AP PRECALCULUS / TOPIC 2.1 • STUDENT GUIDED NOTES

01 | Sequences and explicit rules

Name: _____ Date: _____ Class: _____

Today I am learning to identify and model sequences.

So I can predict values from constant additive or proportional change.

I'll know I have learned it when I can write, evaluate and justify a rule.

Essential question: How does the type of change determine the model?

Prerequisites: integer substitution, signed arithmetic, powers, roots and percent change.

A sequence is an ordered list of numbers. Each number is a term. A finite sequence ends; an infinite sequence continues. Inputs are integer indices, not all real numbers. We use $n = 1$ for the first listed term unless $n = 0$ is specified. A graph consists of separate points.

1. Vocabulary: Each output is a _____. A sequence with a last term is _____. Its graph is _____ (not connected).

2. I DO • Evaluate $a_n = 4n - 3$ at $n = 7$.

Substitute 7: $a_7 = 4(7) - 3 = 28 - 3 = 25$.

3. WE DO • For $a_n = 3n - 4$, the first four terms are _____, _____, _____, _____.

4. YOU DO • List the first four terms of $a_n = 5 + (-1)^n$.

5. CHECK • (a) Next Fibonacci term: 1, 1, 2, 3, 5, 8, ...? (b) Why is $n = 2.5$ not an index?

02 | Arithmetic sequences: constant difference

Arithmetic means adding the same amount d at each step. This is constant additive change, like a linear function at integer inputs.

RULE: $a_n = a_k + d(n - k)$; $a_n = a_0 + dn$; $a_0 = a_1 - d$.

Find d by subtracting the previous term from the next term.

6. I DO · 2, 5, 8, 11, … Find a rule and a_{100}.

$d = 5 - 2 = 3$. $a_n = 2 + 3(n - 1) = 3n - 1$. $a_{100} = 299$. Check $a_1 = 2$.

7. WE DO · 7, 2, −3, −8, … d = _____; a_0 = _____; a_n = _____.

8. YOU DO · Write rules using k = 2 and k = 4 for 44, 36, 28, 20, … .

9. CHECK · (a) Is $s_n = n^2 - 3$ arithmetic? (b) Is $s_n = 6 - 2n$ arithmetic? Explain.

03 | Arithmetic: nonconsecutive terms and graphs

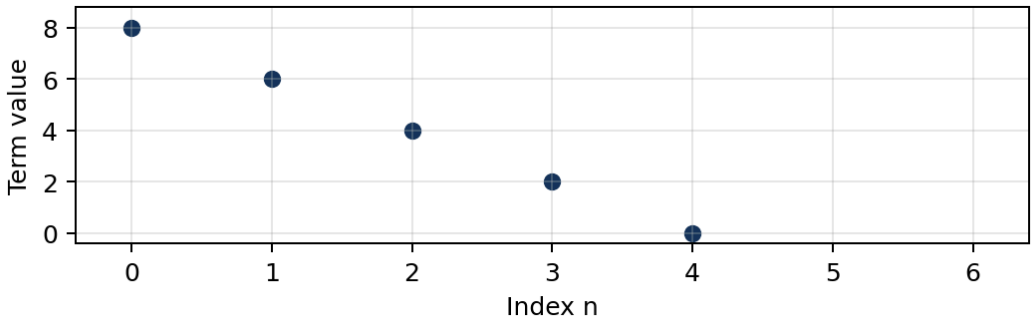
When two indices are not adjacent, divide the change in term value by the change in index. The number of steps is n − k, not n. KEY IDEA: the difference is the slope of the discrete linear pattern.

10. I DO · Given $a_2 = 7$ and $a_6 = 9$, find a rule and a_{24}.

Four steps produce a change of 2, so $d = 2 \div 4 = 0.5$. $a_n = 7 + 0.5(n - 2)$; $a_{24} = 18$.

11. WE DO · Given $a_3 = 8$ and $d = -3$, a_n = _____; a_{12} = _____.

12. YOU DO · Use the graph below to find a_0, d, a_n and a_{17}.



13. CHECK • (a) Why divide by the index gap? (b) If $a_4 = 10$ and $a_8 = 10$, what is d?

04 | Geometric sequences: constant ratio

Geometric means multiplying by the same nonzero factor r each step (for the nonzero terms used here). It is constant proportional change.

RULE: $g_n = g_0 r^n = g_k r^{(n-k)}$; $g_0 = g_1 \div r$. Recursive rule: $g_n = r \cdot g_{n-1}$.

Find r by dividing the next term by the previous nonzero term.

14. I DO • 2, 6, 18, 54, … Write a rule using k = 1 and k = 3.

$r = 6 \div 2 = 3$. $g_n = 2(3)^{(n-1)} = 18(3)^{(n-3)}$. Both give $g_1 = 2$.

15. WE DO • 64, 16, 4, 1, … r = _____; g_0 = _____; g_n = _____.

16. YOU DO • Find a rule and g_6 for 3, −15, 75, … .

17. CHECK • (a) Is 1, 3, 2, 6, … geometric? (b) What does a negative ratio do to nonzero terms?

05 | Geometric graphs and known terms

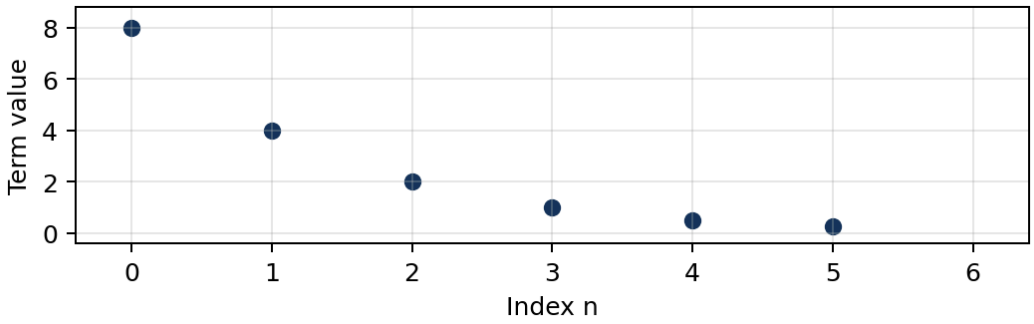
For positive starting values, $r > 1$ means growth and $0 < r < 1$ means decay. A negative ratio alternates signs; do not assume it is monotonic. Geometric graphs are discrete exponential patterns.

18. I DO · Given $g_2 = 24$ and $r = \frac{1}{2}$, find g_n and g_5.

$g_n = 24(\frac{1}{2})^{(n-2)}$. $g_5 = 24(\frac{1}{2})^3 = 3$. There are three multiplication steps.

19. WE DO · Given $g_3 = -3$ and $r = 10$, $g_n =$ _____; $g_5 =$ _____.

20. YOU DO · Use the graph below to find g_0, r, g_n and g_{10}.



21. CHECK · (a) Is the graph connected? (b) What percent is lost at each step?

06 | Geometric: finding a ratio from two terms

Use the earlier term as g_n. Substitute the later term into $g_n = g_n r^{(n-k)}$. Divide by g_n and take the appropriate root. Odd gaps allow one real root; even gaps can allow two signs when the quotient is positive. Context may require $r > 0$.

22. I DO · $g_3 = -2$ and $g_6 = 128$. Find g_n and g_{11}.

$128 = -2r^3$; $r^3 = -64$; $r = -4$. $g_n = -2(-4)^{(n-3)}$; $g_{11} = -2(-4)^8 = -131072$.

23. WE DO · $g_2 = 48$ and $g_7 = 1.5$. $r^5 =$ _____; $r =$ _____; $g_{11} =$ _____.

24. YOU DO · $g_4 = -8$ and $g_{11} = \frac{1}{16}$. Find r and g_1.

25. CHECK · (a) $g_1 = 4$, $g_3 = 36$: possible r? (b) What if the context requires positive quantities at every index?

07 | Applications: fixed amount and fixed percent

Fixed amount → arithmetic. Fixed percentage → geometric. For a positive starting value and percent p written as a decimal, growth factor = $1 + p$ and decay factor = $1 - p$. Always label the index and units.

26. I DO · Row 1 has 20 seats; row 2 has 23. Find seats in row 16.

$d = 3$; $a_n = 20 + 3(n - 1)$; $a_{16} = 65$ seats.

27. WE DO · Starting salary \$37,500; yearly raise 5%. Factor = _____; year 5 salary = _____.

28. YOU DO · In year 0 a farm has 10,000 trees. Each year 10% are harvested. Write a rule and find years 1–4.

29. CHECK • (a) A 7% decrease has factor _____. (b) Why is a $100 raise not the same model as a 5% raise?

08 | Geometric mean and decay tables

For three consecutive geometric terms a, b, c, $b^2 = ac$. For positive terms, $b = \sqrt{(ac)}$. This is the geometric mean. If signs are not restricted, consider both possible signs of b.

30. I DO • A substance has 1320 g at hour 0 and 384 g at hour 2. Find the hourly factor.

$384 = 1320r^2$; $r = \sqrt{(384 \div 1320)} \approx 0.53936$. Choose the positive root for physical mass.

31. WE DO • Complete the decay table below to the nearest gram.

Hour	0	1	2	3	4
Mass (g)	1320		384		

32. YOU DO • Find the positive missing middle term: 9, _____, 144. Write a rule starting at n = 1.

33. CHECK • (a) Why not average 9 and 144? (b) Could −36 also fit without positivity?

09 | AP extension: bouncing ball

A ball starts 30 feet above the ground and rebounds to 80% of its previous peak. Define $h_0 = 30$ before impact; h_n is the peak after impact n. Count each complete rebound as an upward and downward path.

34. I DO · Write h_n and calculate the first two rebound peaks.

$h_n = 30(0.8)^n$; $h_1 = 24$ ft; $h_2 = 19.2$ ft.

35. WE DO · Complete the height table to two decimals.

n	0	1	2	3	4	5	6	7
Height	30							

36. YOU DO · Find the first impact whose rebound is less than 1 ft.

37. CHALLENGE · Find total travel from release through ten complete rebounds (back to ground).

10 | Practice: basic and application

LEVEL 1 · Identify patterns and generate terms. LEVEL 2 · Select a rule and justify the index.

38. Find the first four terms of $a_n = 12n + 3$, starting at $n = 1$.

39. Classify $-4, -1, 2, 5, \ldots$ and write a zero-index rule.

40. Classify $-1, -4, -16, -64, \ldots$ and write a zero-index rule.

41. Find the 87th term of 105, 95, 85, 75, … .

42. Given $a_4 = 11$ and $a_{10} = 26$, find a_0 and a□.

43. Given $g_2 = 18$ and $g_5 = 486$, find g_0 and g□.

11 | Practice: AP-level reasoning

LEVEL 3 · Original AP-style questions. Explain why your model fits; check the index.

44. MCQ · Which rules fit 10, 2, −6, −14, …? I. $10 - 8(n - 1)$ II. $18 - 8n$ III. $2 - 8(n - 2)$.
A. I only B. I and II only C. II and III only D. I, II and III

45. MCQ · First term 4, difference 3. Seventh term? A. 22 B. 25 C. 972 D. 2916

46. MCQ · Which rule fits $\frac{1}{2}$, 2, 8, 32, …? A. $2(4)^n$ B. $\frac{1}{2}(4)^{(n-1)}$ C. $\frac{1}{2} + 4n$ D. $8(4)^n$

47. FRQ • Plan A begins with $200 and adds $40 per month. Plan B begins with $200 and grows 20% per month. Write both models (month 0), compare month 4, and explain why the differences in B change.

48. ERROR ANALYSIS • A student writes $g_n = 5(2)^n$ for 5, 10, 20, ... with first index 1. Correct it and explain.

12 | Lesson summary and exit ticket

WHAT I LEARNED TODAY

49. Summary: Arithmetic has constant _____; geometric has constant _____. The index gap is _____. Sequence graphs have _____ points.

50. EXIT 1 • Basic: classify 16, −8, 4, −2, ... and state the constant.

51. EXIT 2 • Application: $a_5 = 14$, $a_9 = 26$. Write an arithmetic rule and find a_{20}.

S2. EXIT 3 • Reasoning: Two positive geometric terms are $g_1 = 3$ and $g_3 = 27$. Can the ratio be uniquely identified if intermediate terms may be negative? Explain.