MTH401 Assignment 1
Solution 2022

Q1: Solve the Differential equation

$(3x^2 + 9xy + 5y^2)dx - (6x^2 + 4xy)dy = 0$; y(2) = - 6

$$(3x^2 + 9xy + 5y^2)dx = (6x^2 + 4xy)dy$$

$$\frac{dy}{dx} = \frac{(3x^2 + 9xy + 5y^2)}{(6x^2 + 4xy)}$$

$$\frac{dy}{dx} = \frac{x^2\left(3 + \frac{9y}{x} + 5\frac{y^2}{x^2}\right)}{x^2\left(6 + \frac{4y}{x}\right)}$$

$$\frac{dy}{dx} = \frac{3 + \frac{9y}{x} + 5\left(\frac{y}{x}\right)^2}{6 + \frac{4y}{x}} \quad \ldots . (1)$$

The given differential equation is homogeneous,

$$y = vx \ldots . (2)$$

$$Then \quad v = \frac{y}{x} \quad \ldots . (3)$$

Differentiate equation (2)

$$\frac{dy}{dx} = v + x\frac{dv}{dx} \quad \ldots . (4)$$

Put the value of equation (4) and (3) in equation (1)

$$v + x\frac{dv}{dx} = \frac{3 + 9v + 5v^2}{6 + 4v}$$

$$= \frac{3 + 9v + 5v^2}{6 + 4v} - v$$

$$x \frac{dv}{dx} = \frac{3 + 9v + 5v^2 - 6v - 4v^2}{6 + 4v}$$

$$x \frac{dv}{dx} = \frac{3 + 3v + v^2}{6 + 4v}$$

$$\frac{dx}{x} = \frac{6 + 4v}{3 + 3v + v^2} dv$$

Integration on both sides,

$$\int \frac{dx}{x} = \int \frac{6 + 4v}{3 + 3v + v^2} dv$$

$$lnx + lnc = \int \frac{2(3 + 2v)}{3 + 3v + v^2} dv$$

$$lncx = 2ln(3 + 3v + v^2)$$

$$cx = (3 + 3v + v^2)^2$$

Replacing v by equation (3) $cx = (3 + 3 (\frac{y}{x}) + (\frac{y}{x})^2)^2$

$$cx = (3 + \frac{3y}{x} + \frac{y^2}{x^2})^2$$

$$cx = (\frac{3x^2 + 3xy + y^2}{x^2})^2$$

$$cx = (3x^2 + 3xy + y^2)^2$$

x^4

$$x^4.\, cx = (3x^2 + 3xy + y^2)^2$$

$$cx^5 = (3x^2 + 3xy + y^2)^2 \ldots . (5)$$

$$y(2) = -6$$

$$\therefore y(x) = y$$

$$x = 2, \quad y = -6$$

$$(2)^5 c = (3(2)^2 + 3(2)(-6) + (-6)^2)^2$$

$$32c = (12)^2$$

$$32c = 144$$

$$c = \frac{144}{32}$$

$$c = \frac{9}{2}$$

Put the value of c in equation (5)

$$\frac{9}{2} x^5 = (3x^2 + 3xy + y^2)^2$$