

Measurements

Learning Goals

By the end of this lab, you should be able to...

- Estimate the uncertainty in measurements made with common devices.
- Report a measurement as a best guess value with an estimate of the uncertainty.
- Compare measured values using a graphical representation.
- Use a device with a relatively large uncertainty to make a relatively precise measurement.

Part I: Measurements Have Uncertainty

Q1

Question 1: Confirm the suggested tolerance quoted by the manufacturer that includes the length of the machine key.

Manufacturer's Statement:

Your machine key is supposed to be 3 inches long. However, as you know, making something *exactly* a given length is impossible. The company that sold us the machine keys knows this as well and reported a *tolerance* for that length. The company states that the machine key might be 0.01 inches longer or 0.03 inches shorter than the quoted length.

Include in the Lab Work/Lab Participation document:

- All three of your measurements written in the form $x_{best} \pm \delta x$ organized in a neat data table.
- An explanation and/or necessary calculations of the uncertainties chosen for each measurement.
- A graphical representation, *the Discrepancy Diagram*, of your three measurements and the tolerance quoted by the manufacturer accompanied by the three-column data table.
- Analysis of the Discrepancy Diagram you created.
- Confirmation of the tolerance - what measuring tool *confirms* it and *why*.

Useful Notes:

1. Equipment

In the gray plastic case, you have a machine key (that hunk of steel) and three instruments for measuring its length: a ruler with 1-cm divisions, a ruler with 1-mm divisions, and digital calipers.

Gray Case containing:

- a. Machine Key
- b. Centimeter ruler
- c. Millimeter ruler
- d. Calipers

Note: Please see page 2 of the Cleanup! Slideshow before you put the rulers back in the case. The rulers can be damaged if you put them back improperly.

Perform the measurements using each measuring tool and record them in a neat and organized data table. Include in the data table the length stated by the manufacturer as well. Read below about the uncertainties for each tool.

2. Properties of the measuring devices:

- a. *Two rulers:* As you make these measurements, you have to keep in mind the properties of the measuring device. With the two rulers, you will have to perform interpolation. That is, you'll have to estimate where between the lines the start and end of the object lie. How sure are you that, for example, the zero mark on the ruler is lined up perfectly with the edge of the object? Does it contribute to the overall uncertainty?
- b. With the *digital calipers* you'll have to be careful to make sure that they have been properly zeroed. An improperly zeroed instrument can introduce systematic error to an experiment.

3. Properties of the machine key:

- a. How do the properties of the machine key come into this? Notice that the machine key has rounded edges. Therefore, trying to align the rulers in *both edges* will contribute to the uncertainty. Refer to the "[Uncertainties and Error Analysis Tutorial](#)" document to calculate the uncertainty in these cases.
- b. Maybe the last millimeter or so is rounded. The rounded edges won't likely make it difficult to use the ruler with 1-cm markings. Nor will the rounded edges affect how you use the calipers because they can clamp onto the flat ends of the machine key.

However, the rounded edges make it difficult to use the ruler with millimeter markings. This might mean that you estimate the uncertainty to be larger than the 0.5 mm that was used with the pencil in the Pre-Lab reading.

4. **Rules for stating uncertainties and answers** (from Taylor's reading - pg 15.) They are also listed below.

- a. *Rule for Stating Uncertainties:* Experimental uncertainties should almost always be rounded to one significant figure. **One exception:** If the leading digit in the uncertainty is a 1, then keeping two significant figures in it may be better.
- b. *Rule for Stating Answers:* The last significant figure in any stated answer should usually be of the same order of magnitude (in the same decimal position) as the uncertainty.

5. **Discrepancy Diagram:**

- a. Refer to [*Everything About Plotting with Logger Pro Tutorial*](#) document or *Tutorial Video* about creating the Discrepancy Diagram. Include all measurements performed and the manufacturer tolerance. Include also the three-column data table of the Discrepancy Diagram.
- b. When working with digital instruments, the instruction manual should tell you what the uncertainty in the measurement is. The instruction manual for these digital calipers states that their *uncertainty is 0.02 mm*.
- c. To show the manufacturer tolerance in the Discrepancy Diagram in the form of the error bars, you must alter the above estimation in such a way that the error bars are the same. Logger Pro can't show error bars with different length for the same measurement.
- d. Analyze the Discrepancy Diagram you created. Discuss the significance of the discrepancies between the measured values and the manufacturer's value, and the respective uncertainties. Refer to section 2.3 in the Taylor's Reading for the discrepancy analysis. *Note: Regardless of whether the discrepancy is significant or not, make sure you address the question asked above: **confirming the tolerance**.*

Part II: A Clever Measurement

The Story

Mark Twain's immortal character Tom Sawyer is known for being a clever kid. (Just check out Chapter 2 of *The Adventures of Tom Sawyer*, the famous whitewashing scene.) In the upcoming experiment, you'll have to be equally clever in order to measure the thickness of a sheet of Tom's text.

Q2

Question 2: Evaluate the success of a clever measuring trick that you practice to measure the *thickness of one sheet* from the book provided when you don't have access to a more accurate measuring tool, like a caliper.

A successful evaluation should include:

- The thickness of a sheet measured directly using *digital calipers* written as $x_{best} \pm \delta x$.
- Description of the trick
- The thickness of a sheet measured using a *millimeter ruler* written as $x_{best} \pm \delta x$, using *a number of sheets* of your choosing to fulfill the success of the trick (See below).
- A description of how that thickness was found.
- Discrepancy Diagram
- An analysis of the outcome (discuss the discrepancy between the two measurements)
- Steps to improve the trick. Show your work to support your answer.
- Conclusion about the success of the trick along with a short explanation about why you consider the trick successful or not.

Equipment

Mark Twain's *The Adventures of Tom Sawyer*
Millimeter ruler
Digital calipers

Useful Notes:

While directly measuring the thickness of a single sheet of paper is impossible with the millimeter ruler, an indirect measurement can be easily accomplished. By measuring the thickness of many sheets (1 sheet = 2 pages), you can reduce the uncertainty in your measurement so that the measurement becomes meaningful. This is called CLEVER TRICK.

This trick is helpful only if you do not have access to a more precise measuring tool, like a caliper.

Obviously, the caliper is more precise than the millimeter ruler. So, let's assume that you do not have access to the caliper, but ONLY to a millimeter ruler. You want to find the thickness of a sheet using the millimeter ruler with high precision.

1. Use the digital caliper to directly measure the thickness of a single sheet of *The Adventures of Tom Sawyer*. Write your response in the form $x_{best} \pm \delta x$.

Now, it's time to use the only measuring tool you have, the millimeter ruler. Apply the CLEVER TRICK to find the thickness of a sheet.

2. Determine the thickness of a sheet in *The Adventures of Tom Sawyer* by measuring the thickness of many sheets at once with the mm-ruler.
3. How can we assign an uncertainty to your response above? All you have to do is divide the uncertainty in the total thickness by the number of sheets that you measured. For example, let's say you measured the thickness of 20 sheets to be $1.6 \pm 0.5 \text{ mm}$. Then the thickness of a single sheet would be $0.08 \pm 0.03 \text{ mm}$ or See [Appendix A](#) if you are interested in the rigorous proof of this extremely useful trick.
4. How close to the accurate thickness of a sheet are you? Use the discrepancy diagram to answer this question.
5. How many sheets should you measure with the mm ruler to have an uncertainty smaller than what you got from using the caliper? Perform some calculations to support your answer.
6. The trick is successful if **BOTH** of these **Two conditions are fulfilled**:
 - a. The Best Estimate measured value (with the mm-rule) by using the trick is within the range of the calipers measurement (range of error bars).
 - b. The uncertainty is reduced, meaning that it is less or equal to the caliper uncertainty.

Time to Clean Up!



Please clean up your station according to the Cleanup! Slideshow found in the lab module.

References

- [1] Dohrn-van Rossum, Gerhard. (1996). *History of the Hour*. University of Chicago Press, Chicago, IL.
- [2] Danielson, Dennis and Graney, Christopher M. (2014). The Case Against Copernicus. *Scientific American*, January 2014, Volume 310, no. 1, 72-77.

Appendix A: Reducing Uncertainty by Measuring Many Sheets at Once

The Problem at Hand

Consider a measured thickness, x_{total} , that represents the total thickness of some number of sheets N . Mathematically, the thickness of a single sheet, x_{sheet} , can be given by

$$x_{sheet} = \frac{x_{total}}{N}$$

Here the thickness of a sheet is a function of only one uncertain measurement (x_{total}). Before we continue with this specific case, let us investigate the general form of this problem.

The General Case

Consider a quantity q that is a function of a single measured value z . We could write this in a very general way as

$$q = f(z)$$

When we have such a relationship (a function of a single variable), the uncertainty δq in the calculated value q is given by

$$\delta q = \frac{df}{dz} \delta z$$

where δz is the uncertainty in the measurement of z .

Back to the Problem at Hand

In our problem, x_{sheet} is like q , x_{total} is like z , and $\frac{x_{total}}{N}$ is like $f(z)$.

That means to find the uncertainty in x_{sheet} , we just take the derivative of $\frac{x_{total}}{N}$ with respect to x_{total} and then remember to multiply that by δx_{total} . We find

$$\delta x_{sheet} = \frac{1}{N} \delta x_{total}$$

Since δx_{total} does not change as x_{total} increases or decreases, this equation shows that increasing N will reduce δx_{sheet} .

Note that this calculation relies on the assumption that there is very little variation in thickness from sheet to sheet.