

Essential Trig Identities & Unit-Circle Lengths

Name: _____

Period: _____

Date: _____

Part A • Recall the Essential Identities

Complete each identity from memory. Use these identities in Parts B and C.

Family	Identity	Your completed identity
Reciprocal	$\csc \theta =$ _____	
Reciprocal	$\sec \theta =$ _____	
Reciprocal	$\cot \theta =$ _____	
Quotient	$\tan \theta =$ _____ use $\sin \theta$ and $\cos \theta$	
Quotient	$\cot \theta =$ _____	
Pythagorean	$\sin^2 \theta + \cos^2 \theta =$ _____	
Pythagorean	$1 + \tan^2 \theta =$ _____	
Pythagorean	$1 + \cot^2 \theta =$ _____	

Copying answers from the answer key has no value. Think about why each answer is true, and use the relationships among the hypotenuse (hyp), opposite side (opp), and adjacent side (adj) to determine and justify your answers. Show your reasoning.

Part B • Select and Use an Identity

Simplify completely. Show the identity you used; answers without supporting work do not receive full credit.

1. Rewrite using only sine and cosine: $\tan x \cdot \cos x$

2. Rewrite using only sine and cosine: $\sec x / \tan x$

3. Simplify: $(1 - \cos^2 x) / \sin x$

4. Simplify: $(\sec^2 x - 1) / \tan x$

5. Simplify: $(\csc^2 x - 1) / \cot x$

6. Simplify: $(1 + \tan^2 x) \cos x$

7. Simplify: $\sin x / (1 - \cos^2 x)$

8. Simplify: $(\sin^2 x + \cos^2 x) / (\sec x \cos x)$

Part C • Verify the Identity

Transform only the left-hand side until it matches the right-hand side. State the identity used at each important step.

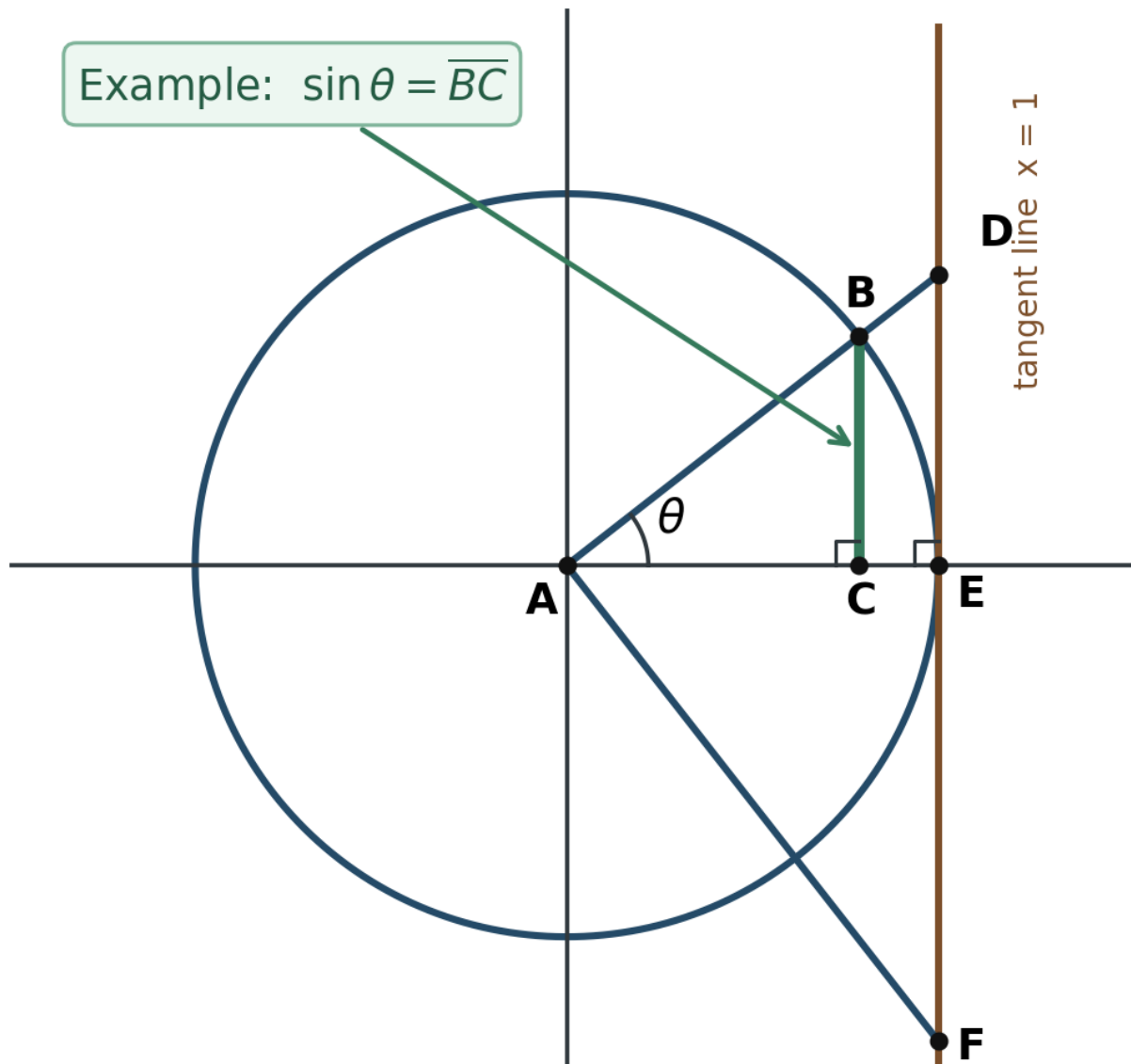
9. $(1 - \sin^2 x) / \cos x = \cos x$

10. $(\sec x - \cos x) / \tan x = \sin x$

11. $\tan x + \cot x = \sec x \cdot \csc x$

Part D • Find the Six Trig Functions in One Diagram

The circle is a unit circle centered at A , so $AB = 1$. The vertical line through E is tangent to the circle at E and has equation $x = 1$. Assume $0^\circ < \theta < 90^\circ$.



Given example: $\sin \theta = BC$ because BC is the vertical coordinate (opposite leg) of point B on the unit circle.

12. Identify the remaining five functions.

$\cos \theta =$

$\tan \theta =$

$\cot \theta =$

$\sec \theta =$

$\csc \theta =$

Unit radius: $AB =$

13. Explain or derive each relationship from the diagram.

a. Why is $\tan \theta = DE$? Use similar triangles ABC and ADE.

b. Use right triangle ADE to show $1 + \tan^2 \theta = \sec^2 \theta$.

c. Use right triangle AEF to show $1 + \cot^2 \theta = \csc^2 \theta$.

d. Explain why $\sec \theta$ and $\csc \theta$ are reciprocal lengths of $\cos \theta$ and $\sin \theta$.

Challenge (optional): Use the diagram to derive $\tan \theta = \sin \theta / \cos \theta$.