Calcolare il limite $\lim\limits_{x\to 0}\dfrac{\operatorname{sen} x-\arctan x}{x^3}$

Ricordiamo gli sviluppi di

$$\operatorname{sen}(x)=x+\frac{x^3}{3!}+\frac{x^5}{5!}+\frac{x^7}{7!}$$

$$\arctan(x)=x-\frac{x^3}{3}+\frac{x^5}{5}-\frac{x^7}{7}$$

$$\lim_{x\to 0}\frac{\operatorname{sen} x-\arctan x}{x^3}=\lim_{x\to 0}\frac{x+\frac{x^3}{3!}+\frac{x^5}{5!}+o(x^5)-(x-\frac{x^3}{3!}+\frac{x^5}{5!}+o(x^5))}{x^3}=$$

$$=\lim_{x\to 0}\frac{x+\frac{x^3}{3!}+\frac{x^5}{5!}+o(x^5)-(x+\frac{x^3}{3}-\frac{x^5}{5}+o(x^5))}{x^3}=\lim_{x\to 0}\frac{\frac{x^3+2x^3}{6}+\frac{x^5-24x^5}{120}+o(x^5)}{x^3}=$$

$$=\lim_{x\to 0}\frac{\frac{3x^3}{6}-\frac{23x^5}{120}+o(x^5)}{x^3}=\lim_{x\to 0}\frac{x^3\left(\frac{1}{2}-\frac{23}{120}x^2+o(x^2)\right)}{x^3}=\frac{1}{2}$$