _____ Explain: _____

Think About It

When would you use a truth table instead of a Venn diagram to analyze an argument? What are the advantages and limitations of each method?

Logical Inference

Key Concept: Logical Inference

Logical inference is _____

Three common logical inferences:

1. Modus Ponens: _____
2. Modus Tollens: _____
3. Hypothetical Syllogism: _____

Practice with Logical Inference

Given the statements:

- If it rains, then the soccer game will be canceled.
- It is raining.

What can you infer? _____

Using which rule of inference? _____

Think About It

How do rules of logical inference help us build valid chains of reasoning? How might these rules be useful in fields like mathematics, computer science, or law?

Types of Logical Fallacies

Key Concept: Using Truth Tables for Arguments

A logical fallacy is _____

Common types of logical fallacies:

1. **Ad hominem**: _____
2. **Appeal to ignorance**: _____
3. **Appeal to authority**: _____
4. **Appeal to consequence**: _____
5. **False dilemma**: _____
6. **Circular reasoning**: _____
7. **Straw man**: _____
8. **Post hoc**: _____
9. **Correlation implies causation**: _____

Practice with Logical Fallacies

Identify the fallacy in each statement:

1. "Senator Smith's tax plan must be bad because Senator Smith is a terrible person."
   Fallacy: _____
   Explanation: _____
2. "No one has proven that ghosts don't exist, so they must be real."
   Fallacy: _____
   Explanation: _____
3. "Either we cut environmental regulations or the economy will collapse."
   Fallacy: _____
   Explanation: _____

4. "Since we installed the new security system, crime has decreased. The security system must be working."
   Fallacy: _____
   Explanation: _____

Think About It

Why are logical fallacies so common in advertising and political discourse? How can awareness of these fallacies help you be a more critical consumer of information?

_____

_____

Key Takeaways

Summarize the most important concepts from each section:

Set Theory Basics:

- _____
- _____
- _____

Logic Basics:

- _____
- _____
- _____

Analyzing Arguments With Logic:

- _____
- _____
- _____

Questions to Ask in Class:

1. _____
2. _____
3. _____