

# Seven Sketches book:

Typos, comments, questions, and suggestions

# Instructions:

- Scroll to the bottom and write your comment in black. If something is written in green, it means we've dealt with it already. The latest version is available <a href="here">here</a>.
- You can leave your **name** if you want to. We will acknowledge those people who significantly contribute to improving the book.
- Indicate issues using **searchable strings** or  $1 + 1 = x^2$  local numberings, rather than page numbers.

# Major suggestions:

- 1. Add a "Summary of notation and symbols" (Ignacio Peixoto)
- 2. Find more compelling, concrete, introduction example for Chapter 5: sell feedback, matrix equations; automatic construction of matrix representation from signal flow graph representation (eg representing a car forms a very sparse matrix); Markov chains?
- 3. CH.7 See if you can bring the beginning car example through the explanation of toposes, so that there's some intuition behind the application as the definitions are being presented. (Adam Theriault-Shay)[It's a good suggestion, but I think it's too difficult to implement well. --David]
- 4. Divide exercises into shorter vs longer problems (eg exercises vs challenges)
- 5. If the book compiles somewhat nicely to A5, then it would be readable on kindle readers
- 6. Provide concrete examples of all notation used in definitions
- 7. Present "big picture" ideas prior to formal proofs

# Typos, comments, questions, and minor suggestions:

- 8. Example 1.16 is about the tree of life. It would be helpful if you would draw it out like in class: mammal, panthera, etc. (Anonymous)
- 9. Following up on 1, please include also the monotone map to the Kingdom->Phylum->Class thing. (Anonymous)
- 10. Should page 8 of Chapter 1, paragraph before Example 1.17, end of second sentence, read " $R \le S \ if \ S \subseteq R$ " instead of " $R \le S \ if \ S \le R$ "?
- 11. Should page 8 of Chapter 1, paragraph before Example 1.17, end of last sentence, instead read "r:  $P \rightarrow Q$  such that  $r \circ f = g$ "?
- 12. Example 1.22 (Upper sets). I get a different answer for UB then the one illustrated. In particular why is {false} not an upper set of B? [Answer: because if x is in the upper set and x≤y then y is in the upper set. Here false≤true, so if false is in the upper set then the upperset is {false,true}.]
- 13. In chapter 2, can you give some examples of adjunctions?
- 14. In the proof to Proposition 1.40, the sentence "But *T* is a maximal element of P" should instead be "But *T* is a maximal element of B." (Also, it may be beneficial to clarify that *T* corresponds to the value "true" in B.)
- 15. In section 1.4, "Back to partitions," the sentence that begins "Suppose given a function between sets" is incomplete. In the same section, "Let's first this about the left adjoint" should be "Let's first think about the left adjoint." Again in the same section, there is a sentence that begins "The right adjoint we defined in" that is unfinished.
- 16. Be clearer about Hasse diagrams and how they generate posets, as well as free/path categories
- 17. In Definition 3.20 (equivalence/isomorphism of monoidal functors), g should go from Q to P
- 18. Pg. 3 of Ch. 4, bullet 1: " $\Phi(p',p)$ " should be " $\Phi(p',r)$ "; also, after reading the following paragraph, I have the impression that the direction of the inequalities in these two bullets are backwards?
- 19. Regarding #11: Agree with 1st correction, but I think the inequalities are in the right direction. What's confusing is the terminology (**produced** would be better than provided), and maybe even more direct translations would help: "1) I can always provide less given the same amount

- of resources. 2) I can always provide the same given more resources"
- 20. In section 2.2.1, the sentence that begins "Concatenating with a length-0" ends abruptly.
- 21. In section 2.2.2, a verb is missing: "we will [missing verb] several more icons in this chapter..."
- 22. In section 2.3, it reads, "However, this really only makes sense when the sets are finite." One of the examples, however, is infinite (the set of prime numbers), and it does make sense to write it out as a one-column table. Perhaps "However, this really only makes sense when the sets are countable" would be more accurate.
- 23. In chapter 5, make matrix composition notation have same order as composition for categories
- 24. In Chapter 5, what you call extended signal flow graphs are what most people call signal flow graphs; what you call signal flow graphs are a simplified form. Maybe better to use the usual terminology.
- 25. In Chapter 6, the pullback diagram after "is then a monomorphism if the square" has the small right-angle in a confusing place; it looks indistinguishable from the pushout diagram above. It might be clearer to put the right-angle in the bottom right corner instead.
- 26. Exercise 2.18 should probably be "give the other value of alpha\_Arrow and the three values of alpha\_Vertex" as opposed to "give the other value of alpha\_Vertex and the three values of alpha\_Arrow." Shortly afterwards, the database instance for the arrows of H show c and d pointing from 4 to 5 and e pointing from 5 to 5, while the diagram shows d and e pointing from 4 to 5 and no arrow c at all.
- 27. What you call "Navigators" could also be called "commutative quantales". Probably worth a mention.
- In 3.2.2 A second glance at wiring diagrams, "The iconography of wiring diagrams for symmetric monoidal posets":
   Condition a. in the definition of monoidal poset, Definition 3.1, says that if x\_1 ≤ y\_1 and x\_2 ≤ y\_2 then x\_1 ⊗ x\_2 ≤ y\_1 ≤ y\_2.
   The last part should be y 1 ⊗ y 2 instead.
- 29. Ch. 1: Overall, the difficulty level seems to increase once Galois connections are introduced. It would help if the concept of Galois connections could be grounded with more concrete examples, perhaps tying it back to the material about partitions; in particular, it would help to make explicit/turn into an exercise that the maps g! and g\* in the "Back to partitions" section are left and right adjoints, as well as how the facts proven for Galois connections like Props. 1.72 and 1.75 and the closure operator material in 1.5.3 apply to these maps. Alternatively, you could make the \*3 / div 3 Galois connection the prototypical example for the section that the reader can check the facts about Galois connections with. More diagrams illustrating the concepts regarding Galois connections can also help. (Nelson Niu)
- 30. Another reason why the Galois connections section is tricky is that many times pairs of maps are stated to form Galois connections without proof or elaboration (or at the very least making it an exercise). For example, Example 1.80 gives defines maps that are claimed to be the left and right adjoints of f\*, but does not actually explain why this is the case (it's not too difficult to check, but some motivation would be nice, especially because Galois connections seem hard to conceptualize without actually playing with the ordering relations). Similarly the maps between Pos(S) and Rel(S) in 1.5.4 are merely stated to be Galois connections without justification. (Nelson)
- 31. For section 1.2.4 on wiring diagrams, it may help to highlight the difference between wiring diagrams, which illustrate facts that can be proven about the poset, and Hasse diagrams, which are representations of the entire poset. (Nelson)
- 32. In section 1.5.4, the top node of the Hasse diagram for Rel({1}) should be labeled {(1, 1)} instead of {1}. (Nelson)

- 33. At the start of section 1.5.3, it is claimed that  $x \le x.f.g$  and that x.f.g.f.g = x.f.g, without justification. It may help to make it an exercise to show these facts are true, using Prop. 1.72. In particular, the proof that x.f.g.f.g = x.f.g involves proving both  $x.f.g.f.g \le x.f.g$  and  $x.f.g.f.g. \ge x.f.g$  using Prop. 1.72. (Nelson)
- 34. Example 1.69 gives a good diagram for how to get a partition on T from a partition on S, but another diagram illustrating a more complicated scenario may be helpful. There should probably also be a diagram for how to get a partition on S from a partition on T. (Nelson)
- 35. After section 1.5.3, which explained that f.g was the closure, I was curious about what kind of map g.f was. Something briefly suggesting that g.f is an interesting map as well would be nice. (Nelson)
- 36. In the proof of the adjointness condition in the proof of Prop. 1.79, it's not entirely clear that iff is proven, since "take the meet of both sides" doesn't appear to be an obviously invertible operation (it may be the case that if A is a subset of B, then meet(A) ≥ meet(B), but the converse is not necessarily true). (Nelson) [Nice catch! --David]
- 37. Example 1.46 says a <u>surjective</u> function f induces a monotone map f\*, but Exercise 1.47 just asks to choose any non-identity function f to find f\*, so at least one of these seems not to be quite right. (Nelson)
- 38. Exercise 1.25, about the ordering on a set of partitions, seems a little out of place, since the ordering on a set of partitions isn't defined until 1.33. (Nelson)
- 39. The proof to Prop. 1.11 references "the first half" and "the second half" of the proposition, even though it's not immediately clear what the two halves of the proposition are. (I think one half is to show that a partition gives an equivalence relation and the other is to show that an equivalence relation gives a partition, but it would be nice to make that explicit.) (Nelson)
- 40. Around exercise 1.4 and anywhere else where the boolean poset came up, it really helped me understand what was going on when I realized that ≤ in the boolean poset was equivalent to implication (=>); that is, A ≤ B when A => B. I think stating that would make it easier to understand how monotone maps into the boolean poset are a good way to model observations. (Nelson)
- 41. Section 1.1.1 (part that introduces join operation) "it is true that a is connected". It should be "it is true that **x** is connected"
- 42. Section 1.1.1 (part that introduces join operation) "we shall say the joined of two systems"
- 43. Section 2.1 typo "Our second goal discuss something called" (Adam Theriault-Shay)
- 44. 2.2.2 ≤ should be a tensor at "We now return to how the properties of monoidal posets....

  Property a. says that if..." (Adam)
- 45. 2.2.2 typo "the associative property says should be thought of as saying that" (Adam)
- 46. Proposition 2.20 "is a symmetric monoidal poset then is" --then so is? (Adam)
- 47. 2.32. Definition You say "three properties" but list 4 (Adam)
- 48. Chapter 2: there is no lemon in the lemon meringue pie.
- 49. What is the difference between icons vs boxes in wiring diagrams?
- 50. In chapter 3: clarify relationship between V-cats and cats.
- 51. (typos) On page 26 it should say "of taking an expression" first line, there is no "n". on page 31 second paragraph it says "can also can address". Page 33 first paragraph "some come are aligned". On page 52 example 2.43 it says "Thus f lets us can convert...". Definition 2.71 says "an matrix" and in that same page in the last paragraph it says "eith 0..."
- 52. I don't get very clearly the "Assuming Bob is in San Diego" example, it could help if you write it in propositional logic language.
- 53. On page 53 after exercise 2.50, in the definition of a dagger V-category you should mention that the functor goes to the opposite category

- 54. Exercise 2.52.2 you wrote  $(X \times Y)((x_2, y_2), (x_2, y_2))$  it should be  $((x_2, y_2), (x_3, y_3))$
- 55. Clarify enrichment terminology. Enriched categories are not a priori categories!
- 56. In Prop 5.4.5 (Mat(R) is isomorphic to a certain presented prop), the equal signs in the equations look like dashes.
- 57. Change poset to preorder
- 58. Emphasise why we need symmetry for enriched products
- 59. Link to Elie Adam's thesis in the bibliography is broken
- 60. Chapter 1, page 2: One might call these surprised {\text{\text{it generative effects}}. Should be: One might call these surprises {\text{\text{it generative effects}}. (Jason Grossman)
- 61. page 5: A ≤ B in the poset if eems to be a grammatical whoopsie. Also, ``poset" hasn't been defined at this point, which I suspect is not what the authors wanted, with this being a math book and all, although personally I rather like words being used before they're defined. (Jason Grossman)
- 62. The existence of the generative effect, however, is captured in the inequality  $\Phi(A) \vee \Phi(B) \leq \Phi(A \vee B)$ . (1.3) This can be a strict inequality ...If I understand correctly, the formula only captures a generative effect if it IS a strict inequality (not just can be one). (Jason Grossman)
- 63. page 5 again: the map  $\Phi$  preserves some structure but not others. should be: the map  $\Phi$  preserves some structures but not others. (Jason Grossman)
- 64. Chapter 1, page 7: In Definition 1.8, right before the exercise, the notation used is a1,a2a1,a2 but becomes a,ba,bby the end of the sentence (Marcello Seri)
- 65. Chapter 1, page 9: Remark 1.16, the surjective function is showing a double headed arrow for the first time, maybe it's worth adding a notation footnote (Marcello Seri)
- 66. Chapter 1, page 9: Definition 1.19, missing period in "for any x∈Xx∈X It is often" (Marcello Seri)
- 67. "Example 1.61. Consider the two-element set P = {p, q, r} with ..." Looks like a 3-element set to me! (David Chudzicki)
- 68. Chapter 1 page 7 Definition 1.8:  $AP \cap AQ$  should P and Q be lowercase  $Ap \cap Aq$ ? (James Brock)
- 69. Chapter 1 page 8 Exercise 1.12 (Ap)p∈P(Ap)p∈P should be curly braces for "partition"? (James Brock)
- 70. Chapter 1 page 9 Exercise 1.15 question 2: second relation is the same as the first, is that deliberate, or are arrows pointing in wrong direction? (James Brock)
- 71. In the beginning of 1.2: "a poset a preordered set" with emphasis on letters "p" and "o" of preordered. I find it a little strange. Maybe the authors meant a "partially ordered set" instead. This mnemonic makes it easier to forget about antisymmetry requirement for posets. Calling a partially ordered set a "poset" and a preordered set a "preset" makes more sense and it's more obvious where the names come from. Update: I read further and now I understand it is not a mistake. The author made a deliberate decision to use the name "poset" for presets "skeletal posets" for posets, "partial order" for a preorder and "preorder" for a partial order. (Thrina Burana)
- 72. Chapter 1 page 3: the definition of 'join' could state more clearly that the qualifier "in at least one of A or B" applies separately to each of the connections mentioned. (Jerry Wedekind)
- 73. Chapter 1 page 4: Since you suppress redundant arrows in Hesse diagrams, you might replace "we draw an arrow from system A to system B if A <= B" with "an arrow from system A to system B means that A <= B". (Jerry Wedekind)
- 74. before exercise 1.80 " ES  $\rightarrow$  ET really is left adjoint to \* : ET  $\rightarrow$  S" should be " $\rightarrow$  ES"
- 75. Exercise 1.15 the second figure in part 2 has arrows pointing 'the wrong way' (if it's trying to

- be an example of something that's not a function)
- 76. Are there any plans for a printed version? (Yes. It's been submitted to a publisher. -- David)
- 77. Are solutions to the exercises available so that students can self-check? (No, but two possibilities for such students: 1. John Baez is running a course on the book, and I believe students are sharing responses to exercises; 2. If the book is accepted for publication, we will likely publish solutions to selected exercises (and leave others behind a professor-only wall) --David)
- 78. In the preface you state that "in 2018 there seems to be no introductory material available on monoidal categories". The beautiful book "Picturing quantum processes" by Coecke and Kissinger contains a wonderful introduction to monoidal categories (chapter 3) .(Alfredo Gómez)
- 79. In proof of Proposition 1.88, second paragraph, Proposition 1.81 is used to show that p0 <= g(f(p0)). But 1.81 requires adjointness, that we are trying to prove. Definition of `f` in 1.7 and the fact that g preserves meets should be used instead. (Nicolò Martini, nicolo@martini.io) n
- 80. Equation 1.4, the P and Q subscripts of A should be p and q. And maybe the big-U notation deserves a footnote explanation. (Robert Smart)
- 81. Chapter 1, page 4: For example, the two constituent systems could be the views of two local authorities on possibly contagion between an infected person and a vulnerable person \*. Language seems off here, "possibly contagion between". (Cam Fulton)
- 82. in 1.5.5 before ex 1.97 "This is actually the right adjoint of a Galois connection. Its right adjoint is ...". Well "its" is not well attached to what it refers to, but it will be ok if one of the "right"s is turned into a "left".
- 83. In ex 1.97, all the orderings should be swapped (i.e. Q<=L, Cl(Q) "<=" <= and similarly for Q') (Thomas Read)
- 84. Chapter 1 page 15 Exercise 1.50: **f\*P** and **f\*Q** should be **f\*(P)** and **f\*(Q)** ?(James Brock)
- 85. At the end of Example 2.24, should say "because f(0.5)+f(0.5) \ngeq f(0.5+0.5)" (Thomas Read)
- 86. What you call quantales are usually called *unital, commutative quantales*, not just commutative quantales. Perhaps this would motivate changing the name back to navigator as in an earlier draft of the book? (anonymous)
- 87. The sentence prior to Definition 3.76 appears to be cut off in midword (it ends with a dangling "i" with no terminal punctuation). The definition itself is also very hard to follow because all the symbols used therein a not defined therein (such as "T", and "c\_\*"). Definitions, as a rule, should be as self-contained as practical. Overall I would suggest giving this stretch of text a complete once-over.
- 88. Add remark about defining symmetric monoidal category as a category equipped with an equivalence (of categories) to a strict symmetric monoidal category
- 89. Compare cartesian closed posets and quantales
- 90. Decide whether to state adjoint functor theorem and similar results for posets or preorders
- 91. Definition 3.4 switches half way through from talking about a natural transformation F => G to talking about one from I => J (also presumably the conditions should be labelled (i) and (ii) rather than (i) and (a)) (Thomas Read)
- 92. Definition 3.56 it doesn't seem clear to me (from the content of the book up to this point) what "natural in c and d" means here especially since none of the examples of adjoints given in this subsection are proven (Thomas Read)
- 93. In 1.5.2 Back to partitions *E*S and *E*T are introduced without description. I suspect that they\ are the codomain of the surjective functions from S and T but it is not entirely clear.
- 94. In 1.5.2 "really is left adjoint to  $g*: \mathcal{E}T \to S$ ", should read "really is left adjoint to  $g*: \mathcal{E}T \to \mathcal{E}S$ " (James Brock)

- 95. In 1.2.1 Remark 1.16 Suggestion: when you introduce the double-headed arrow notation for surjection, "Given a surjective function f: A ->> P", mention explicitly that this notation will be used consistently for surjective functions in the following text. (James Brock) [We currently say "We often use the double arrowhead notation \$F\colon S \surj T\$ to denote surjections." I think that's good enough, because I don't want to commit to always doing it, e.g. if it's not important for some reason. -- David]
- 96. 1.5.1 Exercise 1.80 #3 Is it enough to instruct to choose an e that is "non-coarser"? Since some partitions can be neither courser nor finer, maybe instead say something like "Choose any strictly finer partition"? (James Brock) [Doesn't the exercise work as stated? Are you saying that something is false about it, that something is unclear about it, or that a finer point could be made here? --David]
- 97. Start of Section 3.54 typo: "To expose the reader to this concept, we provide a succinct definition of these using opposite categories and opposite categories." (Thomas Read)
- 98. Section 1.2 now (2018-04-05) that you use "preorder" instead of "poset", the "a <u>preordered</u> set" parenthetical is confusing rather than illuminating. (Dan Schmidt)
- 99. In definition 7.44 (c), should the subscript under the lower arrow be  $\Lambda$  instead of j?
- 100. Section 1.6, second paragraph statement that "monotone maps have adjoints if and only if they preserve meets or joins" should be qualified, e.g. "monotone maps <u>from certain preorders</u>" (John Nolan)
- 101. Definition 1.8, to be strict, shouldn't you require |P| <= |A| (Joan Vazquez) ?[Not sure what you mean. Are you worried that there might be extra elements in P? I don't see how this can happen, given the two conditions in (1.4). Feel free to try to construct a counterexample, or otherwise tell us if we're missing something. --David]
- 102. Example 1.62: the expressions (p join p) and (p meet p), strictly speaking, do not make sense because meet and join have been defined for a subset A of P, and A = {p, p} is not a set (Joan Vazquez).
- 103. Paragraph after Definition 1.13: "For any  $t \in T$ , the preimage of t along F is the subset  $\{s \in S \mid f(s) = t\}$ " should end with " $\{s \in S \mid F(s) t\}$ ". nowhere is lowercase f mentioned (Daniel Wang)
- 104. In example 1.94 you use equality j(j(p))=j(p) to say j(p) is a fixed point but the definition of a closure in 1.92 only guarantees isomorphism  $j(j(p)) \sim p$  for preorders (Juan Manuel Gimeno).
- 105. In chapter 4, add exercise elaborating on why compact closure implies monoidal closure.
- 106. In example 1.36, where partitions are first introduced, the set of partitions of A is called  $\mathcal{E}(A)$ , but thereafter it seems to be called  $\mathcal{E}(A)$ . (It's a bit of a pity that the power set is called P(A) with parentheses; it would be nicer if these notations worked in parallel.)
- 107. Proposition 1.11 seems to be false as stated. It asserts a 1-1 correspondence between partitions of a set and equivalence relations on that set. However, your definition of partition (Def. 1.8) includes "part labels"; two partitions with different part labels count as different according to this definition, and you don't make any remark to fix that before stating Proposition 1.11, so in fact infinitely many different partitions correspond to the same equivalence relation.
- 108. The referencing for Equation 1.6 (pp. 23) is inconsistent, it is sometimes referred to as Proposition 1.81 (pp. 23, 25, 26) (Fred Eisele)
- 109. In Definition 1.8 it says "\text{if }p\neq q\text{ then }A\_P\cap A\_Q=\emptyset", with the subscripts P and Q mistakenly written in upper case.
- 110. Can you add the date (or a version number) of the version when generating the PDF?
- 111. "a cost. **o**bjects such that the measure of relationship" pp. 33, first paragraph, comma? period? Capitalization? (Fred Eisele)
- 112. Definition 1.60 will make people believe that in preorders there is *a single* meet/join for a subset, which is not true in general. For example, example 1.62 uses equality = for the result of

- a join/meet operation. In general, (p join p) will only be isomorphic to p in preorders. (Sebastian Galkin)
- 113. In proposition 1.81 it should be stated as part of the hypothesis (at least for part b)) that the functions f and g are monotone. This fact is used in the demonstration. (Sebastian Galkin)
- 114. P. 20 Example 1.71. The codomain of 3x- needs to be the same as the domain of the left adjoint which is presumably the reals. (David Ellerman)
- 115. Check carefully for any uniqueness of joins/meets issues from proposition 1.86 onwards.
- 116. Definition 5.51 symmetry axiom should have sigma M; also, perhaps this is confusing
- 117. Consider using background colours to indicate different sorts of material, especially what can be skipped on a first/high level reading. (see <a href="http://web.evanchen.cc/notes/MIT-18-218.pdf">http://web.evanchen.cc/notes/MIT-18-218.pdf</a>, use mdframed)
- 118. Equation 5.7: isn't this transposed?
- 119. Exercise 5.55: doesn't make sense. Commutative monoid objects don't define commutative monoids without extra structure
- 120. After Theorem 5.47 explain how a proof that two signal flow graphs are equivalent works (shove black nodes left )
- 121. Example 5.35 typo on differential equation (shows second derivative of y)
- 122. Example 2.47: for monotone maps it reads "F: X  $\rightarrow$  Y such that for every  $x_1, x_2 \in X$ , if  $x_1 \le x_2$  then  $F(x_1) \le F(x_2)$ ." Since these two  $\le$  are for different preorders, in order to maintain the equivalence with V-functors they should be  $\le_X$  and  $\le_Y$  respectively. (Daniel Wang)
- 123. Remark 3.49: "there exists a category **Pos** in which the objects are preorders" should probably be changed to **Preord**, since now we're dealing with preorders, not posets (Daniel Wang)
- 124. It would be insightful if the proof of the third statement in Prop 2.62 referenced Prop 1.81 (Daniel Wang)
- 125. Minor inconsistency: In Def 2.55 the definition uses the symbols a,v,w. But Example 2.60 references a version of the definition with a,p,q instead (Daniel Wang) [I think readers have to be able to change variables; we can't keep the same variables for all applications of a def or thm. -- David]
- 126. In Example 2.65 join is defined by infimum/greatest lower bound, even though join is typically supremum/least upper bound. In this example I'm thinking it's because we're working with **Cost** where order is reversed (≥). I'm not sure though, so perhaps add a comment about this: "Note that although join is typically the *supremum*, or *least upper bound*, the order operation in **Cost** is ≥, not ≤. We define join as the greatest lower bound." (Daniel Wang)
- 127. Example 2.65: "the second condition says that for any **b** ∈ **B**" should be changed to "...**b** ∈ [0, ∞]" (Daniel Wang)
- 128. Exercise 1.15: "Answer the following questions for each of the relations below" is confusing because "the following questions" refers to questions 1 and 2 but not questions 3 and 4. (John Garvin)
- 129. Exercise 1.15, question 4: Why B->A instead of A->B? (John Garvin)
- 130. Exercise 1.1: I'm familiar with "foo" as a metasyntactic variable from computer science, but it might be confusing for others. (John Garvin)
- 131. Example 1.46 typo: "is monotone map" should be "is a monotone map" (John Garvin)
- 132. Example 1.64 (Multiple meets or joins may exist) shows a Hasse diagram where both c and d are less than both a and b; the text reads "c and d are both joins of [{ a, b }]"—should that be "meets" instead of "joins"?
- 133. In Exercise 1.66, if  $P = \{a, b\}$  with  $a \le b$  and  $b \le a$ , then  $^{a}$  can be either a or b, since meets are not necessarily unique in a preorder. This works for posets though. (Jonathan Castello)
- 134. In Example 1.67, p v p need not be unique in an arbitrary preorder, for the same reason as

- above, so it is confusing to use = here. (Jonathan Castello)
- 135. typo Remark 4.11: In the matrix for  $M_X$ , the entry in row A / column C should be 3, not infinity (since there is an arrow from A to C with cost 3). (Daniel Wang)
- 136. typo Example 3.10: "Here are two **more another** examples:" (Daniel Wang)
- 137. typo Second paragraph of "3.2.5 Isomorphisms in a category": "Similarly, if one has a preorder with elements a,b, such that a ≤ b and b ≤ a, i.e. a b, **then are** essentially the same." (Daniel Wang)
- 138. typo Example 3.47: in "a non-decreasing sequence  $(F_0, F_1, F_2)$  of natural numbers", missing ellipsis in  $(F_0, F_1, F_2, ...)$  (Daniel Wang)
- 139. A typo in meet example (start of the subsubsection 1.4.1. If the natural numbers start with 1 then their lower bound (meet) is 1.
- 140. Eq 4.7: should be  $(\Phi.\Psi)(p, \mathbf{r})$ , not  $(\Phi.\Psi)(p, q)$  (Daniel Wang)
- 141. Are you guys planning to release the an e-book on Amazon (or other ebook platforms)? The current PDF does not render well on Kindles. [We'll talk to our publisher about it! Thanks for alerting us to the issue. -- Brendan]
- 142. Suggestion: when you release a new version of the book, add an item here with the book's date so we don't have to scan all the way back to avoid entering duplicate errata. (Pete Morcos) [Good idea! We'll create a new section for every new version; see below. Thanks for being diligent about scanning back, and for giving us feedback about how this process works out for you. We really appreciate all your feedback! -- Brendan]
- 143. The National Airspace System example at the end kind of comes out of nowhere (Adam Theriault-Shay)7.5.3.
- 144. Page 217 should be [a-epsilon, b+epsilon] (Andrew Turner)
- 145. 7.3.2 after definition 7.15 "a *continuous function…*" at the end of the sentence should be Op\_X rather than Op(X) (Adam Theriault-Shay)
- 146. Do the later logic stuff on the two examples of the sheaves given in 7.3.3 (like TM) (Nelson Niu)
- 147. Sections 7.4.6 and 7.5.3 seem to be too disjoint. I would like to see in 7.4.6 an example of the "behind the scenes machinery" going on in a logical statement within a topos, similar to (7.17), perhaps for the car example. (Andrew Turner)
- 148. The database schema is mentioned in the intro to 7.3, but we would like to see a more built out example of how databases fit into toposes and sheaves, etc. Add example of how toposes are used in databases, trivial example of person--mayor database (Andrew Turner)
- 149. @Exercise 1.9 items 1 and 2: "p' \in P" should be "p' \in P' " on both lines. (Pete Morcos)
- 150. @Definition 1.23 line -2: "relation X" should be "relation on X". (Pete Morcos)
- 151. @Remark 1.25 line 7: "v is always" should be "v, there is always". (Pete Morcos)
- 152. @Example 1.48 line -3: "is monotone" should be "is a monotone". (Pete Morcos) John Garvin may have reported this above, but he references Example 1.46, which seems correct.
- 153. @Example 1.64 line 4: "are both joins" should be "are joins". (Pete Morcos) In addition to this typo, an unsigned note above reports that all the "joins" should be "meets".
- 154. @Example 1.75 line -1: Add parens around "3 x -".
- 155. @Example 1.75 line -2: Comment: the expression would be easier to follow if the clauses were swapped, to match (1.5). (Pete Morcos)
- 156. Section 1.5.2 Back to partitions line 3: "Suppose given" should be "Suppose we are given". (Pete Morcos)
- 157. @Example 1.94, defn of  $f^*$ : "f a = y" should be "f(a) = y". (Pete Morcos)
- 158. @"We define an adjunction" middle of section 1.5.4: "fix\_f" should be "fix\_j". (Pete Morcos)
- 159. @middle of section 1.6: "Nonetheless, but" doesn't make sense; perhaps a remnant of a half-edited sentence? Perhaps replace with "These"? (Pete Morcos)

- 160. Definition 2.28, part (ii): I think it should be \$\mathcal{X}(x,y) \in \V\$ rather than \$\mathcal{X}(x,y) \in \mathcal{V}\$. (Dan Oneata) Edit: <u>Jonathan Castello explained to me on the forum</u> that this notation is often used by mathematicians when it's clear from the context what is the underlying set; so you can disregard my correction:-)
- 161. I would suggest you to add the words between # on the summary 1.6: "we thought of preorders as the states of a system"... "and monotone maps as describing a way to use one #STATE OF A# system to observe another " #STATE OF A SYSTEM# ..if this modification really holds indeed (Pierre Prado)

#### Suggestions/Errata for Version 18 May 2018

- 162. 3.2.1 Free categories, first paragraph: "and it goes v → w" should probably be "and it goes v → x" (Sergey Tselovalnikov)
- 163. @Example 2.24 "Recall Bool = ( B , ≤, true, ∧) from Example 2.10 and Cost from Example 2.19." The description for Bool is given but not Cost, i.e. "Cost = ([0, ∞], ≥, 0, +)". (Fred Eisele) [I'm not sure what you mean here; it looks to me like we do describe Cost in Example 2.19 -- Brendan] It just seemed inconsistent to repeat the Bool but not the Cost. [Oh, I see. Good point. I've fixed that now!]
- 164. @Definition 2.23 The \$\le\_Q \$ carries the qualifier 'Q' in 2.23.b but none of the other \$\le \conj = \$ are similarly qualified. It is not confusing just inconsistent so it may be intentional. (Fred Eisele)
- 165. @Exercise 2.41. "Consider the **preorder** W := ( N  $\cup$  { $\infty$ },  $\leq$ ,  $\infty$ , min)." Should this be "**monoidal preorder**"? A simple preorder would be "W := ( N  $\cup$  { $\infty$ },  $\leq$ )". (Fred Eisele)
- 166. 3.4.2 Adjunctions: "**Thus** we can **thus** rephrase" (thus is repeated) (Jason Hooper)
- 167. Prop 3.68 in the proof: b : Z -> Z' should be b : Z' -> Z (swap domain/codomain) (Jason Hooper)
- 168. @3.2 Categories footnote [first] labeled '0': the footnote does not appear to refer to anything "The method works even in the infinite case: one takes the infimum of all powers MnY . The result always defines a Lawvere metric space." (Fred Eisele)
- 169. @Definition 3.2: (b) "the following are **the** equal: " should be **then** . (Fred Eisele)
- 170. @Example 2.49: references to "Lipschitz function" should be "Lipschitz function with Lipschitz constant equal to one or less" see definition of "Lipschitz function" at <a href="https://www.encyclopediaofmath.org/index.php/Lipschitz\_function">https://www.encyclopediaofmath.org/index.php/Lipschitz\_function</a> (Valter Sorana)

#### Suggestions/Errata for Version 25 May 2018

- 171. Do you explain the color scheme anywhere? Definitions = red, Examples = green, Exercises = used to have an open diamond but now nothing? I presume this is a work in progress? (Fred Eisele) [The color scheme is as you say. It's just to help highlight where various things are happening: definitions, theorems, etc. We don't plan to explain it anywhere. However, we've added back in the open diamonds to exercises. Do you have comments on it? Does it help? Thanks --David] Yes, I like the open diamonds.
- 172. @Example 3.33: "...since f.h = g.i in F..." f, g, h, and i are in C not F. (Fred Eisele)
- 173. 3.4.3 "The functor E\_f does **the** what one would most expect", superfluous "the" (Jason Hooper)
- 174. Defn 3.76 a), the property should be "we have  $c_i = c_i \cdot D(f)$ " (swap the i/j) (Jason Hooper)
- 175. Ex 3.83, the label t x in the left diagram should be c x (Jason Hooper)
- 176. @"Wiring diagrams for symmetric monoidal preorders" last sentence "we will several" should be "we will encounter several", perhaps. (Pete Morcos)
- 177. @2.27#2 comment on phrasing. Usually when textbooks say "if not ... if so" they're implying

- that those are the only two possibilities. I'm still working through the chapter and haven't done the problem, but I believe a third possibility exists that it's a monoidal preorder but no monoidal monotone exists. If that's correct, perhaps rephrase "if so, find" to "if so, is there"? (Pete Morcos)
- 178. @"Presenting metric spaces with" line 3 "considere" is misspelled. (Pete Morcos)
- 179. @2 lines above Ex. 2.38 "by repeated a" should be either "by repeating" or "by repetition of" (Pete Morcos)
- 180. @Ex. 2.40#2 "a M-category" should be "an M-category" (Pete Morcos)
- 181. @Ex.2.41#2 "a interpretation" should be "an interpretation" (Pete Morcos)
- 182. @Constr. 2.42#a C W should be C f (Pete Morcos)
- 183. @Ex. 2.43 line 5 the term "monoidal functor" is used, but up to this point you've been saying "monoidal monotone", as in Constr. 2.42. (Pete Morcos)
- 184. @2.5 line 4 "condition as" should be "condition is" (Pete Morcos)
- 185. @Prop 2.70 eqn in the proof the x should be a w (Pete Morcos)
- 186. Chapter 2 footnotes 5&6 are bumped forward a page even though there's room. Footnote 7 is totally borked clicking the link goes to chapter 3, and the actual footnote 7 seems to be missing completely. (Pete Morcos) [Fixed 6 and 7. Footnote 5 is an issue with the coloured boxes: the footnote either appears at the end of the box or at the bottom of the page where the box ends. I'm not sure how to fix it, but perhaps our publisher will know. -- Brendan]
- 187. @Section 2.6 "the Kelly" should be "Kelly" (Pete Morcos)
- 188. Suggestion I'm glad that you use a single numbering for all examples, definitions, etc, to make it easier to scan for them. It turns out that you often go several pages without a numbered equation. Consider including equations in your unified numbering scheme. When this book gets printed, having pages with no numbered equations at all breaks all the binary search algorithms people use to find things in books. (Pete Morcos)
- 189. In the proof of Proposition 2.62 and Remark 2.57 it states that Eq. (2.22) says we have a Galois connection in the sense of Definition 1.74, 'however this isn't quite the case because the definition is between a pair of monotone maps and it doesn't appear to be mentioned that the map taking an element to its hom-element is monotone. I think this requires proof or at least mention. (David Ledvinka)
- 190. @Remark 2.2: "The preferred notation a given setting" -> "The preferred notation <in> a given setting" (MTM)
- 191. @Section 3.4.3 paragraph 5 "The functor  $\Sigma$  F does **the** what one would most expect..." extraneous "the". (Fred Eisele)
- 192. @Section 3.4.3 paragraph 6 "one **imagine** that there should be no such" should be "imagines". (Fred Eisele)
- 193. @Example 3.71: it is not clear whether  $(X, Y) = (\underline{4}, \underline{6})$  or  $(X, Y) = (\underline{6}, \underline{4})$ . I believe it is the former? (Fred Eisele)
- 194. @Definition 3.76 (i): "an object  $C \in C$ ;" given that C is a category shouldn't that be "an object  $C \in Ob(C)$ ;" [Similarly for (ii)] [Was there a note saying that based on context C might mean Ob(C)?] (Fred Eisele)
- 195. @Definition 3.76 (a): "for each  $f: i \rightarrow j$  in J, we have,  $c_i = c_j$ .D(f)" shouldn't that be "...,  $c_j = c_i$ .D(f)"? (Fred Eisele)
- 196. @Exercise 3.84: "how should F^op **do** on objects" sounds funny, maybe "how should F^op **act** on objects" (Fred Eisele)
- 197. @Definition 3.76: Suggestion: The concept of cones and their relationship to diagrams is fairly sophisticated and foundational. I would expect there to be more diagrams. The two diagrams provided are fine but it is not clear that they are both of category C; they need a label. Category

- J has no diagram and D is inferred. Here is a sketch to suggest what I mean. (Fred Eisele)
- 198. @Section 3.2.2 Your main example of presenting categories with path equations is "degenerate" in the following sense: the equation you impose does not imply any other interesting equations. Exercise 3.9 is also degenerate in this sense. Only when we hit the next two examples do we get equations that imply other equations. You might want to make it easier for people to understand that when we declare f = q, we are forcing efh = egh for a bunch of e,h. People who already know presentations of groups will, of course, get this. (John Baez)

#### Suggestions/Errata for Version 1 June 2018

- 199. Sec 4.4.2 "But since we **have replace** the fact" (Jason Hooper)
- 200. Sec 4.4.2 "Suppose given additional morphisms g: A -> B" this should be g: B -> C I think (Jason Hooper)
- 201. Ex 2.44 "the above preorder **shown left**" there is only one preorder shown (Jason Hooper)
- 202. Ex 5.13, in the internal flow graph, the subscript on the top arrow **c1** should be **c,1** (Jason Hooper)
- 203. Ex 5.39, in the signal flow graph, the second D label should be D^2 (Jason Hooper)
- 204. Prop 5.48, in the proof, "the amplification that i contributes to j is" should be to k (Jason Hooper)
- 205. In Def 5.40, the scalar (a) has white space around it.
- 206. Ex 5.60 "A monoid is a monoid object in (Set, X, 1)" if this is the category it should be (Set, 1, X) to match Defn 5.59 (C, I, (X)) (Jason Hooper)
- 207. Defn 6.9: "together with a pair morphisms" (missing "of") (Jason Hooper)
- 208. Ex 6.24 "to **can** prove that the pushout" (extra can) (Jason Hooper)
- 209. Ex 6.37 "The monoidal product is given by **just by** the..." (Jason Hooper)
- 210. @Ex 3.94: "The pullback selects pairs (i, j)  $\subseteq 4 \times 6$ " would imply the pairs in the drawing are reversed. E.g. the red dot in the upper left labeled "(1,4)" should be "(4,1)". Given that this is similar to the drawing in Ex 3.82 the definitions of X and Y should probably match. (Fred Eisele)
- 211. Sec 2.2.4, p. 46. "because there is another monoidal structure on (B, <=, true)". If I understand correctly, the true should be deleted, because the other monoidal structure on Bool has false as its unit? (Rif A. Saurous)
- 212. Sec 2.2.5, p. 49. "Monoidal monotones are examples of monoidal monotones." (I should certainly hope so.) (Rif A. Saurous)
- 213. Sec 2.2.5, p 49. "Throughout the book Definition 6.58" is missing some connecting word. (Rif A. Saurous)
- 214. Example 2.44, "Recalling the monoidal unit I of Bool is false", I believe you mean true here (assuming we're referring to "Booleans with AND" described in Example 2.24, which seems very likely but could be made clearer with someting like "Recalling from Example 2.24 that". (Rif A. Saurous)
- 215. Sec 4.2, p. 109. "Along the way me will develop some abstract machinery..." should be we.
- 216. After @Exercise 4.4: "as mentioned in Exercise 2.81, Bool is a quantale" it is mentioned in "Exercise 2.90. Show that Bool (B, ≤, true, Λ) is a quantale." (Fred Eisele)
- 217. @Example 1.44: Suggestion, the notion of upper sets has been a bit confusing for me. I think an addition to the drawing may be helpful. There are two ways to arrive at the upper set. One way is to think of the powerset of P and eliminate those elements that do not meet the condition. Another way, that appeals to my intuition is to notice that each member of P can be seen as generating an upper set. Here is a drawing https://docs.google.com/drawings/d/e/2PACX-1vRB05MxB\_BDV-G7KaZCRuLduGbn6SQ9cWY

- mxRdSiLLTiefAcqfd7 1zBYaUMoaRrpqtks01L8l2SW6B/pub?w=304&h=195 (Fred Eisele)
- 218. @Section 4.1: "But these **effect** neighboring teams" I frequently get this wrong but I think you want "affect", the verb, not "effect" the noun. (Fred Eisele)
- 219. @Section 4.2.3: in the last diagram, did you intend to imply by no mapping to \$ (g/n, funny) that such movies are impossible, or that is has no cost? Is this a valid profunctor? Is the infinite cost implied? Later in 4.18 the 'e' node has no dependency? (Fred Eisele) [Good catch! That was a little inside joke: it's a perfectly valid profunctor, just one that says good natured, funny movies are not on the cost scale (perhaps you can interpret this as impossible). We've added an exercise to make others think about this too. With regards to 4.18, the node 'e' doesn't need a dependency: any directed acyclic graph generates a preorder. Remember, for example the discrete preorder. --Brendan]
- 220. In Exercise 1.9, primed labels should belong to primed label sets. [My 1.9 is a definition, not an exercise, and I can't find a nearby exercise with primes, other than 1.13 which seems to have primes in the right places. Please check the latest version here <a href="http://math.mit.edu/~dspivak/teaching/sp18/7Sketches.pdf">http://math.mit.edu/~dspivak/teaching/sp18/7Sketches.pdf</a> and check the numbering. --David]
- 221. @sentence after Def. 2.38: "examples of monoidal monotones" should say functors, I guess? (already noted above #210). Also, the end of that sentence is ungrammatical, perhaps it should say "throughout the book (see Definition 6.58)"? (Pete Morcos)

#### Suggestions/Errata for Version 8 June 2018

- 222. @proof of Prop. 2.93: last symbol should be "a" not "v" (Pete Morcos) [I don't think I agree. --David]
- 223. Suggestion @Def. 2.38: When I see an unadorned term like "monoidal monotone", I assume the definition is the most basic one. It threw me for a loop to see a bias toward smaller in that definition. Later in John's class he explained lax & oplax monoidal monotones. It would have helped me to see a footnote mentioning those terms and that the book would always use lax alone, much as you did for closure/interior operators in Chapter 1. (Pete Morcos)
- 224. In definition 3.67 (adjunction), footnote 6, in the naturality square, lower right corner, G shouldn't be there, I mean, D(Fc', Gd') wants to be to D(Fc', d'). (Jesús López).
- 225. Page 5: "In other words, A ≤ B if A implies B." doesn't make sense to me. False implies true but true doesn't imply false? You seem to have jumped from bare truth values to now having an order on propositions; this appears to have its own generative effect of confusing me. (Martin MacKerel)
- 226. Page 7: You probably want "We can now define partitions more formally." to be on its own line. Similarly elsewhere in the book for these terms pulled out in bold, on eg pages 9, 37, and 38. (Martin MacKerel) [Thanks for the input, but we're fine with these being 'paragraph titles' -- B]
- 227. Page 7 has a superscript for footnote 3, but the footnote itself is on page 8. (Martin MacKerel) [The convention is that footnotes within coloured boxes appear at the end of the page on which the box ends. It's unfortunate, but I haven't be able to change it. -- B]
- 228. The empty set symbol ∅ looks like a zero in the font you are using, which I find distracting. (Martin MacKerel)
- 229. Page 9, Definition 1.19. You use the term "bijection" a few times later on in the book, so now might be the best time to introduce it. (Martin MacKerel)
- 230. Page 10: A little more explanation or clarity around introducing the function i when discussing isomorphism might be helpful to the kinds of readers for whom you need to define "injective". (You also mention the composite function before you define it.) Or, perhaps, toss that paragraph altogether. (Martin MacKerel)

- 231. Page 10, Example 1.23. Unless the inconsistency is deliberate, it'd be better to either collapse to "f(22) = f(23) = d" or expand to "f(11) = a, f(12) = a" for clarity and consistency. (MMK)
- 232. Page 11: "Suppose given a function F:  $X \to Y$  and an element of X, thought of as a function x:  $\{1\} \to X$ ." doesn't scan well. Perhaps "Suppose we are given ..."? (MMK)
- 233. Remark 1.29: "Indeed, Hasse diagram" → "Indeed, a Hasse diagram" (MMK)
- 234. Remark 2.68: At the end of it, another remark is referenced, but an incorrect one. I assume it should be Remark 3.58 instead of Remark 2.68 referencing itself. (Bruno Gavranović)
- 235. Example 1.51: "mammal" is listed twice in the definition of the set. (MMK)
- 236. Example 1.52: You use P(X) here, by which I assume you mean the power set, but that was introduced as PX in Example 1.40. Similarly, the upper set is introduced as UP in Example 1.44, but referred to as U(P) in Example 1.53. (MMK)
- 237. Example 1.52: For clarity, it would be helpful to refer explicitly to the "power set" or reference Example 1.40. Also mention that you are using the same order relation introduced in Example 1.40. (Also in current Example 1.45.) If the power set is **always** given the order of subset inclusion, then that should be stated in Example 1.40. (MMK)
- 238. I suggest you introduce the product preorder (Example 1.46) before the upper set preorder (Example 1.45), then add an exercise to find the upper set preorder on the result of the product preorder exercise (here, Exercise 1.47). This helped me understand the upper set preorder structure and its relationship to the structure of the original preorder. Addendum: I see you did a portion of this in Exercise 1.54, but I do think it'd be nice for the student to more fully understand upper set preorders before moving onto monotone maps. (MMK)
- 239. In Exercise 1.54, the link to Exercise 1.47 goes to page 14, but Exercise 1.47 is on page 15. (MMK) [Couldn't reproduce the error; could have been a .pdf issue that fixed itself.. -- David]
- 240. Exercise 1.55: what do you mean, "are just functions"? (MMK)
- 241. Remark 1.30. "3." should be "(c)" to match the labeling of the conditions in Definition 1.27. (MMK)
- 242. Just after Definition 1.62: "a relabeling the elements." → "a relabeling of the elements." (MMK)
- 243. Either when you introduce ≤ in Exercise 1.1 or when you next reference it in section 1.1.2, can you tell us how to say it in English? If p ≤ q, do we say "p is less than or equal to q"? Seems kind of wordy; is there something pithier? Addendum: I see in Exercise 1.5 you use the term "greater than" (without the "or equal to") and starting in Example 1.69 you say "less than". It would be nice to state this explicitly, esp because it conflicts with the common use of ≤. (MMK) [You can use "A is at most B". I tried to add something about pronouncing ≤, but I found it broke up the flow too much to be worthwhile in the end. --David]
- 244. Exercise 1.71, Example 1.72: "poset" → "partial order" (MMK)
- 245. Example 1.35: perhaps the definition of "total order" should be pulled out as a Definition? (MMK)
- 246. Section 1.4.2: "that the we see something" → "that we see something"
- 247. Section 1.4.2: G / capitalization inconsistency on Φ. The first sentence uses \Phi while the Definitions use \phi; this matches some inconsistencies I've seen between boxed elements and main text, except that later in the main text you use \phi. It seems like for the most part in the book you use \Phi so perhaps it's best to standardize on that. (In Chapter 6, you actually use \varphi, but from what I can tell that's specific to a thing I don't know about yet called morphisms.) (MMK)
- 248. Example 1.81: "greater than" → "greater than or equal to", "smaller than" → "smaller than or equal to". Perhaps this is an example of your unusual use of "less than" for ≤ contaminating your use for ordinary numbers? (MMK)
- 249. The page numbers in the text do not match the page numbers in the PDF. It is possible to have

- a page number of "i", "ii", "v", etc. for early pages, and even to have a page number of "Cover" for the first page. (MMK) [This may be dependent on your pdf viewer. It works in most versions of Acrobat, doesn't seem to work in Preview. --Brendan]
- 250. Page 26, after Exercise 1.97: superfluous green bit at the bottom (bleeding over from the next page). (MMK) [We'll just have to watch out for this as we continue to edit; it's difficult to diagnose in the .tex. --David]
- 251. Example 1.107: "the inclusion of sub-preorders". This is confusing. It'd be less confusing if you mentioned that term earlier, say, "This is a subset of P, and inherits an order as a result, which we will call a sub-preorder.", and then also use the singular: "the inclusion of this sub-preorder". (MMK)
- 252. "The lessons we have learned in this category" → "The lessons we have learned in this chapter" lol (MMK)
- 253. "showing different models are closely related" → "showing that different models are closely related" (or maybe *how*?) (MMK)
- 254. Section 2.2: "In Section 1.2 we introduced preorders." → "In Section 1.2.2 we introduced preorders." (MMK)
- 255. Example 2.6. "if h' beats h"  $\rightarrow$  "if h' beats or ties h" (otherwise we don't have h  $\leq$  h for any h) (MMK)

#### Suggestions/Errata for Version 15 June 2018

- 256. Ex 1.82, the floor/ceiling functions should be below or equal to, and greater than or equal to, not strictly below/greater (Jason Hooper)
- 257. Ex 1.90, 'and **turns** "pulls it back"', turns should be removed (Jason Hooper)
- 258. 1.6 Summary, "the idea of a pair of inverse map" should be maps (Jason Hooper)
- 259. Ex 2.6, "just throw them away" should be throw just one of them away, not both (Jason Hooper)
- 260. Sec 2.2.2 "But how exactly are symmetric monoidal preorders and wiring diagrams **are** connected", extra "are" (Jason Hooper)
- 261. Ex 2.11, the 5 on the middle output wire has a white background (Jason Hooper)
- 262. Sec 3.1 The example FQL code, "Department.Secr.worksIn" WorksIn should be capitalized (Jason Hooper)
- 263. Sec 3.2 footnote on page 73, "throughout **ones** interaction" should be "one's" (Jason Hooper)
- 264. Page 209., typo: "Two use a function f..." (Bruno Gavranović)
- 265. Sec 3.5.4 "Just like upper bounds have as a dual concept", extra "as" (Jason Hooper)
- 266. You still have the inconsistency between UP in Example 1.45 and Exercise 1.55 and U(P) in Example 1.54. (MMK)
- 267. Re: #238 above. Now in Exercise 1.55, the link to Exercise 1.48 goes to page 14, but Exercise 1.47 is on page 15. This is the link in part 3 of the exercise (not the intro sentence). This bug shows up in both Acrobat Reader and Preview, but only if continuous scrolling is off (viewing single pages). My guess is that what is happening is that your markup is showing that Exercise 1.48 actually starts at the very bottom of page 14 with a blank line or two (similar to the problem with #249 above). (MMK) [Yeah, that happens to me too. I guess it's what you say. We're not sure how to fix it, but thats for letting us know. -- Brendan]
- 268. Exercise 1.56 (very minor): awkward break in which the right paren is on a separate line. (MMK)
- 269. Section 1.4.2: "The preservation of meets implies that the map \phi behaves well" probably want to use \Phi here. (MMK)
- 270. Ex 3.94, in the last paragraph, the domain for g should be 4, not 5 (Jason Hooper)

- 271. Example 1.103: A key point that is not obvious is that f\_\*(A) contains Y \ Image(f). As Valter Sorana says in <a href="https://forum.azimuthproject.org/discussion/comment/17240/#Comment\_17240">https://forum.azimuthproject.org/discussion/comment/17240/#Comment\_17240</a>, 'f\_\*(A) does not just give you "all the buckets that only contain balls from A", but also all the buckets that never get any ball from X'. It would be helpful to mention this. (MMK)
- 272. @Exercise 3.8: "these obeyed" should be "these are obeyed" (Pete Morcos)
- 273. @Remark 3.25: missing hyphen in second mention of "V category" (Pete Morcos)
- 274. @Exercise 3.33: It might be good to explicitly mention that these "internal arrows" are not morphisms. As I move into your "primordial soup" in this chapter, I think confusions between conceptual levels are going to get more likely. (Pete Morcos)
- 275. @section 3.3.3 para 2: I don't think you ever explain the term small category. It's also used in the bulleted list in 3.2.4. I guess a footnote is needed somewhere, presumably in the definition of category at the time when you say "collection". (Pete Morcos)
- 276. @Def 3.50: There is some awkward and confusing phrasing here. The text "from any category D: J -> C" really suggests at first glance that D is a category. (English is not associative, causing parser ambiguity here.) Even though I know what it's supposed to say, every single time I read it I stumble on that issue. A big issue is that D is used both as a script category name in the previous definition, and now as an unrelated functor here. I have 2 suggestions, then. (1) If D is not a very traditional name for a diagram, change it to something like A or M. (2) rephrase the fragment to "a functor A: J -> C from any category I to C itself", to make the English structure unambiguous. (Pete Morcos)
- 277. @Def 3.50 and footnote 6: Usually, you use parentheses when functors act, e.g. D(f). In these two places (maybe more), you omit the parentheses: D f. I like the clean look especially when the functor appears inside other parentheses, but the parens can't always be omitted, due to ambiguity. I wanted to check with you whether this was an accident, or a deliberate choice to sometimes trade consistency for readability. The brevity works well in the tight confines of the footnote, but it seems that the Definition at least should use parens for consistency with other top-level text. (Pete Morcos) [We agree. A lot of people are complaining about parenthesis inconsistencies. We've made the change.]
- 278. @proof of Prop 2.93: I mentioned this in #221 above, which you disagreed with. I see now that my erratum was phrased in an incorrect way, but I do think there is still an issue here. Forgive me if I'm still wrong. The formula says "join\_someSet v". I don't think this makes sense, because v is a single fixed parameter value, not a dummy variable. I expect to see one of two notations for this type of construction: "join\_condition dummyVar", where dummyVar sweeps over all values satisfying the condition, or "join someSet". I believe what you meant to say here was simply "join {a in V | a otimes v <= w}", i.e. remove the set from below the join, and replace v with that set. I hope I was clearer this time! (Pete Morcos) [Nice attention to detail! Sorry for the misunderstanding. We made two errors. The 'v' on the right hand side should be an 'a'. But as you point out, we also neglected to mention the dummy variable below the join symbol. Since the dummy variable 'a' is clear from context, this is actually a standard notation; but we didn't define it in Definition 1.76. We've added a comment. -- Brendan]
- 279. Section 2.2.2, after "Wiring diagrams for symmetric monoidal preorders", it'd be better to reference Eq. (2.9) rather than Eq. (2.12) because it's a short look back instead of a longer look forward. (MMK) [Nice, thanks. -- Brendan]
- 280. "But how exactly are symmetric monoidal preorders and wiring diagrams are connected?" (MMK)
- 281. Example 2.11 (incredibly minor): it is slightly aesthetically displeasing that the 5 has white area around it. (MMK)
- 282. Section 2.2.2, "Finally, the symmetry condition (d), that  $x \otimes y = y \otimes x$ ,"  $\rightarrow$  "Finally, the symmetry

- condition (d), that  $x \otimes y \setminus cong y \otimes x$ ," (MMK)
- 283. Section 2.2.3 "we can keep axiom (e) from" → "we can keep axiom (e) from page" (MMK)
- 284. Exercise 2.32, Exercise 2.91, and Example 2.92: this is yet another notation for the power set. Previously, in Example 1.41 and elsewhere, you used P(X) where the P was simply not italic. This P is in the "double struck" style a la \mathbb{P}. (MMK)
- 285. Exercise 2.32, Exercise 2.91, and Section 2.6: you use the term "powerset", but everywhere else you write "power set". (MMK)
- 286. Exercise 2.32: "as discussed in ??." → "as discussed in Example 1.41." (MMK)
- 287. Exercise 2.42: remove part 1, and reword part 2 to just refer to Exercise 2.28. (MMK)
- 288. Exercise 2.70: as pointed out by Ignacio Viglizzo in <a href="https://forum.azimuthproject.org/discussion/comment/19664/#Comment\_19664">https://forum.azimuthproject.org/discussion/comment/19664/#Comment\_19664</a> it is not clear how the result for part 1 follows given that "There is no order on \mathcal{X}, the order is in the poset [0,∞]." Perhaps "skeletality" has a more fleshed-out (haha) meaning than presented in Remark 1.30, where it seems to require an order relation? (MMK) [I posted the amended exercise in the thread linked above -- Brendan]
- 289. Example 2.82: The remark about all joins seems like a non-sequitur; I just worked out all the possibilities à la <a href="https://forum.azimuthproject.org/discussion/comment/18764/#Comment\_18764">https://forum.azimuthproject.org/discussion/comment/18764/#Comment\_18764</a> (MMK)
- 290. Example 2.92: the link to Exercise 2.59 is another example of a misfire when in single (or double) page mode. (MMK)
- 291. Proposition 2.93, comment #221 above is correct AFAICT. v is fixed. We want the join of all a matching the selection criteria under the \bigvee sign. (MMK) [Yeah, you were right. Sorry for the misunderstanding. -- Brendan]

#### Suggestions/Errata for Version 20 July 2018

- 292. Referring back to 219, regarding Ex.1.13 part 1, "...there is at most one p' in P with..." I think it should be "...there is at most one p' in P' with ..."

  https://forum.azimuthproject.org/discussion/2097/exercise-13-chapter-1#latest (Fred Eisele)
- 293. Sometimes you write cartesian closed category, and sometimes you write Cartesian closed category. Be consistent on capitalization?
- 294. Minor stylistical note: Most of the definitions use roman numerals to enumerate the constituents of the defined notion with the exception of definitions 2.48, 2.71 and 4.41, which use arabic numerals, (Dan Oneata)
- 295. Typo in Exercise 5.5, 1) Draw two morphisms (Bruno Gavranović)
- 296. Exercise 2.59 (2): How can we find the \mathcal{M}-category that corresponds to the graph from part 1? It seems that this should be formally defined (e.g., we needed Theorem 2.46 to say there is a correspondence between preorders and Bool-categories). Or, is the question asking to do it intuitively based on the proposed interpretation? (MTM)
- 297. In Chapter 1, do bool enriched yoneda, "upper set" form. Add an exercise on how to connect this to usual statement. (Paolo Perrone)
- 298. @footnote 8 of Def 3.69: I don't yet fully understand this diagram, but I believe you want F and G there to be L and R, to match the adjoint names used in the definition. (Pete Morcos)
- 299. @footnote 8 of Def 3.69: the arrow f should be of type c'->c, not c->c', because of the contravariance of Hom in the first variable. (Ivo Dell'Ambrogio)
- 300. @3.4.3 para 4: "mired in technical idea" should be "mired in technical ideas". (Pete Morcos)
- 301. @3.5.3 line 7: "limits are concern" should be "limits are concerned" (Pete Morcos)
- 302. Example 4.59: "is simpler to look at that to write" should be "[...] than to write" (Dan Oneata)

- 303. Add comment to Remark 4.45 about how strictification allows us to pretend any monoidal category is strict, and this is what we do when we draw diagrams. Also note that we will not make this formal.
- 304. Rewrite introduction of wiring diagrams in 2.2.2

#### Suggestions/Errata for Version 24 August 2018

- 305. Solution to exercise 1.42 comes before the solution for exercise 1.40 (Bruno Gavranović)
- 306. Solution to exercise 1.46 is mislabeled as solution for exercise 1.44 (Bruno Gavranović)
- 307. Solution to exercise 2.50 b), the reference for proof is missing "...by the proof of ??..." (Bruno Gavranović)
- 308. Solution to exercise 3.18, missing word: "There are four morphisms..." (Bruno Gavranović)
- 309. Def 2.46: hom-obj X(x,y) in V, 'V' is the set, the font used indicates the monoidal preorder (cal)V (Fred Eisele)
- 310. Section 4.1: "But these effect neighbouring teams" => these affect?
- 311. Section 5.5: PawelSobocinski -> add space between first and last names
- 312. Definition 4.8 (V-profunctor) denotes  $\Phi$ : X^op × Y  $\rightarrow$  V as  $\Phi$ : X -> (crossed) Y which is the reverse of the definition on <a href="https://ncatlab.org/nlab/show/profunctor">https://ncatlab.org/nlab/show/profunctor</a> (Leo Gorodinski) [This is just a matter of convention -- Brendan]
- 313. Section 4.5.1 defines a compact closed category as a generalization of a symmetric monoidal category whereas according to <a href="https://ncatlab.org/nlab/show/compact+closed+category">https://ncatlab.org/nlab/show/compact+closed+category</a> it is a specialization. (Leo Gorodinski)
- 314. In Exercise 1.64, part 3 currently states "Show that if  $p \le p$ ' if and only if  $U(p') \subseteq U(p)$ ", I think it should read "Show that  $p \le p$ ' if and only if  $p' \subseteq p$ " because U has not been defined for a given  $p \in P$  and the statement is not true for all upper sets but is true for  $p' \in P$  (Margo Crawford)
- 315. Explain "generative effects" terminology better
- 316. Emphasise structure and coherence ideas throughout text
- 317. Check uses of term "poset"
- 318. Fill in reference to Fong Sarazola paper

#### Suggestions/Errata for Version 24 August 2018

- 319. Footnote 4 on := notation for assigning is not attached to the first time that notation is used (on the previous page the null set is defined using :=) (Margo Crawford)
- 320. Under example 2.1.4, "This corresponds to the idea that we may stacking any two valid boxes in parallel is still valid:" (Jared Briskman)
- 321. Under example 2.1.4, the second wiring diagram of  $x \le y \le z$  should be composed into one diagram, not a copy of the first diagram. (Jared Briskman)
- 322. Example 2.6, at the bottom, (Disc(M),= ,\*, e) should be (Disc(M), =, e, \*), to be in the order introduced in 2.2 (Margo and Jared)

# Suggestions/Errata for Version 12 October 2018 (arXiv v3)

To get the latest PDF, see Releases · applied-category-theory / shereeven-sketches. Issues fixed in v3.0.1:

- 1. Chp. 1
  - a. On p. 260 (solutions for ch. 1) exercises 1.11 and 1.10 are out of order. (Zack Newman)

### (fix commit)

- b. In Question 1.11, why is there an ending diamond after part 3? (Chris Dutchyn) (fix commit)
- c. On p. 261, solution to exercise 1.17 has \$(12, 23)\$ where it should instead have \$(22, 23)\$ (Zack Newman) (fix commit)
- d. Section 1.4.2, bottom of page 28: "there exist s\_1, s\_2\in S with such that" unnecessary "with"? (Tomasz)
- e. Paragraph right above 1.109. Typo: 'We want show' → 'We want to show' (Lisa Bylinina)
- f. Exercise 1.109, item 2. If [eqn] **holds**, then **holds** yadda iff yadda **holds**, for all yadda yadda. given that "holds" is short for "is true" and as such can be omitted anytime (cf "if a holds then b holds" vs "if a then b"), there are too many "holds" in a single sentence. They don't add much to it, while making it more difficult to parse by forcing the reader to put mental parentheses where in fact the right strategy would be to ignore them altogether. (Alexey Filippov)
- g. Proof of Theorem 1.115: "By Proposition 1.111" -> "By Proposition 1.107"? (Ivan Smirnov). The proof in 1.115 claims "By Proposition 1.111, it suffices to show that p0 ≤ 1( f (p0)) and that f (1(q0)) ≤ q0 for all p0 ∈ P and q0 ∈ Q." I think it is actually referencing 1.107, and not 1.111. (Nicholas Fazzio). (fix commit)
- h. In exercise **1.114**, the solution on **page 268** has an error. In row 2, we have f(p) less than/equal to q labeled as **no**, when the inequality is 1 less than/equal to 2. **No** translates to untrue here, hence the error. Similarly, for checking the right adjoint we have p less than/equal to g(q) also labeled as **no** when the inequality is also 1 less than/equal to 2. The bicondiditonal is still satisfied with these both being yes's, but the means to get there are incorrect. (Macy Aiken).

### 2. Chp 2

- a. Exercise 2.45, point 3: Typo?  $(Z, \leq, *, 1)$  should be  $(Z, \leq, 1, *)$ , same in the solutions. We easily came up with '1' as an operation, but '\*' as an integer looked kind of suspicious! (Lisa Bylinina)
- Solution to Exercise 2.63 is wrong, it seems? In particular, take M(A,B). The matrix in the solution says 6, but it should be 10. It looks like the lines and rows are flipped (because M(B,A)=6) (Lisa Bylinina and Alexey Filippov). (fix commit)

#### 3. Chp 4

- Typo: On page 129, section 4.3, just below the co-design wiring diagram, the third sentence reads "On other hand the boxes", but should read "On **the** other hand" (+1 David VandeBunte)
- Example 4.43. The table Col(Phi) should have infinity instead of zero on the way back, e.g. Col(Phi)(x, A) = infinity. This might be related to the use of the emptyset symbol in the definition of Col, which evidently can be confusing in the context of a general quantale V. (Joel Sjögren)

### 4. Chp 5

- a. Page 149 of the printed version. In 5.2 the second part of the definition of f+g should be m'+g(i-m). Juan Manuel Gimeno.
- b. Exercise 5.23, section 3: Swap **Mor(C)** and **Ob(C)** for referring to a correct graph structure. (Konstantin Nisht)

#### 5. Chp 6

a. P.198: "A cocommutative cocomonoid" should read "A cocommutative comonoid". A "cocomonoid" would be a monoid. (Only the nut gets bigger and more exotic when twice-dualized) (+1 David VandeBunte)

- b. The solution to 6.57 seems to have a typo it should be that Morphism **2** (rather than 3) is not equal to any other depicted morphism (Bruno Gavranović)
- c. Ex 6.79: Page 208, "A decoration functor for circuits", the definition of the functor Circ on objects mislabels the labeling function as "E -> C" instead of "A -> C" (Bruno Gavranović)

#### 6. Chp 7

- a. On page 225, section 7.2.1, in the "Limits and colimits" paragraph, there is some weird looking text "and taking quotients.object 0". (Valentin Robert)
- b. Exercise 7.4, it says "Prove Proposition 7.3 using the definition of a limit from Section 3.4.2." but limits were defined in Section 3.5.2. (Bruno Gavranović)
- c. On page 228, section 7.2.1, in the "Subobject classifier" paragraph, the "fully glory" should be "full glory" (Valentin Robert)
- d. Exercise 7.80 incorrectly refers to the example 7.78 instead of example 7.79 (Bruno Gavranović)

#### Unresolved issues:

#### 323. Chp. 1

- a. The heading of Exercise 1.6 is separated from its body by a page break.
- b. In Definition 1.14, you might want to explain the need for "non-empty" (Chris Dutchyn)
- c. In Question 1.17, why is "let's allow ourselves ... in the same part" there? (Chris Dutchyn)
- d. Before Definition 1.22, you might want to dispose of the "partial" vs "total" function question ... and tell us that "function" := "total function". You use it to avoid vacuous cases later. (Chris Dutchyn)
- e. "Alice's observation fails to preserve the join operation." Shouldn't this be "The join operation fails to preserve Alice's observation"? Edit: I now see what it's saying, but it was very confusing to me at first. It means that phi(x v y)!= phi(x) v phi(y), but "v" hasn't been introduced for Booleans yet, so I never even considered that interpretation. And "preserving order" means something completely different: x <= y => phi(x) <= phi(y), so I didn't see the relationship. See my guestion here. (+1 David VandeBunte)
- f. "Finally, we may want to take a disjoint union of two sets, even if they have elements in common. Given two sets X and Y, their disjoint union X tY is the set of pairs of the form (x,1) or (y,2), where x ∈ X and y ∈ Y." Having very little math background, this doesn't explain what disjoint unions are. This quote from Wikipedia is short, and found it more useful: "the disjoint union (or discriminated union) of a family of sets is a modified union operation that indexes the elements according to which set they originated in." (With that said, I love the book, and thank you for putting so much time and effort into it, and for making the topic approachable, even for a lay person. Attila)
- g. Exercise 1.20, "let P be the set of ( $\sim$ )-closed and ( $\sim$ )-connected subsets  $\{A_p\}$   $p \in P$ "—we introduced P as the set of labels; it does not contain subsets  $A_p$ . (Alexey Filippov)
- In definition 1.22 you could use the contrapositive definition of a injective function (<a href="https://en.wikipedia.org/wiki/Injective\_function">https://en.wikipedia.org/wiki/Injective\_function</a>), it could be more natural to interpret as function that preserves distinctness. Alternatively you could add a conclusion at the end of formal notation. (Pawel Sawicz)
- i. Definition 1.22: "A function from S to T is a subset  $F \subseteq S \times T$  such that for all  $s \in S$  there exists a unique  $t \in T$  with  $(s, t) \in F$ " I suspect this definition should have the word "unique" removed; as it stands, it is the definition of an injective function.
- j. In remark 1.35, you use "~=" stacked, but I can't find any discussion of it before that point ... it shows up in 1.49. Please help. (Chris Dutchyn)
- k. Example 1.37, after table: instead of "... from 4 to 4 ..." there must be "... from 2 to 2 ..."

- (Henry Chern)
- I. Exercise 1.37: we say there is one path 2 to 3, but there are really infinitely many. (Alexey Preobrazhenskiy)
- m. Example 1.54: The definition of an upper set is wrong. See: https://davidvandebunte.gitlab.io/executable-notes/notes/ssc-what-is-order.html
- n. Exercise 1.57: it would be easier if you put "b" left of "c". (Chris Dutchyn, +1 David VandeBunte)
- o. Solutions for 1.57 and 1.66: the empty set is missing from the upper set preorder (Joachim Hotonnier)
- p. In the solution to exercise 1.57, {(b,1),(b,2),(c,1),(c,2)} is also an upper set (Joeri van Eekelen)
  - Seconded. I too noticed this was missing. Should the empty set be there as well? (lan Coulter)
- q. Example 1.58: italics for "opposite preorder" since it's a term? (Chris Dutchyn)
- r. on p. 20, exercise 1.65 refers to exercise 1.51 and claims we drew the Hasse diagram for \$\mathsf{P}(\mathbb{B})\$. We drew the Hasse diagram for a \*different\* 2-element set (\$\{1, 2\}\$) (Zack Newman)
- s. Exercise 1.66: part 4 refers to Example 1.56 (product preorder), whereas it seems like it should refer to Exercise 1.57 (the one with a diagram). (Ivan Smirnov)
- t. Example 1.66, last line, should refer to 1.57 not 1.56 for the picture of \uparrow? (Chris Dutchyn)
- u. Exercise 1.66 (Yoneda lemma for preorders): "Show that if p≤p' in P if and only if ↑(p')⊆↑(p)." The construction is a little awkward: "show that if ... if and only if". Maybe omit the first "if"? (Zack Newman)
- v. Question 2. Upper and the Yoneda lemma. ↑Pop:>U(P)

  Pop is a tuple (P,<=op) not a set, so is the signature wrong?
- w. Exercise 1.66: This question is extremely hard for outsiders to follow (David VandeBunte). See:
  - https://math.stackexchange.com/questions/3307889 https://davidvandebunte.gitlab.io/executable-notes/notes/ssc-what-is-order.html
- x. 1.69: replace "non-identity" with "non-isomorphism", otherwise I'll use two three-element sets with the obvious (trivial) isomorphism {a,b,c} and {1,2,3} ... I know, I'm evil... (Chris Dutchyn)
- y. Beginning of Section 1.3.2, through the paragraph after Definition 1.93: When reading this for the first time, it was a bit confusing that the function indicating a generative effect was sometimes called f, and other times called phi (Daniel Irving Bernstein)
- z. Proposition 1.78. In the last sentence of the proof, instead of 'f', there must be  $f_{_{U}}$  (Henry Chern)
- aa. Exercise 1.79. Instead of "Let P and Q be preorders" there must be "Let P and Q be sets with a preorder relation" (Henry Chern)
- bb. Discussion after exercise 1.66, "to know an element is the same as knowing its upper set—that is, knowing its web of relationships with the other elements of the preorder"—it is not quite clear that we introduced "its upper set" for ↑ p. (Alexey Filippov)
- cc. Exercise 1.79 If the reader is not familiar with the 'f'' notation from elsewhere, they (like me) might not remember from 1.26 what it is and get confused (like me). This might make the exercise look more difficult than it actually is. A quick remark about preimages would help here. (Lisa Bylinina)

- dd. In Example 1.97 (Oct 12 version, p.27), x is first used as an element in  $\mathbb{Z}$  and then as an element in  $\mathbb{R}$ ; similarly, y is first used as an element in  $\mathbb{R}$  and then as an element in  $\mathbb{N}$ . The change in notation is a bit reader-unfriendly as we are still in the same example. (Julio Song)
- ee. Page 263 the explanation for exercise 1.98 says "largest" when it means "smallest".
- ff. Exercise 1.101 solution to part 2 could use the adjoint functor theorem, rather than just brute calculation.
- gg. Exercise 1.101: Since the given function preserves meets, it should have a left adjoint by the adjoint functor theorem (Theorem 1.115). See also: <a href="https://davidvandebunte.gitlab.io/executable-notes/notes/ssc-galois-connections.html">https://davidvandebunte.gitlab.io/executable-notes/notes/ssc-galois-connections.html</a>
- hh. In Exercise 1.104, is there a reason to reuse labels 11, 12, 13, etc. for the partitioned T? I feel like it makes it a little harder to reason about what's happening, with no benefit. (Valentin Robert)
- ii. Proposition 1.107, minor note: it might make sense to have a colon after "equivalent". (Ivan Smirnov)
- jj. Proposition 1.111, minor wording: "Suppose ... any subset" -> "Suppose that ... is any subset" or "Let ... be any subset". Also, "Right adjoints preserve meets" may not be the best label for the whole proposition since it also handles the left adjoints as well. (Ivan Smirnov)
- kk. The second row (p=1, q=2) for the solutions for Exercise 1.114 appear to be wrong. It is true that both  $f(p) \le q$  and  $p \le f(q)$ . (Nicholas Fazzio)
- II. In Example 1.117, the names of the sets change from A and B to X and Y and then back. mm. Example 1.117: This is a bit confusing. It could be clarified that there are two adjoint pairs here, (1) f\_! and f^\*, and (2) f^\* and f\_\*. (+1 David VandeBunte)
- nn. Just before 1.135, did you want to change the font of "Cl" as the function from Rel(S)->Pos(S)? You use sans-serif in part 3 of exercise 1.135. (Chris Dutchyn)

#### 324. Chp. 2

- a. Definition of symmetric monoidal preorder should possibly be (strict) symmetric monoidal preorder?
- b. In section 2.2.2, "We say that a wiring diagram is valid if the monoidal product of the elements on the left is less than the monoidal product of those on the right" I think this should say that the monoidal product of the elements of *each box* have to be less than (or equal to) the monoidal product of the elements on its right. (This seemed to really confuse my colleagues, who interpreted it in a consistent but weird way in which a diagram indicating something like 2 <= 10 <= 4 is considered valid.)</li>
- c. Related to the above, in section 2.2.2 you say what it means to draw a wire above a wire, but you don't explicitly say in that section what it means to draw a box above another box, or a box above a wire I think it would be helpful to say explicitly that drawing a box above a box means both inequalities are true.
- d. 2.2.2, definition of a valid wiring diagram: should "less than" be "less than or equal to"? (Ivan Smirnov)
- e. Section 2.2.4 ... the Booleans: only two different structures: can't I build some out of NAND and NOR too? (Chris Dutchyn)
- f. Example 2.32: use "1|n for all n" so you're using the same letter/variable on the right-hand side of | (you previously said "m|n ...") (Chris Dutchyn) (+1 David VandeBunte)
- g. 2.2.5, Definition 2.41: why drop "map" as in "monoidal monotone map" ... now monotone is an adjective (as in "a monotone map") and a noun (as in "a monoidal monotone") and the reader needs to take undue care. Alternatively, go back and change all the other "a

- monotone map" uses to just "a monotone". (Chris Dutchyn)
- h. Example 2.42: The floor function is  $\mathbf{R} \Rightarrow \mathbf{Z}$ , rather than  $\mathbf{R} \Rightarrow \mathbf{N}$  (consider floor of  $-\pi$ ). Perhaps it might make sense to rephrase the whole example in terms of  $\mathbf{Z}$  and  $\mathbf{R}$ ? That seems to work better. (Alexey Filippov) (+1 David VandeBunte)
- i. Exercises 2.43 and 2.44 don't they presuppose that ∞ is a member of the set of real numbers, a real number as well? Seems like it's a silent assumption here that one might want to make explicit, saying we're dealing with an affinely extended real number system. Also, what would happen to these structures if we had the regular set of real numbers, without ∞? It feels like there's a need for a short clarifying discussion here. (Lisa Bylinina)
- j. This is with regard to a possible mistake in the solution to Exercise 2.45. I believe that the uniqueness part of this claim is false. Namely, wouldn't f(n) = 2<sup>n</sup> be another monoidal monotone between these two monoidal posets? Or perhaps I'm unclear on something in the definition. (Daniel Irving Bernstein) (+1 David VandeBunte)
- k. I had exactly the same remark about 2.45: any  $m^n$  seems to work since it's not just me I hope you give an explanation here. I would also add that many exercises are of the short form "Why? Why not?" and it doesn't seem like a good idea to end an answer with another "Why?" Unless this is an exercise in level shifting and you plan to add "Answers to Exercises in Answers to Exercises". (Tomasz)
- I. Me three ... 2.45 seems to be incorrect (ChrisDutchyn)
- m. The footnote after exercise 2.52 (about Hausdorff distance) seems to be split across pages 60 and 61, leading to a seemingly unnumbered and irrelevant footnote on page 61. (Luke Worth) (+1 David VandeBunte)
- n. Exercise 2.62: In part 2 of this exercise it's claimed that you can get an M-Category where the hom-object from *x* to *y* is computed as follows: for each path *p* from *x* to *y*, take the intersection of the sets labelling the edges in *p*. Then, take the union of these sets. However, if we imagine a graph where there is no path from a vertex A to itself, wouldn't this violate condition (a) in the definition of V-categories?
- o. Exercise 2.63: It is not immediately obvious that the diagonal should be infinity, and that if there is no path between A,B then M(A,B)=0; it seems that establishing the W-category in part 3 of the question is what gives rise to the solution to part 2 (Lisa Bylinina and Alexey Filippov).
- p. After Exercise 2.78. There is no edge (only the path) from A to C, so the matrix for X in equation (1.77) must be: (then you need to correct the matrix  $X \times Y$ ) (Henry Chern)

X	Α	В	С
Α	0	2	8
В	8	0	3
С	8	8	0

q. Exercise 2.84: Additional explanation for coming up with an idea of what the hom-element might be: if we assume Bool is monoidal closed, we can then conclude how the hom-element has to look: We have  $a\&b \le c$  iff  $a \le b$  -o c, in particular for a=true we see:  $b \le c$  iff true  $\le b$  -o c, i.e.  $b \le c$  iff b -o c. So we choose (b -o c) :=  $(b \le c)$  (which is the same as b => c). (Hero Wanders)

- r. Remark 2.89: Replace X(x,x) with V(x,x) (Hero Wanders)
- s. There is double repetition of "the" occurs several times in the text. (Konstantin Nisht)
- t. 2.96: <a href="https://math.stackexchange.com/questions/3846876/joins-and-meets-in-preorder">https://math.stackexchange.com/questions/3846876/joins-and-meets-in-preorder</a> (David VandeBunte)
- u. Remark 2.97. The correspondence with the Hausdorff distance here is: sup -> meet, inf -> join, but in Example 2.91 the join is the supremum in the actual order of Cost, as explained in the footnote. Is the confusion (again) due to reverse orders? That is, the join is the standard infimum on [0,∞], and that's what we want to recover? (Tomasz)
- v. Proof of Proposition 2.98 uses the adjoint functor theorem and states that (v -) preserves joins. However, (v -) is a right adjoint. Shouldn't we use the left adjoint,  $(- \otimes v)$ , instead? (Zhengqun Koo)
- w. In the proof of proposition 2.98, "We need to show that <u>if</u> Eq. (2.88) holds then − ⊗ v : V → V has a right adjoint.... (Louise Nielsen)
- x. 2.104: https://math.stackexchange.com/g/4213631/245548
- y. Ch. 2.6, pg 76 a typo, "Enrichment is a fundamental notion in category theory, and **we will we** return to it in Chapter 4,.." (Alexey Filippov)
- z. Exp 2.27 Boolean with And and true as the monoidal unit, the truth table seems to be wrong as false AND true is false but it has been written true

#### 325. Chp. 3

- a. There seems to be inconsistency of notation throughout the whole book when referring to objects of a category. Sometimes it's referred as "c \in Ob(C)" (definition 3.6), sometimes as "c \in C" (page 247., when specifying that a predicate is a sheaf morphism) edit: Although now I see this is explicitly mentioned on page 93 when defining a category. (Bruno Gavranović)
- b. Example 3.5: For m+1  $\leq$  i  $\leq$  m+n instead of setting (f+g)(i) = m' + g(i) you should probably set (f+g)(i) = m' + g(i-m) or alternatively (to avoid using subtraction) define it by m' + g(j) with  $1 \leq j \leq n$  being the unique natural number satisfying j + m = i; Reason: g is not defined for n  $\leq$  i (Hero Wanders)
- c. Example 3.42: It must send morphisms in  $\mathcal{N}$  to morphisms in  $\mathbb{N}$ . The second N should be the same as the first, because the first N represents the pre-order ( $\mathbb{N}$ ,  $\leq$ ), but the  $\mathbb{N}$  only represents a set, i.e. it has no morphisms (Harmon Nine). (+1 David VandeBunte)
- d. On page 281, the solution to part 2 of exercise 3.48 doesn't sound right: D(f) is not an inclusion because it sends a set of people to a set of the gifts they give themselves.
- e. Remark 3.59: This claims that F;G is a natural isomorphism. If this is the case, where is the natural transformation (with an inverse) on which this natural isomorphism is based? Is it between F and G, is this the composition of F and G (F;G), and if it is the latter, what are the two functors that it is between? (Harmon Nine) (+1 David VandeBunte)
- f. The paragraph after Exercise 3.76 seemed to be confusing. an Instance on 1 is the same thing as a set (not Category Set). So let's let's identify 1-Inst with Set(the category). Are we confusing set with Set here? (Hao Deng)
- g. Exercise 3.78 solution: change name Emory to Emmy

#### 326. Chp. 4

- a. Immediately preceding Def 4.2: Recall that if X = (X, <=) is a preorder, then its opposite Xop = (X, >=) has x >= y iff y <= x. It is not immediately clear what is intended. I see now that the definition of op implies that if x <= y in X then y <= x in Xop, but the notation for the opposite preorder makes use of >= rather than <= which makes sense iff x >= y is equivalent to y <= x.
- b. Solution to Exercise 4.9: The author almost meant to refer to Definition 2.69 rather than

- Definition 2.41 (which is specific to monotone maps) (David VandeBunte).
- c. Definition 4.21: It isn't immediately obvious (to me) that this construction indeed produces a profunctor. Maybe it's worth including a proof, or an exercise asking for a proof? (Daniel Irving Bernstein)
- d. Typo: The definition of 4.21 stipulates X,Y,Z as categories, and then uses the same equation as 4.20 (which used P,Q,R); specifically, it says to take the join over all q \in Q, which should read over all q \in Y (or better yet, match the letters to the categories or vice versa). (+1 David VandeBunte)
- e. Lemma 4.27: The arrow between P and Q indicating the profunctor phi is missing its vertical line (Daniel Irving Bernstein)
- f. Typo: The arrows giving the profunctors in lemma 4.31 are missing the vertical bar.
- g. Rough Definition 4.45: it would be nice to see "well-behaved" precisely defined in a remark after this (Daniel Irving Bernstein)
- h. Example 4.49, fourth bullet point. It seems like this should say  $(f \times g) \circ (f' \times g') = (f \circ f') \times (g \circ g')$ . See a longer explanation under "Commutative diagram for snake equations" in <a href="https://davidvandebunte.gitlab.io/executable-notes/notes/ssc/4-5.html">https://davidvandebunte.gitlab.io/executable-notes/notes/ssc/4-5.html</a>.

#### 327. Chp. 5

a. Page 193, the diagram in example 6.37 is printed in black while text references blue and red arrows. Juan Manuel Gimeno. (David VandeBunte: likely refers to Example 6.42 on pg. 194, but this issue seems to have been fixed)

#### 328. Chp. 6

- a. Theorem 6.77: "...and whose morphisms are *equivalence* classes..." should be "...and whose morphism are *isomorphism* classes...", because we do not have any equivalence relation. Also, ncatlab refers to isomorphism classes in the page about decorated cospans. (Konstantin Nisht)
- Definition 6.97, section iii: Probably function composition sign (circle) in the right side of the equation should be replaced with semicolon sign for composition of morphisms, because we are still using categorical terms. (Konstantin Nisht)
- c. Definition 6.52, (typo) the 3-tuple of symmetric monoidal category is in an inconsistent order (C, \otimes, I) w.r.t. the rest of the book (C, I, \otimes). (Jean)

#### 329. Chp. 7

- a. On page 227, section 7.2.1, in the "Cartesian closed" paragraph, the sentence starting with "So now you've transformed a two-player game..." is very hard to read, and possibly nonsensical around the "valued in..." part. (Valentin Robert)
- b. Example 7.54: Quotes behave strange in the last paragraph. (Konstantin Nisht)
- c. (file not found :) ) Example 7.54: But if it is not in H, the mathematics requires us to ask more questions: is its source in H? is its target in G"? both? neither? we are talking about the subobject classifier of graphs; the morphism (0, V; 0) surely corresponds to the target being in H, the subgraph, rather than G? (AKoziell)
- d. Example 7.58: Is it really true that  $V \rightarrow U = U$ ? We can make a union of that answer with the interval (4; +infinity). (Konstantin Nisht)
- e. 7.61: <a href="https://math.stackexchange.com/g/3912771/245548">https://math.stackexchange.com/g/3912771/245548</a>
- f. Just above Eq. 7.63 the set of S-predicates is denoted as "\Omega^E". Was this meant to be "\Omega^S", since E is never mentioned again? (Bruno Gavranović); In the first paragraph of 7.4.5 they denote it correctly. (Rostislav Svoboda)

#### 330. General

- a. How about linking the number of an exercise to its solution and vice versa in the PDF file? (Marcin Ciura)
- b. How about using three digits after the period in the numbering? For instance, Exercise
   1.1 would become Exercise 1.001. This way, searching the PDF file will be easier.
   (Marcin Ciura)

## **Errata for Cambridge University Press Print Version**

- 1. Solution to Exercise 1.52 (page 261): Hasse diagram should also contain {b1, b2, c1, c2} (Asad Saeeduddin)
- 2. Page 26-27: "The preservation of meets implies that phi behaves well when restricting..." should have f instead of phi, to be consistent with the rest of the page. (Carmen Constantin);
- 3. Exercise 7.30: 3.  $V_3 = \{a,b,d,e\}$  should be  $\{a,b,c,e\}$ .
- 4. (suggestion) In the text after Exercise 1.2, it's not clear how Alice combines her observations, and therefore almost gives the impression that what is important is that f(x) = f(x+y). -MTM
- 5. In addition to 2.: There should be 3 replacements "Φ -> f". Also, as a minor detail please consider using "monotone map f" instead of "map Φ" in the sentence "The preservation of meets implies that the map Φ...", page 27. (Rostislav Svoboda)
- 6. (suggestion / question) Definition 1.25: To my mind, remark 1.26 implies that the reflexivity condition (a) of definition 1.25 should read as: " $x \le x$  for all  $\in X$ " (?) (Björn Schemmann)
- 7. In Remark 1.38, it refers to "Example 1.30" but 1.30 is a Remark. (Aaron Schumacher)
- 8. Solution to Exercise 3.46 (page 280): the lower arrows in the commutative diagram should read \alpha \c'\ and \beta \c'\.
- 9. Definition 3.56: the notation \alpha\_{F c} should be written as \alpha\_{F(c)} for clarity, since the object in D is notated elsewhere as F(c), namely the component of alpha being used in the definition
- 10. Exercise 6.64 (page 207): Perhaps I'm confused about the notation, but shouldn't the Circ functor have the type Circ: FinSet -> Set? If Circ refers to the C->D functor component of the (Circ, phi) symmetric monoidal functor which has the type (FinSet, +) -> (Set, ×).
- 11. Solution to Exercise 7.2 (page 313): minor typo: r; h 2 = q; f (not g; f)