Linear Algebra Lesson 12:

Nullity, Rank, and Range

Please be sure to mark down the date and time that you start this lesson. Carefully take notes on pencil and paper while watching the lesson videos. Pause the lesson to try classwork before watching the video going over that classwork. If you work with any classmates, be sure to write their names on the problems you completed together. Please wear masks when meeting with classmates even if you meet off campus.

You will cut and paste the photos of your notes and completed classwork and a selfie taken holding up the first page of your work in a googledoc entitled:

MAT313F21-lesson12-lastname-firstname

and share editing of that document with me <u>sormanic@gmail.com</u> and with our graders. If you have a question, type QUESTION in your googledoc next to the point in your notes that has a question and email me with the subject MAT313 QUESTION. I will answer your question by inserting a photo into your googledoc or making an extra video.

Watch the Playlist 313F21-12-1to5

Nullity and Null Space of a Matrix Defn: Nullity is the dimension of the null space. Classwork: What is the nullity of $\begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 3 & 1 & 0 \end{bmatrix}$ Start $\begin{bmatrix} 1 & 0 & 2 & 1 & 0 \\ 0 & 3 & 1 & 0 \end{bmatrix}$ $\begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 3 & 1 & 0 \end{bmatrix}$ Start $\begin{bmatrix} 1 & 0 & 2 & 1 & 0 \\ 0 & 3 & 1 & 0 \end{bmatrix}$ $\begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 3 & 1 & 0 \end{bmatrix}$ $\begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 3 & 1 & 0 \end{bmatrix}$ Space $\begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 3 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ so far we have three leaders Since there are four columns There is one free variable Null space = $\begin{cases} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} + x_3 \begin{pmatrix} 2 \\ 1 \end{pmatrix} & x_3 \in \mathbb{R} \end{cases}$ Null space = span of directions

one free variable

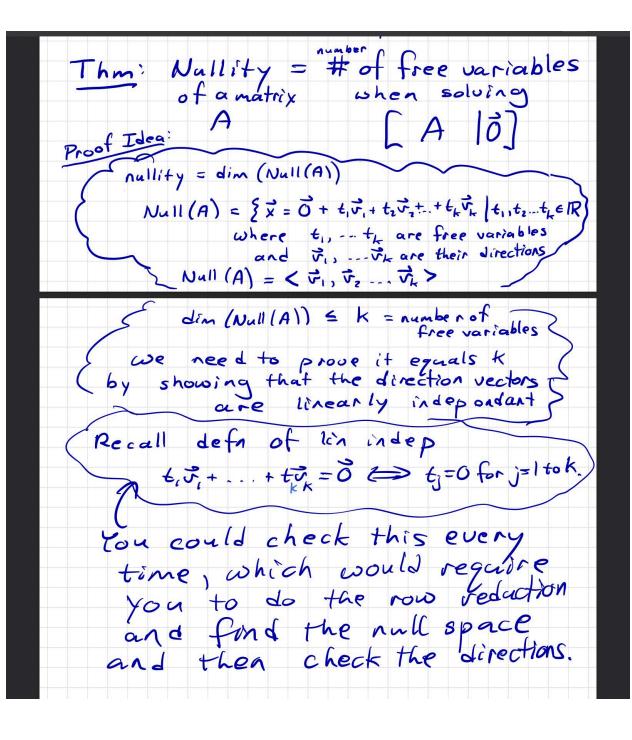
only one direction

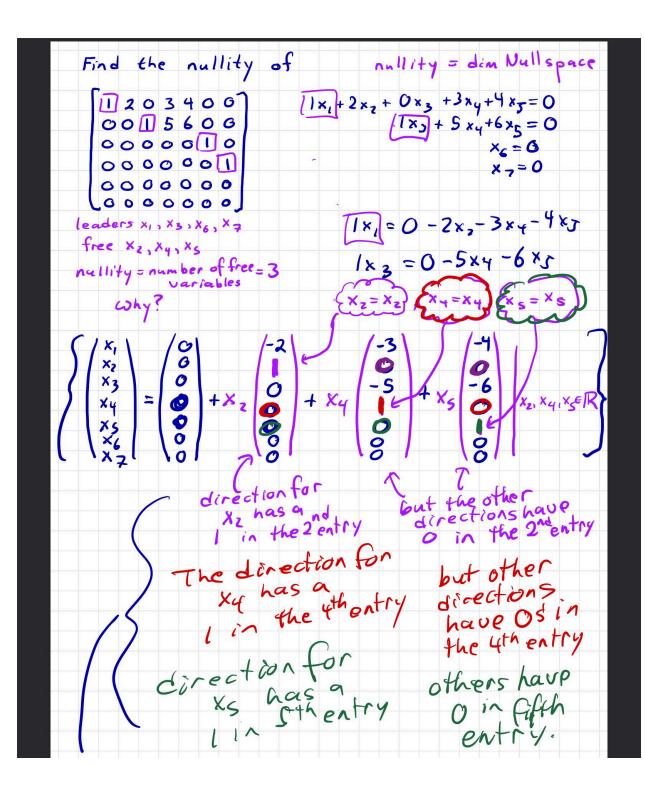
Dimension of null space = 1 = nullityThm: Wallity = # of free variables
of a matrix when solving

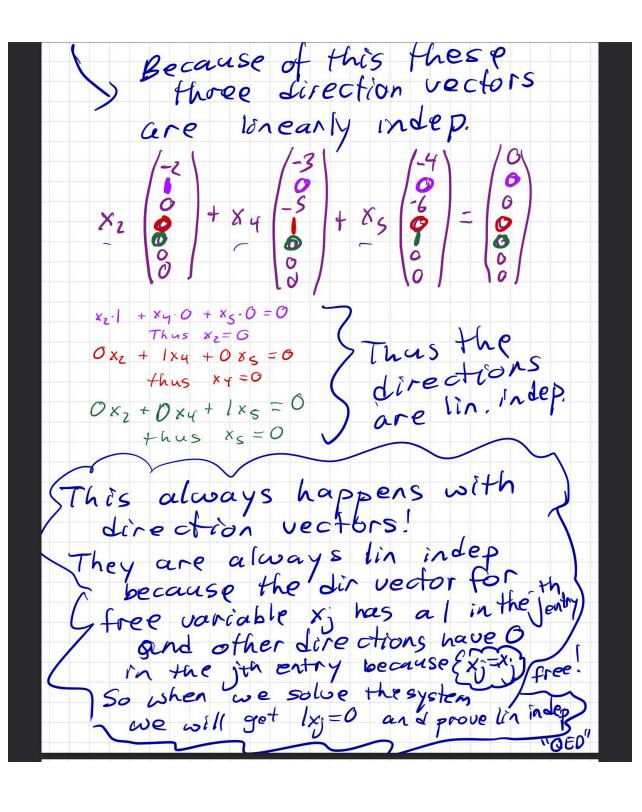
[A 10] sproof of this than is difficult

Rank and Range of a Matrix Range of a Matrix is the Span of its columns. Range 0 3 1 0 = < (0), (3), (7), (6) > Rant of a matrix = dim of the Range Recall that the columns may not be linearly independent, use pivot columns to find linearly independent columns To find pivot columns do row reduction and find which columns have leaders [1021 0] Echelon
CO 0 0 0 0 enough pivot columns have Columns 1, 2 and 4. 50 rank =3. Thm: Rank = # of leaders
of matrix A when solving [A | o]

Thm: Rank + Nullity = number of variables x,,x2 ... xm If a matrix has m columns then rank + nullity = m. Rank + Nullity = dimension of domain' Amcolumns A & Maxm $A \stackrel{>}{\times} = \stackrel{>}{y} \qquad input \stackrel{>}{\times} \in \mathbb{R}^m$ $\stackrel{>}{\times} \in \mathbb{R}^m \qquad \stackrel{>}{y} \in \mathbb{R}^n$ $\stackrel{>}{\times} \in \mathbb{R}^m \qquad \stackrel{>}{\times} \in \mathbb{R}^n$ Remember always follow the row reduction algorithm taught in the course.

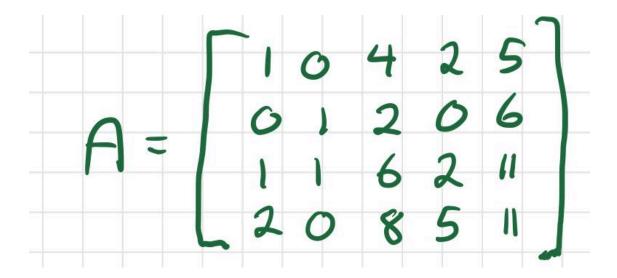






David the leaders Style of the columns Co
Classwork: find a basis of brearly independent
vectors for the range (pause + try)
Recall that the proof columns are interry in
These are the columns with leaders.
$Range = \left\langle \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \right\rangle$
These vectors are a basis for range (A) be cause they are linearly indep
be cause they are linearly indep
Classwork: verify this basis is lin. indep.
Check ((()
has only one solution ti=Ofor
J=1to 4
10000 0 00 000
o o o o o reduction
18000 0 (easy temo for ass)
no free vardables so it 15 living.

Homework: For the following matrix A



Homework:

(Note Exam 2 is very similar to this homework but with easier matrices)

Part I of HW:

- 1) Row Reduction to Echelon Form (boxing leaders)
- 2) Find Nullity and Rank
- 3) Write the Range as the span of a basis of linearly independent pivot columns.
- 4) Verify the basis of the Range is linearly independent

Part II of HW:

- 1) Continue Row Reduction of A to Reduced Echelon Form (boxing leaders)
- 2) Find Null Space of A
- 3) Check the directions in the null space using matrix multiplication
- 4) Write the Null Space as a span of a basis of linearly independent directions.
- 5) Verify the directions of the null space are linearly independent.

After completing the homework check the homework solutions <u>here</u>. Email me your corrected solutions and ask any questions you have. You must also include a <u>selfie</u> with your work.

After that you may study and then practice the Sample Exam in a timed setting <u>here</u>. You do not submit your sample exam to me.