

Name: _____

IN THIS LESSON, YOU WILL:

- Use the compound interest formula given a financial situation and interpret its results
- Evaluate expressions given variable definitions
- Demonstrate how compound interest can affect future loan balances
- Explain the difference between APR and APY
- Calculate the future value of periodic investments



INTRO

CALCULATE: Calculating Interest

Interest on a loan can be calculated in a variety of ways. Examine the two tables and use them to answer the questions below.

Table 1

Month	Beginning Balance		5% Interest		Ending Balance
1	\$100	+	$0.05 \times \$100 = \5	=	\$105
2	\$105	+	$0.05 \times \$100 = \5	=	\$110
3	\$110	+	$0.05 \times \$100 = \5	=	\$115
4	\$115	+	$0.05 \times \$100 = \5	=	\$120
5	\$120	+	$0.05 \times \$100 = \5	=	\$125

Table 2

Month	Beginning Balance		5% Interest		Ending Balance
1	\$100	+	$0.05 \times \$100 = \5	=	\$105
2	\$105	+	$0.05 \times \$105 = \5.25	=	\$110.25
3	\$110.25	+	$0.05 \times \$110.25 = \5.51	=	\$115.76
4	\$115.76	+	$0.05 \times \$115.76 = \5.79	=	\$121.55
5	\$121.55	+	$0.05 \times \$121.55 = \6.08	=	\$127.63

1. Briefly describe how each month's interest is calculated in Table 1.
2. How is Table 2's calculation different?
3. Would you prefer Table 1 or Table 2 if you were earning interest on an investment? Explain your answer.
4. Would you prefer Table 1 or Table 2 if you were being charged interest on a credit card or loan? Explain your answer.



EXPLORE IT

Compound Interest

The process of including interest in the balance when calculating new interest, like we did in Table 2 above, is called **Compounding Interest**. In that table, interest was compounded one time each year. What happens if we decide to compound the accumulated interest more frequently, like monthly or daily? Is there an easy way to calculate your balance when compounding? Let's explore these questions.

Let's start with the simple interest equation that you've used many times before:

$$A = P(1 + r)^t$$

Where:

- A = Total Balance after interest has accrued
- P = Principal (This may be referred to by other terms such as starting balance or loan amount)
- r = the annual interest rate
- t = the amount of time in years

1. If the annual interest rate was 8%,
 - a. How would you calculate the monthly interest rate?
 - b. How would you calculate the daily interest rate?

2. Write a general expression for converting an annual interest rate into other time intervals where n is used to represent the time interval.

If we update the formula above to include this time interval, it looks like this now:

We have one more problem. The variable t represents years, but if we're compounding monthly or daily, we need the exponent to reflect the number compounds, NOT the number of years.

3. If you are compounding monthly over 3 years, how many times do you compound?
4. If you are compounding daily for 3 years, how many times do you compound?
5. If n represents the time interval and t represents time in years, write an expression that calculates the number of times you compound in t years.

Ok, we're almost there. Let's update our formula one more time.

The equation above is called the **compound interest formula** and it can be used to calculate the account balance when interest is compounded n times per year.



LEARN IT

Using the Compound Interest Formula

Now that we have a formula that can be used to calculate balances that use compound interest, let's put it into practice.

If Alana has a loan balance of \$12,000 that is compounded monthly at a rate of 4.5%. Assuming that she

makes no payments on the account, what will her balance be in 4 years?

1. Review the completed example below.

1 Write the compound interest formula:
It's often useful to write down the equation with variables before substituting

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

2 Substitute values:
In the situation above,

- $P = \$12,000$
- $r = 4.5\%$ (0.045 when we plug it in)
- $n = 12$ because compounding is done monthly
- $t = 4$ years

$$A = 12000\left(1 + \frac{0.045}{12}\right)^{12 \cdot 4}$$

3 Simplify:
Start by simplifying the parentheses and exponent

$$A = 12000(1.00375)^{48}$$

4 Raise the base to the exponent:
Remember that order of operations tells us to apply exponents before we multiply. That means we'll wait to multiply by the principal until we simplify the exponent.

$$A = 12000(1.19681)$$

5 Multiply:
Finish simplifying by multiplying the remaining values together

$$A = 14361.72$$

2. Follow your teacher's directions to complete each compound interest problem.

Example 1

$P = \$5000$

$r = 6.25\%$

Compounded daily for 5 years

1 Formula:

2 Substitute:

3 Simplify:

Example 2

Carmine takes a loan for \$11,500 at a rate of 8% that is compounded quarterly. Assuming she makes no payments for the first 2 years, what is her loan balance?

4 Apply the Exponent:

5 Multiply:



PRACTICE IT

5-Point Practice: Compound Interest Formula

Complete any number of problems that add up to 5 points.

1 point each

$P = \$8,500$

$r = 3.175\%$

Compounded monthly for 10 years

$P = \$28,000$

$r = 9\%$

Compounded annually for 5 years

$P = \$275$

$r = 6.25\%$

Compounded daily for 20 years

2 points each

Marko has a credit card balance of \$875 that is compounded daily at an annual percentage rate (APR) of 19.62%. If he makes no payments, what will his balance be in 2 years?

Bethany has \$37,525 in student loans that accumulate interest at 3.73% annualized interest that is compounded monthly. If she starts making payments in 4 years, what will her loan balance be?

**3 points
each**

Camilla took a \$23,500 secured auto loan that allows her to make no payments for 18 months. If interest is still compounded monthly at a rate of 6.3%, what will her balance be in 18 months when she starts making payments?

Shawn's mortgage balance of \$263,000 is compounded daily at a rate of 4.075% annualized interest. If no payments are made for 6 months, what will the balance be after this time has passed?



LEARN IT

VIDEO: [APR vs APY](#)

When shopping for a loan, you may see the terms APR and APY. What are these terms and how do they apply to compound interest? Watch the video at the top of the linked article, then use it to answer the questions.

1. What do APR and APY stand for?
2. What is the key difference between APR and APY?

Example

If a loan charges 1% interest per month, what is its APR and APY?

$$APR = r \cdot n$$

Where:

- r = periodic interest rate
- n = number of periods in a year

$$APR = 1\% \cdot 12 = 12\%$$

APR is 12%

$$APY = \left(1 + \frac{r}{n}\right)^n - 1$$

Where:

- r = Annual Percentage Rate, as a decimal
- n = number of times the account is compounded in a year

$$APY = \left(1 + \frac{.12}{12}\right)^{12} - 1$$

APY = 0.1268

APY is 12.68%.

3. Shawn knows that his student loan interest is calculated at a rate of 0.35% per month.

- What is Shawn's APR?
- What is Shawn's APY?
- Which of the percentages represents the actual cost that Shawn will pay for his student loans if he makes no payments?

4. Take a look at the following advertisements for financial products.

1.23% APY*
for 12 months
WITH A FLEXIBLE SAVINGS ACCOUNT.

First National Bank
One with You.[®]

Own it – Whenever, Wherever
Give yourself some credit and save while you shop.

9.88% APR*

COMMUNITY POWERED
FEDERAL CREDIT UNION

VISA® CREDIT CARD
rates as low as

Why does the savings account advertise using APY while the credit card advertises using APR?



APPLY IT

Follow your teacher's directions to complete the Application Problems.

Teachers, you can find the Application problems linked in the Lesson Guide.