Name:	Class: Sec _____	Index No.: _____

4E

MERIDIAN SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2020

ADDITIONAL MATHEMATICS **Paper 2**

4047/02 1 September 2020

SECONDARY 4 EXPRESS **2 hours 30 minutes**

Additional Materials: Nil

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The total marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is **100**.

For Examiner's Use	
Total Marks	

This question paper consists of **22** printed pages and **2** blank pages.

Setter: Mdm Shermane Yeo **[Turn over**

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2\sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

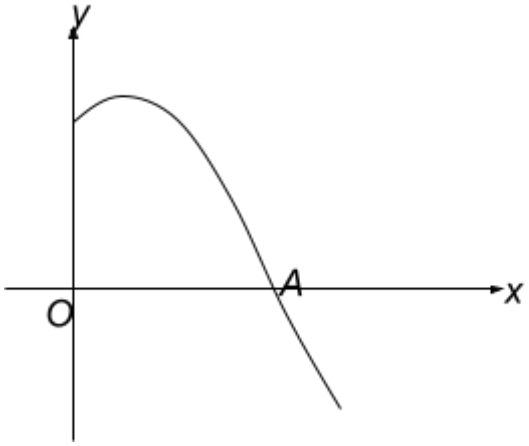
Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}ab\sin C$$

1 The graph below shows part of the graph of $y = \sin x + \sqrt{3}\cos x$, where x is in radians. The graph intersects the x-axis at point A.



(i) Find the exact coordinates of point A. [2]

(ii) Find the rate of change of y at point A given that x is increasing at a constant rate of $\frac{\pi}{10}$ units per second. [3]

2 **(i)** Factorise $x^3 + 8$. [1]

(ii) A pyramid has a base area of $\left(x^3 + 8\right)$ cm^2 and volume $\left(x^3 + \frac{1}{3}x^2 + \frac{14}{3}x + 4\right)$ cm^3.

(a) Show that the height of the pyramid can be expressed as

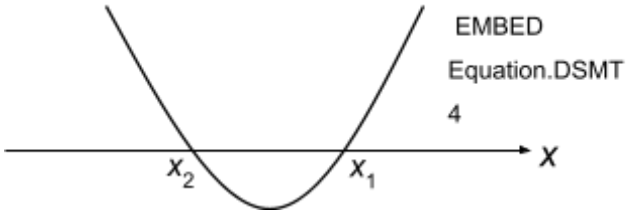
$A + \frac{x^2 + Bx + C}{x^3 + 8}$, where A, B and C are constants. [2]

(ii) **(b)** Using your results in part **(i)** and part **(ii)(a)**, express the height of the pyramid as partial fractions. [4]

3 **(a)** **(i)** Sketch the graph of $2y^2 + 3x = 0$. [2]

(ii) It is given that the line $2y = k^2 - x$, where k is a constant, meets the curve $2y^2 + 3x = 0$. Show that $-\sqrt{m} \leq k \leq \sqrt{m}$, where m is a rational number. [4]

(b) The quadratic curve $y = x^2 + x - \frac{1}{2}$ cuts the x-axis at x_1 and x_2, as shown in the diagram below. Show that $\frac{x_1}{x_2} = \sqrt{3} - 2$. [4]



4 **(a)** Write down, without simplifying, the first three terms of the expansion, in ascending powers of x, of $\left(2-\frac{x}{5}\right)^n$. Hence find the value of n if the first two non-zero terms in the expansion of $(2+x)\left(2-\frac{x}{5}\right)^n$ are $a+bx^2$. [4]

(b) **(i)** Expand and simplify $\left(1+\frac{x}{100}\right)^{10}$ in ascending powers of x up to and including x^4. [2]

(b) **(ii)** A sum of money, \$$A$, increases in such a way that after one year, it amounts to $\$A \times \frac{103}{100}$, after two years to $\$A \times \frac{103}{100} \times \frac{103}{100}$, and so on. Use your expansion in **part (i)** to estimate the sum obtained at the end of 10 years, given that an initial amount of \$10 000 was invested. [3]

(iii) Peter claimed that this is a good estimate of the sum of money obtained. Do you agree with him? Justify your answer. [1]

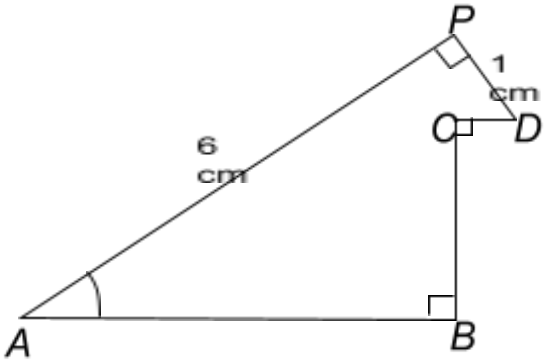
5 **(a)** Solve the equation $\dfrac{4}{\cot^2(x-60°)}-1=0$ for $0° \le x \le 360°$. [3]

(b) **(i)** Show that $\cos^4\frac{1}{4}x-\sin^4\frac{1}{4}x=\cos\frac{1}{2}x$. [2]

(ii) Hence, show that

$$\int_0^{\frac{\pi}{2}} \left[\cos^4 \frac{1}{4}x - \sin^4 \frac{1}{4}x + \frac{1}{7-2x} \right] dx = \sqrt{2} + \ln \sqrt{\frac{7}{7-\pi}} \,.$$ [4]

6 In the diagram, $\angle PAB = \theta°$, where θ is acute. $\angle APD = 90°$, $PA = 6$ cm and $PD = 1$ cm. C is vertically below P such that AB and CD are each perpendicular to BC.



L is the total length of AB, BC and CD.

(i) Find the values of a and b for which $L = (a\sin\theta + b\cos\theta)$ cm. [3]

(ii) Hence express L in the form $R\sin(\theta + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. [2]

(iii) State the maximum value of L and the corresponding value of θ. [2]

(iv) Find the value of θ for which $L = 6$ cm. [2]

7 **(i)** The line $x = 4$ passes through the highest point of a circle, with centre C. A tangent to the circle at $A(7, -2)$ intersects the y-axis at $-\frac{29}{4}$. Find the equation of the circle. [6]

(ii) Another circle has an equation $x^2 + y^2 + 8x - 10y + 29 = 0$. By considering the radii of the two circles, explain whether it intersects the circle in part **(i).** [4]

8 A circular cylinder, open at one end, is constructed of thin sheet metal with a total surface area of 420π cm^2. The cylinder has a base radius of r cm and a height of h cm.

(i) Assuming that the thickness of the metal sheet is negligible, show that the volume of the cylinder, V cm^3, is given by $V = \frac{\pi r}{2}\left(420 - r^2\right)$. [3]

(ii) Find the value of r for which V is stationary given that r can vary. [3]

(iii) Evaluate the stationary value of V and determine, with working, whether this value is a maximum or a minimum. [3]

9 The table below shows some experimental values of two variables x and y. It is known that x and y are related by the equation $y = px + \frac{q}{x} - 1$.

x	1	2	3	4	5
y	1	−1.68	−3.24	−5.28	−5.84

One of the values of y is subjected to an unusually large error.

(i) Draw the graph of $(xy + x)$ plotted against x^2 on the grids on the next page. [2]

(ii) Use your graph to

(a) determine the incorrect value of y and estimate its correct value, [2]

(b) estimate the values of p and q. [2]

(iii) Explain how the graph could be used to estimate the values of x when $y + \frac{1}{4}x + 1 = 0$. Find the positive value of x. [3]

10 A particle starts from a fixed point A and travels in a straight line. As it passes a point B, the brakes are applied and the particle slows down at a constant rate of $\frac{1}{3}$ m/s^2, coming to rest at point C. For the motion from A to B, the velocity, v m/s, of the particle, t s after leaving A, is given by $v = 1 + 3e^{\frac{1}{2}(t-4)}$ for $0 \leq t \leq 4$.

(i) Find the time taken for the object to reach a velocity of 3 m/s. [2]

(ii) Find the initial acceleration of the particle. [2]

(iii) Find the velocity of the particle at point B. [1]

(iv) Find the total distance travelled from A to C. [6]

11 A curve is given by the equation $y = \frac{x-2}{2x+1}$ where $x \neq -\frac{1}{2}$.

(i) Explain why the curve is defined for all values of x except $x = -\frac{1}{2}$. [1]

(ii) Explain, with working, why the curve has no stationary point. [2]

(iii) The curve crosses the x-axis at point P. The normal to the curve at P meets the curve again at point Q. Find the area of triangle OPQ, where O is the origin. [8]

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