"NOTA" stands for None Of These Answers. For this test, unless otherwise specified, all quadrilaterals are simple (their sides do not cross), and all kites are convex.

Use the following information for questions 1, 2, and 3:

Trapezoid $GLHF$ has height 4 and bases of lengths 8 and 4.

1. What is the area of $GLHF$?
   A. 16 B. 32 C. 24 D. 48 E. NOTA

2. If one angle measures 90°, what is the perimeter of $GLHF$?
   A. $4\sqrt{2}$ B. $4 + 4\sqrt{2}$ C. $12 + 4\sqrt{2}$ D. $16 + 4\sqrt{2}$ E. NOTA

3. If $GLHF$ is isosceles, what is the perimeter of $GLHF$?
   A. $12 + 2\sqrt{5}$ B. $12 + 4\sqrt{5}$ C. $12 + 6\sqrt{5}$ D. $12 + 8\sqrt{5}$ E. NOTA

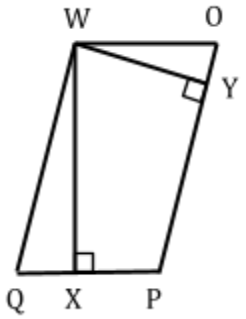
4. Super Radz loses a round of Pictionary after trying to draw a parallelogram $AHSP$. He labeled only $AH \| SP$, but Richard protests that more information is required to classify the quadrilateral. Which additional fact would prove that $AHSP$ is a parallelogram?
   A. $AH = SP$ B. $AP = HS$ C. $AS = HP$ D. $AS \perp HP$ E. NOTA

5. In rhombus $ABCD$, $AB = AC = 4$. What is its area?
   A. $2\sqrt{3}$ B. $4\sqrt{3}$ C. $8\sqrt{3}$ D. $16\sqrt{3}$ E. NOTA

6. In rhombus $ABCD$, $AC = BD = 4$. What is its area?
   A. $2\sqrt{2}$ B. $4\sqrt{2}$ C. $8\sqrt{2}$ D. $16\sqrt{2}$ E. NOTA

Use the following information for questions 7 and 8:

Given parallelogram $QWOP$ and its two perpendiculars from W intersecting lines QP and OP at X and Y respectively, $QX = 18, XP = 7, PY = YO = 15$, and $WY = 20$.



7. What is WX?
   A. 24 B. $\frac{49}{2}$ C. 25 D. $\frac{300}{7}$ E. NOTA

8. What is XY?

A. $\frac{117}{5}$ B. $\frac{103}{5}$ C. 20 D. 24 E. NOTA

9. A trapezoid has a circumcircle with radius $\frac{35\sqrt{6}}{24}$, an incircle with radius $\sqrt{6}$, and a leg of length 5. What is its area?

A. $8\sqrt{6}$ B. $10\sqrt{6}$ C. $12\sqrt{6}$ D. $14\sqrt{6}$ E. NOTA

10. What is the area of a parallelogram with a base of $1009^{1/3}$ and a height of $2^{1/3}$?

A. 2018 B. $2018^{1/9}$ C. $2018^{1/3}$ D. $2018^{2/3}$ E. NOTA

11. Which of the following could be the degree measures of the angles of a parallelogram (not necessarily in the same order as given)?

A. $107^\circ, 83^\circ, 107^\circ, 83^\circ$ B. $75^\circ, 105^\circ, 75^\circ, 115^\circ$

C. $120^\circ, 60^\circ, 110^\circ, 70^\circ$ D. $116^\circ, 116^\circ, 64^\circ, 64^\circ$ E. NOTA

12. Given a rectangle whose sides are in the *silver ratio*, if we remove two congruent squares with side length equal to the shorter side, then we are left with another smaller silver rectangle. What is the value of the silver ratio (ratio of longer side to shorter side)?

A. $1 + \sqrt{2} : 1$ B. $1 \pm \sqrt{2} : 1$ C. $\sqrt{2} : 1$ D. $\frac{1+\sqrt{5}}{2} : 1$ E. NOTA

13. Quadrilateral congruence is more complicated than triangle congruence. For example, SSSSA is a valid congruence theorem for convex quadrilaterals only. Otherwise, two quadrilaterals with four congruent sides and one equal angle may be different. As an example, what is the sum of the two possible areas of a kite $ABCD$ (not necessarily convex) with $\angle A = 60^\circ, AB = AD = \sqrt{6}, BC = CD = \sqrt{3}$? (Hint: These constraints imply that $BD = \sqrt{6}$.)

A. $\frac{3\sqrt{3}}{2}$ B. $\frac{3\sqrt{3}+3}{2}$ C. $3\sqrt{3}$ D. $3\sqrt{3} + 3$ E. NOTA

14. Cyclic kite $NOPE$ has side lengths $NO = NE = \sqrt{6} - \sqrt{2}$ and $PO = PE = \sqrt{6} + \sqrt{2}$. Additionally, $\angle E = 90^\circ$. What is its area?

A. 2 B. 4 C. 6 D. 8 E. NOTA

15. What is the side length of a nondegenerate square whose area is numerically equal to the length of its diagonal?

A. 1 B. $\sqrt{2}$ C. 2 D. 4 E. NOTA

16. Kevin is infinitely bored and decides to paint a convex quadrilateral of area 64 black and white with this process: First, paint the whole quadrilateral white. Then paint the quadrilateral defined by the white quadrilateral's midpoints black. Then paint the quadrilateral defined by the last quadrilateral's midpoints white. Repeat the process of connecting midpoints and painting to infinity. What is the total area Kevin will paint?

A. 64 B. 96 C. 108 D. 128 E. NOTA

17. Adam is new to geometry and is playing with two tiles shaped like congruent isosceles triangles. He lays them on a table side by side, with two sides of the triangles joined together, to form a quadrilateral. Which of the following could *not* be the shape he formed?
A. Rectangle B. Kite C. Rhombus D. Trapezoid E. NOTA

Use the following information for questions 18 and 19:

Michelle invented a dog food bowl for portion control in the shape of a rhombus with rigid wooden sides of length 2 feet each and negligible thickness. She can translate the frame and open/close the frame using hinges at the vertices, but it must fit in a rectangular 2 feet by 3 feet space.

18. What is the maximum possible area the frame covers, in square feet?
A. $2\sqrt{5}$ B. 4 C. $2\sqrt{3}$ D. $\frac{3\sqrt{7}}{2}$ E. NOTA

19. What is the minimum possible area the frame covers, in square feet?
A. $2\sqrt{3}$ B. $\frac{\sqrt{39}}{2}$ C. $\frac{\sqrt{26}}{2}$ D. 0 E. NOTA

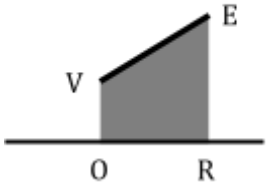
20. Which of the following is false?
A. If three angles of a quadrilateral are acute and equal to one another, then it is concave.
B. If both diagonals of a quadrilateral bisect opposite angles, then it is a rhombus.
C. If both diagonals of a quadrilateral bisect each other, then it is a parallelogram.
D. If both diagonals of a trapezoid are equal, then it is isosceles.
E. NOTA

21. Zoe decides to overlay two 3×5 inch food photos such that two corners of each photo coincide with two of the other photo, and the overlapping region is in the shape of a rhombus. What is the area of the overlap, in square inches?
A. $\frac{24}{5}$ B. $\frac{34}{5}$ C. $\frac{51}{5}$ D. $\frac{63}{5}$ E. NOTA

22. Vivian folds a rectangular strip of paper with a short dimension of $\sqrt{3}$ cm such that when the opposite corners are folded to meet each other, the edges of the paper (angle marked) make an angle of 30°. What is the area of the unfolded paper strip in square cm?

A. $3\sqrt{3}$ B. $3\sqrt{3}+3$ C. $6\sqrt{3}$ D. $6\sqrt{3}+6$ E. NOTA

23. McCree has a window in the shape of an upward-opening parabola, which has equation $y-1=3(x-4)^2$. His sniper rifle is a line segment of length 2, and he rests it on the window such that it passes through the focus of the parabola. It's high noon, so the rifle casts a shadow on the directrix $\overleftrightarrow{OR}$ such that $OV \| ER$ below. What is the area of $OVER$? (Hint: A parabola is the locus of points equidistant from a point called the focus and a line called the directrix.)

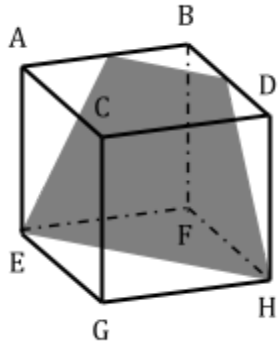


A. $\frac{\sqrt{6}}{3}$ B. $\frac{\sqrt{6}}{2}$ C. $\frac{3\sqrt{3}}{2}$ D. $\frac{2\sqrt{3}}{3}$ E. NOTA

24. In isosceles trapezoid $GUPT$ with $\overline{UP} \parallel \overline{GT}$, ΔGUP is isosceles with $GU = UP$ and ΔGPT is isosceles with $PG = GT$. What is the measure of the acute angles of the trapezoid?

A. 36° B. 48° C. 60° D. 72° E. NOTA

25. In the cube below, with edge length 1, the plane passing through E, H, and the midpoints of $\overline{AB}$ and $\overline{BD}$ intersect the cube in a trapezoid. What is the area of the trapezoid?



A. $\frac{9}{8}$ B. $\frac{9}{16}$ C. $\frac{3\sqrt{2}}{4}$ D. $\frac{3\sqrt{6}}{8}$ E. NOTA

26. Quadrilateral $CATH$ has diagonals of equal length. Which of the following statements is true?
A. $CATH$ cannot be a kite.
B. If $\overline{CH}$ is parallel to $\overline{AT}$, then $\overline{CA}$ must be parallel to $\overline{TH}$.
C. If the diagonals also bisect each other, then $CATH$ must be a square.
D. If $CATH$ is also a rhombus, then $CATH$ must be a square.
E. NOTA

27. Giorgio A. Tsoukalos's wild hair has told him that an ancient alien civilization is communicating from the four-star constellation Carnegii. His star map is worn out and only contains stars at $(-6, 0), (6, 0), (0, 8)$, but he knows that the fourth star is in the first quadrant, and the stars of Carnegii form a convex quadrilateral with area 51 *at most*. What is the area of the locus of all possible locations for the fourth star?
A. $\frac{51}{8}$ B. $\frac{99}{16}$ C. $\frac{193}{32}$ D. 6 E. NOTA

28. In a rhombus with diagonals of lengths 6 and 8, what is the area of the inscribed circle?
A. $\frac{81}{25}\pi$ B. $\frac{25}{4}\pi$ C. $\frac{576}{25}\pi$ D. $\frac{2304}{25}\pi$ E. NOTA

29. In a rhombus with diagonals of lengths 6 and 8, what is the area of the inscribed square, where every side of the rhombus contains one of the square's vertices, and a side of the square is parallel to the shorter diagonal?
A. 12 B. $\frac{576}{49}$ C. $\frac{288}{25}$ D. $\frac{25}{2}$ E. NOTA

30. A cyclic quadrilateral has circumradius $\sqrt{6}$ and side lengths $\sqrt{7}, \sqrt{11}, \sqrt{13}, \sqrt{17}$, in that order. What is its area? (Hint: Look for right triangles.)
A. $\frac{\sqrt{77}+\sqrt{221}}{2}$ B. $\frac{\sqrt{91}+\sqrt{187}}{2}$ C. $\frac{\sqrt{143}+\sqrt{119}}{2}$ D. $\frac{\sqrt{42}+\sqrt{66}+\sqrt{78}+\sqrt{102}}{2}$ E. NOTA