Unit-1 MECHANISMS AND VELOCITY & ACCLERATION ANALYSIS

1. **What is a mechanism?**
   - o A mechanism is a system of rigid bodies connected by joints to produce a desired motion. The primary purpose is to transmit or transform motion.
2. ***Define degrees of freedom in a mechanism.***
   - o ***Degrees of freedom refer to the number of independent movements allowed in a mechanism. It is calculated using Gruebler's equation: DOF = 3(n - 1) - 2j - h, where n is the number of links, j is the number of joints, and h is the number of higher pairs.***
3. **What is a kinematic chain?**
   - o A kinematic chain is a series of links connected by joints that allow relative motion between them. When one link is fixed, it becomes a mechanism.
4. **Differentiate between a machine and a mechanism.**
   - o A mechanism is a system designed to transmit or transform motion, while a machine is a combination of mechanisms that transmit power to perform useful work.
5. **Explain the concept of inversion in a mechanism.**
   - o Inversion in a mechanism is obtained by fixing a different link of the kinematic chain each time, resulting in different configurations of the same mechanism.

Velocity Analysis

6. **What is the velocity of a point in a mechanism?**
   - o The velocity of a point in a mechanism is the rate at which the point changes its position with respect to a reference frame, often expressed as a vector.
7. **Explain the relative velocity method.**
   - o The relative velocity method involves finding the velocity of different points in a mechanism by considering the velocity of one point relative to another, using vector diagrams.
8. **Define the Coriolis component of acceleration.**
   - o The Coriolis component of acceleration occurs in rotating systems when there is relative sliding motion along a rotating path. It is given by 2ωvr, where ω is the angular velocity, and vr is the relative velocity.
9. **State the Instantaneous Center of Rotation (ICR).**
   - o The Instantaneous Center of Rotation (ICR) is a point in a mechanism where the velocity is zero at a particular instant, and all other points rotate around it.
10. ***What is a velocity polygon?***
    - o *A velocity polygon is a graphical representation of the velocities of various points in a mechanism, obtained using vector diagrams based on the relative velocity method.*

Acceleration Analysis

11. **What is acceleration in a mechanism?**
    - o Acceleration in a mechanism is the rate of change of velocity of a point concerning time, consisting of tangential and radial components.
12. **Explain the tangential and radial components of acceleration.**

- Tangential acceleration is the component of acceleration perpendicular to the radius vector, while radial (centripetal) acceleration is directed towards the center of rotation.

13. **What is the purpose of an acceleration diagram?**
    - An acceleration diagram is used to determine the acceleration of various points in a mechanism, considering both tangential and radial components.
14. **Define the term "Jerk" in kinematics.**
    - Jerk is the rate of change of acceleration with respect to time, representing the third derivative of displacement concerning time.
15. **How do you find the acceleration of a point in a rotating link?**
    - The acceleration of a point in a rotating link is found by combining the tangential acceleration (αr) and the radial (centripetal) acceleration ($\omega^2 r$), where α is angular acceleration, ω is angular velocity, and r is the radius.

UNIT I

Explain the theory of the instantaneous center of rotation and Kennedy's theorem. Discuss the significance of the instantaneous center in the analysis of planar mechanisms and derive the conditions for locating the instantaneous center in different linkages. Provide examples of how Kennedy's theorem is applied in mechanical engineering.

Answer:

1. Introduction to Instantaneous Center of Rotation:

The **Instantaneous Center of Rotation (ICR)**, also known as the Instantaneous Center (IC) or Pole, is a fundamental concept in kinematics, particularly in the analysis of planar mechanisms. It is a point in or outside a body at which the velocity of all points on the body is momentarily zero. This concept is critical for understanding the motion of a rigid body, as it simplifies the analysis by reducing the problem to a simpler rotational motion about a single point.

In a planar mechanism, every link or component can have an instantaneous center of rotation relative to another link. Knowing the location of the IC allows for the determination of velocities of various points on the mechanism without requiring complex calculations.

2. Theory of Instantaneous Center of Rotation:

For any rigid body undergoing planar motion, the velocity of any point on the body can be considered as a rotation around some point (the instantaneous center). Mathematically, the velocity VAV_AVA of any point AAA on the body is given by:

VA=ω×rAV_A = \omega \times r_AVA=ω×rA

Where:

- ω\omegaω is the angular velocity of the body.
- rAr_ArA is the distance from the instantaneous center to point AAA.

The key characteristic of the instantaneous center is that at this point, the velocity is zero, meaning it is a point about which the body is momentarily rotating.

3. Locating the Instantaneous Center:

To locate the instantaneous center of rotation in a mechanism with multiple links, one can use the following methods:

- **Perpendicular Bisector Method:** For two points AAA and BBB on a link with known velocities, the instantaneous center lies on the intersection of the perpendicular bisectors of the velocity vectors of points AAA and BBB.
- **Intersection of Velocity Lines:** For two points AAA and BBB on a link, the lines drawn perpendicular to the velocity vectors at AAA and BBB will intersect at the instantaneous center.

For example, in a four-bar linkage, each link will have an instantaneous center with respect to each other. There are generally four links in a four-bar mechanism, resulting in six possible instantaneous centers. Out of these, three can be directly located using the intersection of the velocity lines, and the remaining three are located using **Kennedy's theorem**.

4. Kennedy's Theorem:

Kennedy's theorem (or the Three-Center Theorem) states that if three bodies (or links) in a plane are moving relative to each other, the three instantaneous centers of rotation relative to each pair lie on a straight line.

This theorem is significant in planar mechanism analysis as it simplifies the process of finding the instantaneous centers, especially in complex linkages like the four-bar linkage.

Application of Kennedy's Theorem:

Consider three links L1L_1L1, L2L_2L2, and L3L_3L3 in a mechanism:

- The instantaneous center of L2L_2L2 relative to L1L_1L1 is denoted as I12I_{12}I12.
- The instantaneous center of L3L_3L3 relative to L2L_2L2 is denoted as I23I_{23}I23.
- The instantaneous center of L3L_3L3 relative to L1L_1L1 is denoted as I13I_{13}I13.

According to Kennedy's theorem, these three centers I12I_{12}I12, I23I_{23}I23, and I13I_{13}I13 lie on a straight line. If two of these centers are known, the third can be easily located by ensuring it lies on the line connecting the other two.

5. Significance of Instantaneous Center and Kennedy's Theorem:

The concept of the instantaneous center and Kennedy's theorem are significant in the analysis of planar mechanisms for several reasons:

- **Simplification of Velocity Analysis:** The use of the instantaneous center reduces the complexity of velocity analysis. Once the IC is located, the velocity of any point on the link can be determined using simple rotational motion formulas.
- **Determining Relative Motion:** The relative motion between different links can be easily analyzed by locating the IC and applying Kennedy's theorem.
- **Design and Analysis of Mechanisms:** In designing mechanisms like four-bar linkages, slider-crank mechanisms, and cam-follower systems, the IC provides a straightforward way to understand the motion of different parts of the mechanism.

6. Example Application:

Consider a four-bar linkage with links ABABAB, BCBCBC, CDCDCD, and DADADA. To analyze the motion:

1. **Identify Known Instantaneous Centers:**
   - I12I_{12}I12: Instantaneous center between links ABABAB and BCBCBC.
   - I23I_{23}I23: Instantaneous center between links BCBCBC and CDCDCD.
   - I34I_{34}I34: Instantaneous center between links CDCDCD and DADADA.
2. **Use Kennedy's Theorem:**
   - If I12I_{12}I12 and I34I_{34}I34 are known, locate I13I_{13}I13 (instantaneous center between ABABAB and CDCDCD) on the line connecting I12I_{12}I12 and I34I_{34}I34.
3. **Calculate Velocities:**
   - With the IC located, the velocity of any point on the links can be calculated using the rotational motion about the IC.

Explain the theory of graphical methods for displacement in planar mechanisms. Discuss the procedures and techniques involved in analyzing displacement using graphical methods, including the construction of displacement diagrams and the use of vector polygons. Provide examples of typical applications of these methods in mechanical engineering and discuss the advantages and limitations of graphical methods compared to analytical methods.

Answer:

1. Introduction to Graphical Methods for Displacement:

Graphical methods are used in planar mechanism analysis to determine the displacement of various links and components. These methods offer a visual and intuitive approach to solving complex kinematic problems, making them useful in understanding and designing mechanisms where analytical solutions might be cumbersome. The primary graphical technique used for displacement analysis involves constructing displacement diagrams and vector polygons.

2. Theory of Displacement Analysis Using Graphical Methods:

Graphical methods for analyzing displacement involve the following key techniques:

1. **Displacement Diagrams:**
   - Displacement diagrams are graphical representations that show the relative displacement of different links in a mechanism. They are constructed by plotting the positions of various points or links at different instances of time or crank angles.
2. **Vector Polygons:**
   - Vector polygons are used to represent the magnitudes and directions of displacement vectors in a mechanism. By constructing vector polygons, one can visualize the resultant displacement and relative positions of different links.

3. Procedures for Graphical Displacement Analysis:

Step 1: Construct the Mechanism Diagram

- Start by drawing a scaled diagram of the mechanism, including all the links and joints. Identify the fixed frame and the moving links that need to be analyzed.

Step 2: Define the Initial Positions

- Determine the initial positions of the moving links or points. Mark these positions on the diagram.

Step 3: Plot Displacement Vectors

- For each link or point whose displacement needs to be determined, plot the displacement vector. The length of the vector represents the magnitude of displacement, and the direction represents the direction of displacement.

Step 4: Construct Vector Polygons

- If analyzing the displacement of multiple links, construct a vector polygon by connecting the tips of the displacement vectors. The polygon will help visualize the relative displacements and check for closure.

Step 5: Analyze Displacement

- Use the vector polygons to determine the relative displacements and positions of the links. By comparing the initial and final positions, you can evaluate the displacement of individual links.

Step 6: Check Consistency

- Verify that the displacement analysis is consistent with the mechanism's constraints and motion requirements. Ensure that the vector polygons close properly and that the displacement vectors align with the expected motion.

4. Example of Graphical Displacement Analysis:

Example Problem: Consider a simple four-bar linkage with known lengths. Use graphical methods to determine the displacement of a point on one of the moving links when the input link rotates by a specified angle.

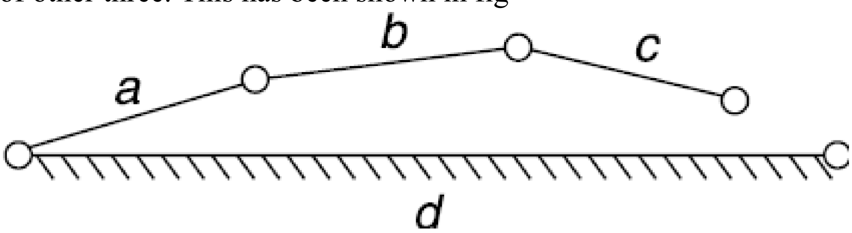
Solution:

1. **Draw the Mechanism:**
   - Sketch the four-bar linkage, including the fixed frame, input link (crank), coupler, and output link.
2. **Define the Initial Positions:**
   - Mark the initial positions of the links and the point of interest on the moving link.
3. **Plot Displacement Vectors:**
   - For each link, draw the displacement vectors corresponding to the rotation angle of the input link.
4. **Construct the Vector Polygon:**
   - Connect the tips of the displacement vectors to form a closed polygon. The polygon's sides represent the displacements of the links.
5. **Analyze Displacement:**
   - Measure the length and direction of the resulting vector to determine the displacement of the point on the moving link.
6. **Verify Results:**
   - Ensure the vector polygon closes properly and that the displacement analysis aligns with the mechanism's constraints.

1.Explain inversions of a four bar chain in detail?

The FourBar chain

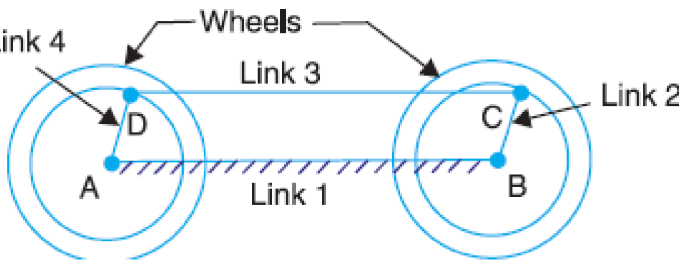
A four bar chain is the most fundamental of the plane kinematic chains. It is a much proffered mechanical device for the mechanisation and control of motion due to its simplicity and versatility. Basically, it consists of four rigid links which are connected in the form of a quadrilateral by four pin-joints.
When one of the link fixed, it is known as mechanism or linkage. A link that makes complete revolution is called the crank. The link opposite to the fixed link is called coupler, and the forth link is called a lever or rocker if it oscillates or another crank if it rotates.
It is impossible to have a four-bar linkage if the length of one of the link is greater than the sum of other three. This has been shown in fig



Inversion of Four Bar chain

First inversion: coupled wheel of locomotive

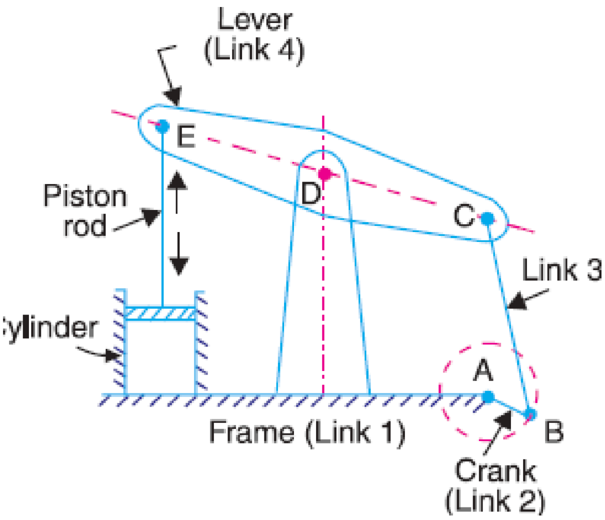
The mechanism of a coupling rod of a locomotive (also known as double crank mechanism) which consists of four links is shown in Fig.



In this mechanism, the links *AD* and *BC* (having equal length) act as cranks and are connected to the respective wheels. The link *CD* acts as a coupling rod and the link *AB* is fixed in order to maintain a constant centre to Centre distance between them. This mechanism is meant for transmitting rotary motion from one wheel to the other wheel.

Second inversion
Beam Engine

A part of the mechanism of a beam engine (also known as cranks and lever mechanism) which consists of four links is shown in Fig. 1.10.
In this mechanism, when the crank rotates about the fixed centre A, the lever oscillates about a fixed centre D. The end E of the lever CDE is connected to a piston rod which reciprocates due to the rotation of the crank.

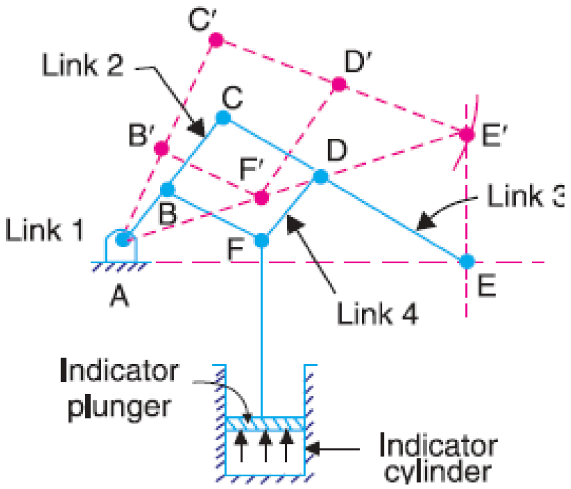


In other words, the purpose of this mechanism is to convert rotary motion into reciprocating motion.

Third inversion: watts indicator mechanism

A Watt‟s indicator mechanism (also known as Watt's straight line mechanism or double lever mechanism) which consists of four links is shown in Fig.

The four links are: fixed link at A, link AC, link CE and link BFD. It may be noted that BF and FD form one link because these two parts have no relative motion between them. The links CE and BFD act as levers.
The displacement of the link BFD is directly proportional to the pressure of gas or steam which acts on the indicator plunger. On any small displacement of the mechanism, the tracing point E at the end of the link CE traces out approximately a straight line.
Fig.



2. Explain the working of any two inversions of a single slider crank chain with neat sketches.

The slider crank chain

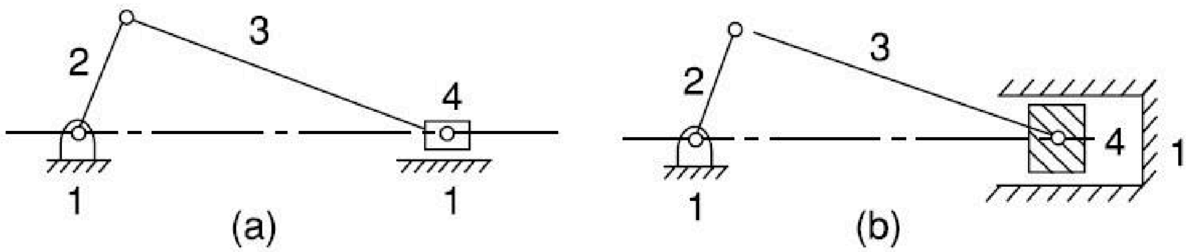
When one of the turning pairs of a four-bar chain is replaced by a sliding pair, it becomes a single slider-crank chain or simply a slider-crank chain.
It is also possible to replace two sliding pairs of a four-bar chain to get a double slider-crank chain. In a slider-crank chain, the straight line path of the slider may be passing through the fixed pivot O or may be displaced.
The distance e between the fixed pivot O and the straight line path of the slider is called the offset and the chain so formed an offset slider-crank chain.
Different mechanisms obtained by fixing different links of a kinematic chain are known as its inversions.

First inversion

This inversion is obtained when link 1 is fixed and links 2 and 4 are made the crank and slider respectively. (fig.a)

a Reciprocating engine

b Reciprocating compressor



Second inversion

Fig. 1.12 first inversion

- Fixing of the link 2 of a slider-crank chain results in the second inversion.
- **Applications:**

a Whitworth quick-return mechanism

b Rotary engine

Third Inversion

By Fixing of the link 3 of the slider-crank mechanism, the third inversion is obtained. Now the link 2 again acts as a crank and the link 4 oscillates.

Applications:

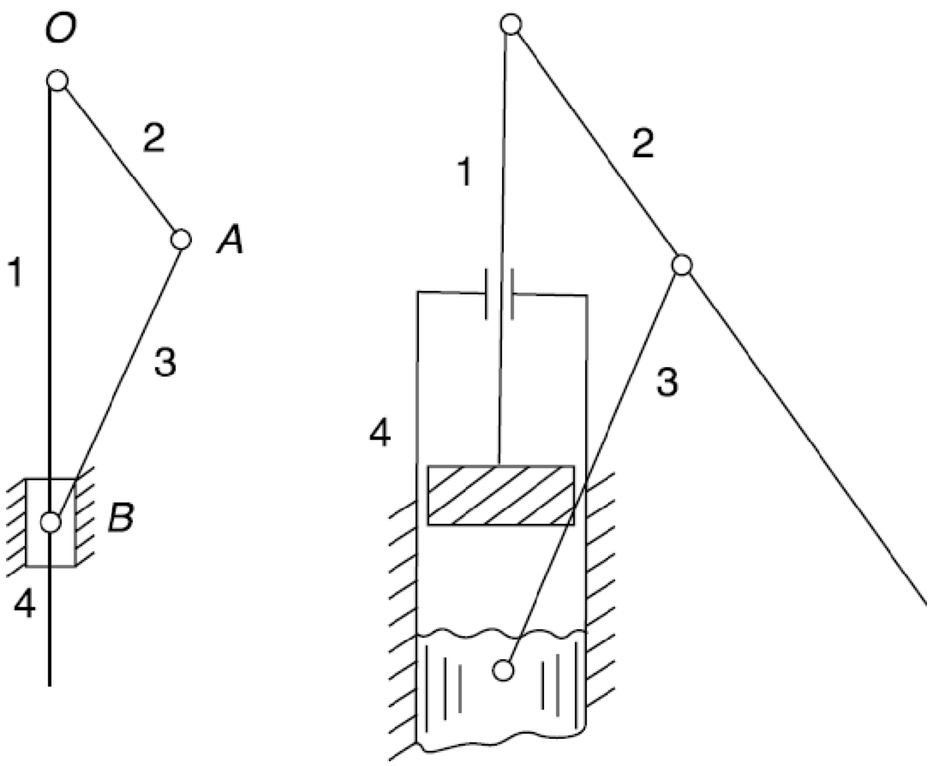
a Oscillating cylinder engine

b Crank and slotted-lever mechanism

Fourth Inversion

If the link 4 of the slider-crank mechanism is fixed, the fourth inversion is obtained. Link 3 can oscillates about the fixed pivot B on the link 4. This makes the end A of the link 2 to oscillate about B and the end O to reciprocate along the axis of the fixed link 4.

Application: Hand Pump



(a) (b)

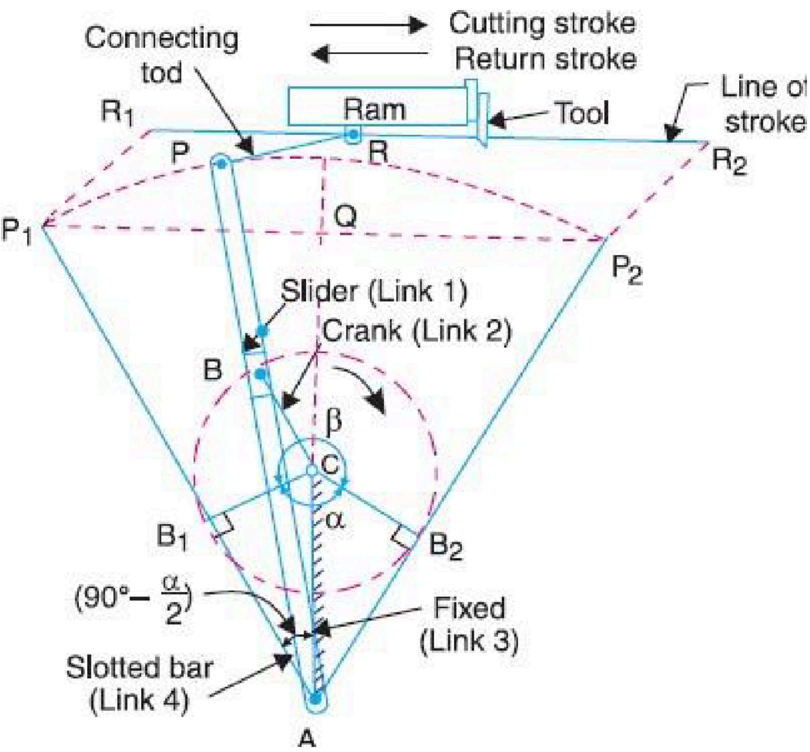
3. Explain with neat sketch the working of crank and slotted lever quick return motion mechanism. Deduce the expression for length of stroke in terms of link lengths.

Crank and slottedlever Mechanism

This mechanism is mostly used in shaping machines, slotting machines and in rotary internal combustion engines.

- In this mechanism, the link AC (i.e. link 3) forming the turning pair is fixed, as shown in Fig. The link 3 corresponds to the connecting rod of a reciprocating steam engine. The driving crank CB revolves with uniform angular speed about the fixed center C. A sliding block attached to the crank pin at B slides along the slotted bar AP and thus causes AP to oscillate about the pivoted point A.

- A short link PR transmits the motion from AP to the ram which carries the tool and reciprocates along the line of stroke R1R2. The line of stroke of the ram (i.e. R1R2) is perpendicular to AC produced



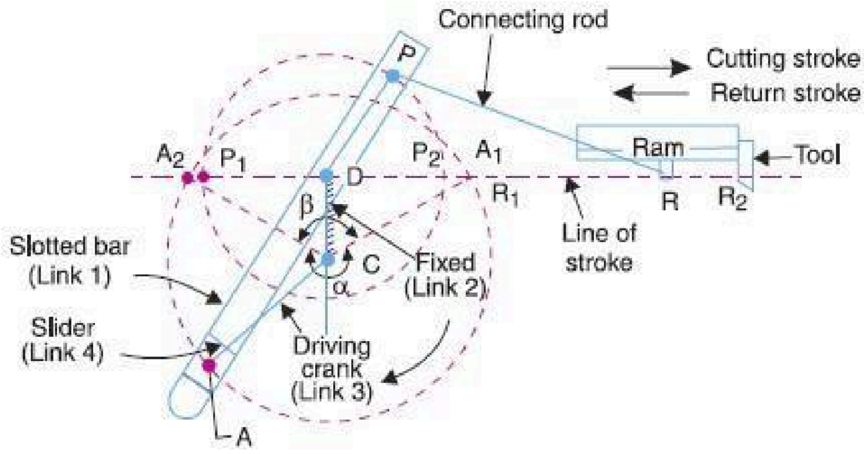
In the extreme positions, $AP1$ and $AP2$ are tangential to the circle and the cutting tool is at the end of the stroke. The forward or cutting stroke occurs when the crank rotates from the position $CB1$ to $CB2$ (or through an angle β) in the clockwise direction. The return stroke occurs when the crank rotates from the position $CB2$ to $CB1$ (or through angle α) in the clockwise direction. Since the crank has uniform angular speed,

4. State and explain Whitworth quick return mechanism. Also derive an equation for ratio of time taken for return strokes and forward strokes.

Whitworth Quick Return Mechanism

This mechanism used in shaping and slotting machines.

- In this mechanism the link CD (link 2) forming the turning pair is fixed; the driving crank CA (link 3) rotates at a uniform angular speed and the slider (link 4) attached to the crank pin at a slides along the slotted bar PA (link 1) which oscillates at D.
- The connecting rod PR carries the ram at R to which a cutting tool is fixed and the motion of the tool is constrained along the line RD produced



5. Define Kinematic pair and discuss various types of kinematic pairs with example

Kinematic pairs according to nature of contact:

a. Closed Pair:

○ When the elements of a pair are held together mechanically, it is known as a closed pair. The two elements are geometrically identical; one is solid and full and the other is hollow or open. The latter not only envelops the former but also encloses it. The contact between the two can be broken only by destruction of at least one of the members.

b. Unclosed Pair:

○ When two links of a pair are in contact either due to force of gravity or some spring action, they constitute an unclosed pair. In this, the links are not held together mechanically, e.g. cam and follower pair.

Kinematic pairs according to Nature of Relative Motion:

a. Sliding pair:

○ When two links have a sliding motion relative to another; the kinematic pair is known as sliding pair.

b. Turning pair:

○ When one link is revolve or turn with respect to the axis of first link, the kinematic pair formed by two links is known as turning pair.

c. Rolling pair:

○ When the links of a pair have a rolling motion relative to each other, they form a rolling pair.

d. Screw pair:

○ If two mating links have a turning as well as sliding motion between them, they form a screw pair.

e. Spherical pair:

○ When one link in the form of sphere turns inside a fixed link, it is a spherical pair.

Example: 3.1

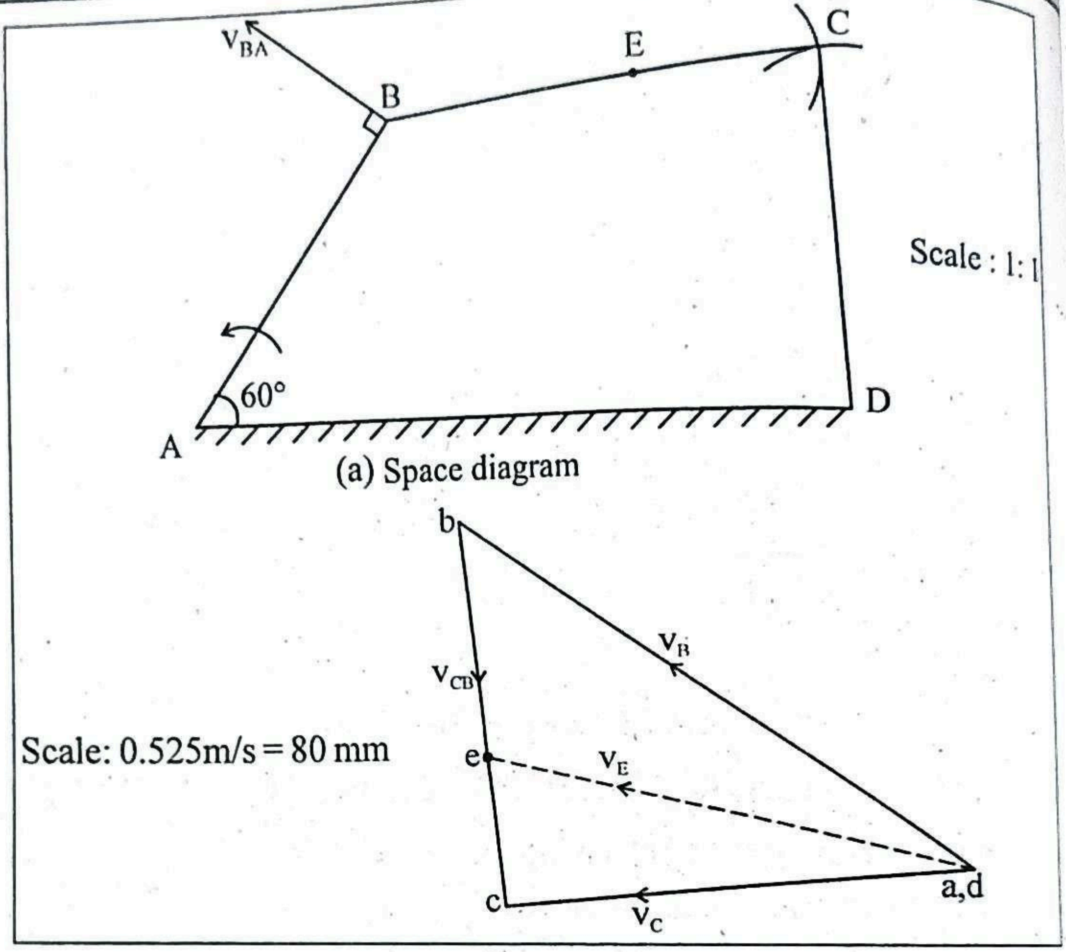
Following data refer to the dimensions of the links of a four-bar mechanism: AB = 50mm; BC = 66mm; CD = 56mm and AD (fixed link) = 100 mm. At the instant when $\angle DAB = 60^o$, the link AB has an angular velocity of 10.5 rad/s in the counter clockwise direction. Determine the velocity of point C, velocity of point E on the link BC while BE = 40mm and the angular velocities of the links BC and CD. Also, sketch the mechanism and indicate the data. ***[AU, May/June 2013]***

☞ ***Solution:***

Given:

AB = 50*mm* = 0.05*m*; BC = 66*mm* = 0.066*m*; AD = 100*mm* = 0.1*m*; $\angle DAB = 60^o$;

ω_{AB} = *10.5 rad/s*; BE = 40*mm* = 0.04*m*; CD = 56*mm* = 0.056*m*.



(a) Space diagram

Figure 3.13 (b) Velocity diagram

✓ Velocity $v_{BA} = \omega_{AB} . AB = 10.5 \times 0.05 = 0.525$ *m/s*.

✓ Taking a suitable scale; the space diagram is drawn.

✓ The crank rotates counter clockwise with an angular velocity of 10.5 rad

✓ $\therefore$ v_{BA} is perpendicular to the link BA in the direction as shown.

Velocity diagram

$$v_{BA} = v_B = 0.525 \; m/s$$

✓ The fixed points A and D are taken as a single point at a suitable location as a

A suitable scale is taken [0.525 *m/s* = 80*mm*]

✓ From *a*; vector *ab* is drawn perpendicular to the link AB.

vector *ab* = 0.525 *m/s* = 80*mm*

✓ From *b*; vector *bc* is drawn perpendicular to the link BC.

✓ From *d*; vector *dc* is drawn perpendicular to the link DC.

They intersect at point c.

✓ **By measurement** : $cd = 60mm = v_{CD}$

$$cb = 52mm = v_{CB}$$

✓ **By scale conversion :** $v_{CB} = \frac{0.525}{80} \times 52 = 0.34125\ m/s$

$$v_{CD} = v_C = \frac{0.525}{80} \times 60 = 0.39375\ m/s$$

✓ ∴ Velocity of point C, $v_C = 0.39375\ m/s$ ... (Ans)

Velocity of point E, v_E

$$\frac{BE}{BC} = \frac{be}{bc} \text{ and } be = \frac{BE}{BC} \times bc = \frac{0.04}{0.066} \times 52 = 31.515\ mm$$

✓ The point e is marked in the vector cb at 31.515mm from b.

✓ Now, e and a are connected and ea represents the velocity $v_{EA} = v_E$.

✓ By measurement: $ea = 63mm$ and;

✓ Velocity of point E, $v_{EA} = v_E = \frac{0.525}{80} \times 63 = 0.4134\ m/s$ ... (Ans)

Angular velocities of links BC and CD

Angular velocity of link BC, $\omega_{BC} = \frac{v_{CB}}{CB} = \frac{0.34125}{0.066} = 5.17\ rad/s$ ... (Ans)

Angular velocity of link CD, $\omega_{CD} = \frac{v_{CD}}{CD} = \frac{v_C}{CD} = \frac{0.39375}{0.056} = 7.03\ rad/s$ ... (Ans)

Example: 3.2

In a slider crank mechanism, the length of crank OB and connecting rod AB are 125mm and 500mm respectively. The centre of gravity G of the connecting rod is 275mm from the slider A. The crank speed is 600 r.p.m clockwise. When the crank has turned 45° from the inner dead centre position, determine velocity of the slider A, velocity of the point G and angular velocity of the connecting rod AB. ***[AU, May/June 2014]***

☞ ***Solution:***

Given:

OB = 125mm = 0.125m; AB = 500mm = 0.5m; AG = 275mm = 0.275m; $\theta = 45^o$; N = 600 $r.p.m$;

Angular velocity of crank, $\omega = \frac{2\pi N}{60} = \frac{2\pi(600)}{60} = 62.83\ rad/s$ (clockwise)

Velocity, $v_{BO} = v_B = \omega_{BO} \times BO = 62.83 \times 0.125 = 7.85375\ m/s$

The space diagram is drawn with a suitable scale (1:5)

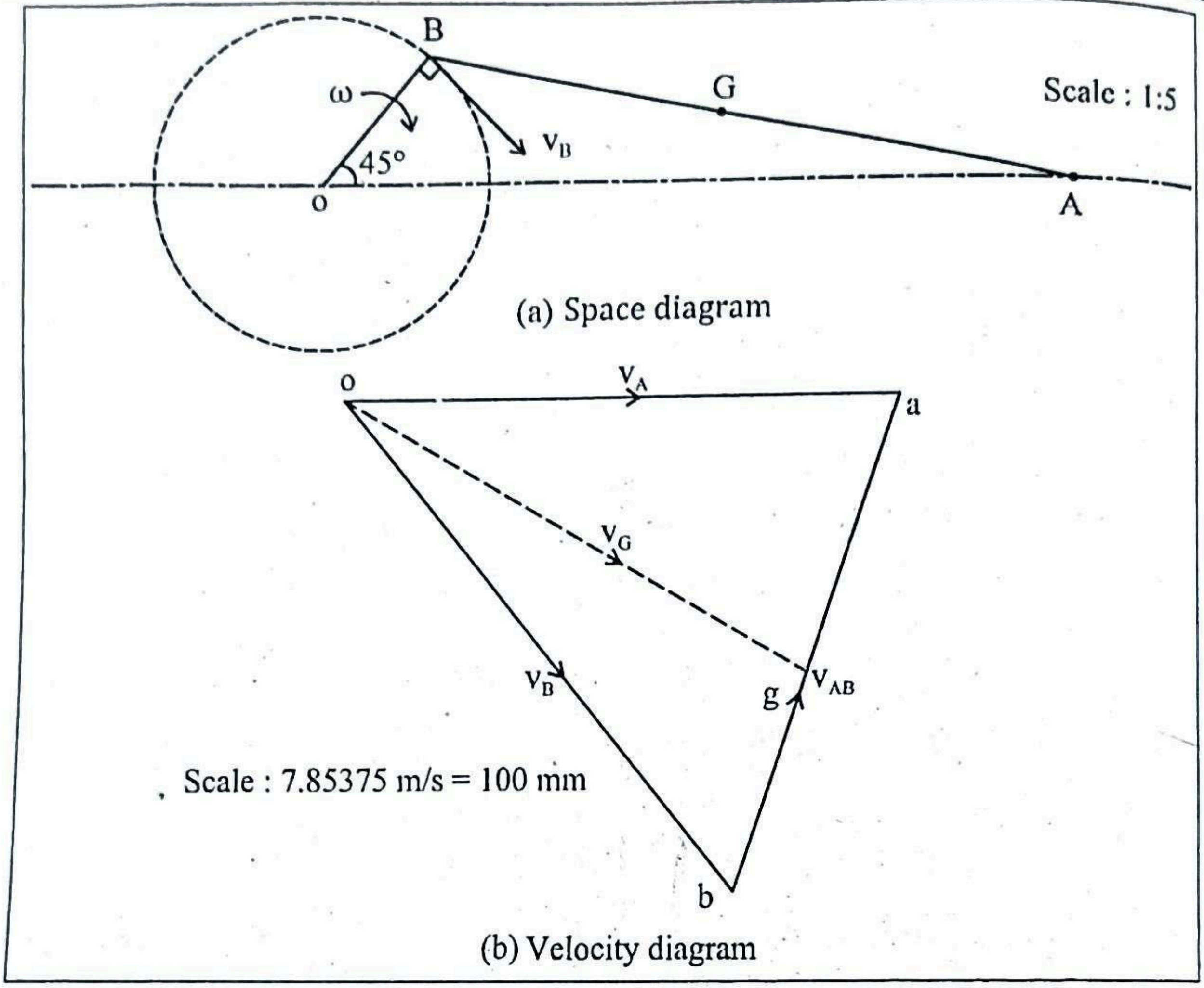


Figure 3.14

Velocity diagram

- ✓ A suitable scale is taken [7.85375 *m/s* = 100*mm*]
- ✓ The fixed point *o* is taken randomly at a suitable location.
- ✓ From *o*; vector *ob* is drawn perpendicular to the link OB; *ob* = 100*mm*.
- ✓ From *b*; vector *ba* is drawn perpendicular to the link BA.
- ✓ We know; the line of stroke of the piston is horizontal-given
- ✓ ∴ A horizontal vector *oa* is drawn from *o* representing the velocity of piston.
- ✓ The vectors *oa* and *ba* meet at the point *a*.

By measurement : $oa = 82mm = v_{AO}$

$ba = 72mm = v_{AB}$

By scale conversion :

Velocity of slider A, $v_{AO} = v_A = \dfrac{7.85375}{100} \times 82 = 6.44\ m/s$ ... (Ans)

Velocity of connecting rod, $v_{AB} = \dfrac{7.85375}{100} \times 72 = 5.6547\ m/s$

Velocity of point G, $v_{GO} = v_G$

$$\frac{AG}{BA} = \frac{ag}{ba} \text{ (or) } ag = \frac{AG}{BA} \times ba = \frac{0.275}{0.5} \times 72 = 39.6\ mm$$

∴ Point g is marked at 39.6*mm* from 'a' in the velocity diagram in vector *ba*. Connect the points *g* and *o*; by measurement *go* = 86*mm*.

By scale conversion:

Velocity of G, $v_{GO} = v_G = \frac{7.85375}{100} \times 86 = 6.7542\ m/s$ ... (Ans)

Angular velocity of connecting rod, ω_{AB}

$$\omega_{AB} = \frac{v_{AB}}{AB} = \frac{5.6547}{0.5} = 11.3094\ rad/s$$ ... (Ans)

Example: 3.3

The crank AB of four bar mechanism shown rotates at 60 r.p.m clockwise. Determine the relative angular velocities of the coupler to the crank and the lever to the coupler. Find also the rubbing velocities at the surface of pins 25mm radius and the joints B and C. [AU, Nov/Dec 2013]

Solution:

Given :

AB = 40*cm* = 0.4*m*; BC = 70*cm* = 0.7*m*; CD = 60*cm* = 0.6*m*; Horizontal distance between A and D = 100*cm* = 1*m*; Vertical distance between A and D = 20*cm* = 0.2*m*; N = 60 *r.p.m*; *r* = 25*mm* = 0.025*m*.

Angular velocity of crank AB, $\omega_{BA} = \frac{2\pi N}{60} = \frac{2\pi(60)}{60} = 6.2832\ rad/s$ (clockwise)

Velocity of B with respect to A, $v_{BA} = v_B = \omega_{BA} \cdot BA = 6.2832 \times 0.4 = 2.51327\ m/s$

The space diagram is drawn to a suitable scale (1:10)

Velocity diagram

✓ A suitable scale is selected [2.51327 *m/s* = 100*mm*]

✓ The fixed points A and D are marked as a single point *a, d* at a suitable place.

✓ From a; vector *ab* is drawn perpendicular to the link AB.

vector ab represents v_{BA} = 2.51327 *m/s* = 100*mm*

✓ From the point *b*; vector *bc* is drawn perpendicular to the link BC.

✓ From the point *d*; vector *dc* is drawn perpendicular to the link DC.

vectors *bc* and *dc* intersect at point c.

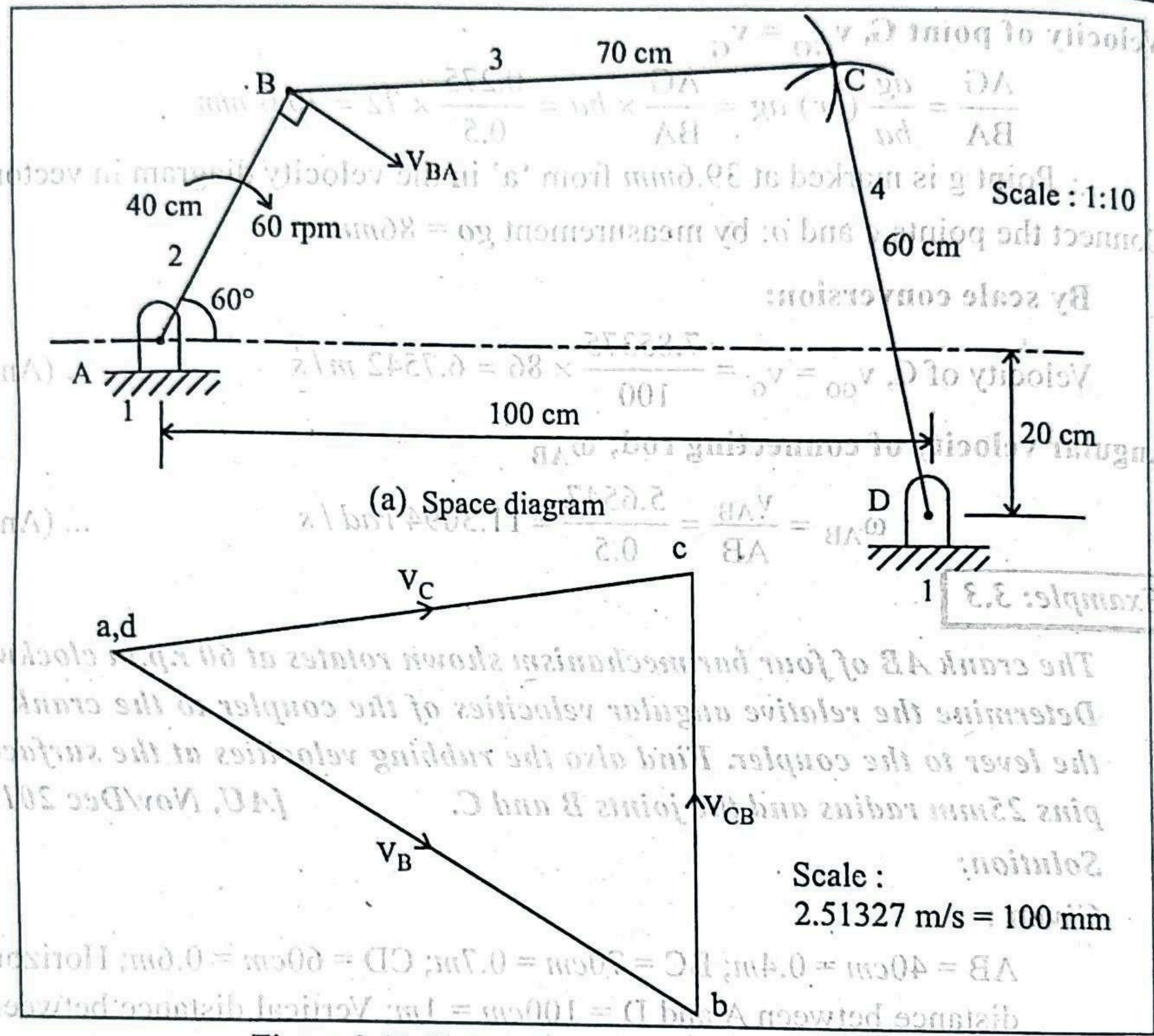


Figure 3.15 (b) Velocity diagram

✓ **By measurement :** $dc = 82mm = v_{CD} = v_C$

$bc = 66mm = v_{CB}$

✓ **By scale conversion:** velocity of C, $v_c = \frac{2.51327}{100} \times 82 = 2.0609\ m/s$

velocity of CB, $v_{CB} = \frac{2.51327}{100} \times 66 = 1.6588\ m/s$

Relative angular velocities

Angular velocity of link CD, $\omega_{CD} = \frac{v_{CD}}{CD} = \frac{2.0609}{0.6} = 3.4348\ rad/s$ (clockwise)

Angular velocity of link CB, $\omega_{CB} = \frac{v_{CB}}{CB} = \frac{1.6588}{0.7} = 2.37\ rad/s$ (counter clockwise)

Relative angular velocity of coupler to the crank is:

$\omega_{BA} + \omega_{CB} = 6.2832 + 2.37$ ..... (+ve since both are opposite to each other)

$= 8.6532$ rad/s ...(Ans)

Relative angular velocity of lever to the coupler is:

$\omega_{CB} + \omega_{CD} = 2.37 + 3.4348$ ..... (+ve since both are opposite to each other)

$= 5.8048$ *rad/s* ...(Ans)

Rubbing velocities

At joint - B

Rubbing velocity at B $= r_B \cdot (\omega_{BA} + \omega_{CB})$

$= 0.025 \times 8.6532$

$= 0.21633$ *m/s* ...(Ans)

At joint - C

Rubbing velocity at C $= r_C \cdot (\omega_{CB} + \omega_{CD})$

$= 0.025 \times 5.8048$

$= 0.14512$ *m/s* ... (Ans)

DO D 100 ... (Ans)

Example: 3.7

The crank and connecting rod of a theoretical steam engine are 0.5m and 2m respectively. The crank makes 180 r.p.m in the clockwise direction. when it has turned 45° from the inner dead centre position, determine : (i) velocity of piston (ii) angular velocity of connecting rod (iii) position and linear velocity of any point on the connecting rod which has the least velocity relative to crank shaft. ***[AU, April/May 2015]***

☞ ***Solution:***

Given:

OA = 0.5*m*; AB = 2*m*; θ = 45°; N = 180 *r.p.m.*

Angular velocity of crank, $\omega_{AO} = \frac{2\pi N}{60} = \frac{2\pi(180)}{60} = 18.85\ rad/s$

Velocity of A with respect to O, $v_{AO} = v_A = 0.5 \times 18.85 = 9.4248\ m/s$

The space diagram is drawn to a suitable scale (1:20)

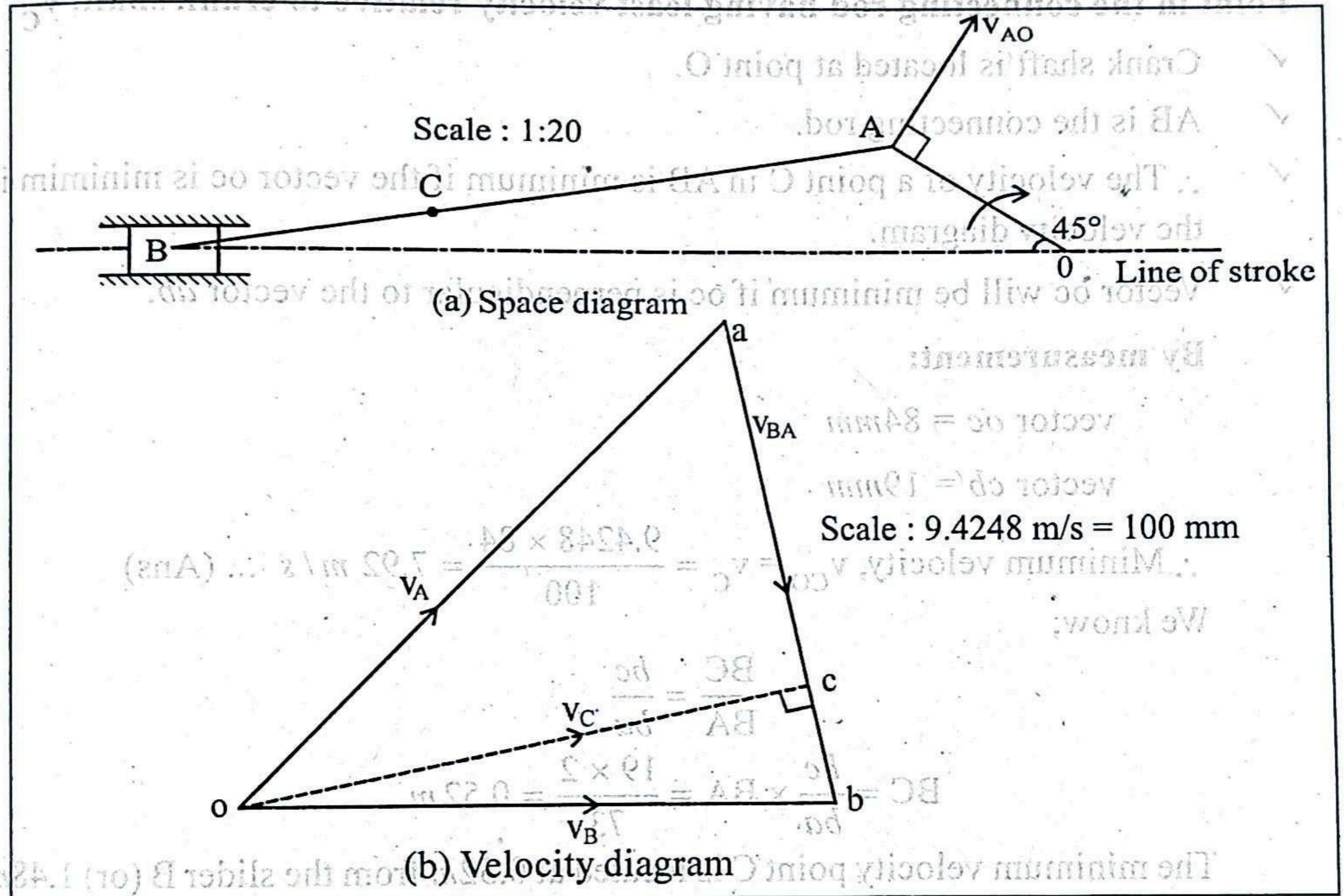


Figure 3.19

Velocity diagram

✓ The fixed point *o* is located arbitrarily in the space.

✓ From the point *o*; vector *oa* is drawn in the direction of v_{AO} perpendicular to the link OA.

vector $oa = v_{AO} = v_A = 9.4248\ m/s = 100mm$.

✓ From the point *a*; vector *ab* is drawn perpendicular to the link AB.

✓ From the point *o*; vector *ob* is drawn parallel to the line of stroke of the slider B.

vector *ab* and *ob* intersect at point *b*.

By measurement:

Vector *ob* = 86*mm*

Vector *ab* = 73*mm*

By scale conversion:

✓ Velocity of slider (piston), $v_{BO} = v_B = \frac{9.4248 \times 86}{100} = 8.11\ m/s$ ... (Ans)

✓ Velocity of connecting rod, $v_{BA} = \frac{9.4248 \times 73}{100} = 6.88\ m/s$

✓ Angular velocity of connecting rod, $\omega_{BA} = \frac{v_{BA}}{BA} = \frac{6.88}{2} = 3.44\ rad/s$ ... (Ans)

Point in the connecting rod having least velocity relative to crank shaft, v_C

✓ Crank shaft is located at point O.

✓ AB is the connecting rod.

✓ ∴ The velocity of a point C in AB is minimum if the vector oc is minimim in the velocity diagram.

✓ Vector oc will be minimum if oc is perpendicular to the vector *ab*.

By measurement:

vector $oc = 84mm$

vector $cb = 19mm$

∴ Minimum velocity, $v_{CO} = v_C = \frac{9.4248 \times 84}{100} = 7.92\ m/s$ ... (Ans)

We know;

$$\frac{BC}{BA} = \frac{bc}{ba}$$

$$BC = \frac{bc}{ba} \times BA = \frac{19 \times 2}{73} = 0.52\,m$$

The minimum velocity point C is located at $0.52m$ from the slider B (or) $1.48m$ from the crank pin A. ... (Ans)

What is a cam?

- A cam is a mechanical component that converts rotational motion into linear motion or vice versa. It is typically used in machinery to control the movement of a follower.

🎬 **Define the term 'follower' in cam mechanisms.**

- A follower is a component in a cam mechanism that moves in response to the cam's shape and motion. The follower translates the cam's rotational motion into linear or oscillatory motion.

🎬 **Differentiate between radial and cylindrical cams.**

- Radial cams have a profile in a plane perpendicular to the axis of rotation, causing the follower to move in a straight line. Cylindrical cams have a profile on the surface of a cylinder, causing the follower to move in an oscillating manner.

🎬 ***What is a displacement diagram in cam design?***

- *A displacement diagram is a graph that shows the relationship between the follower's displacement and the cam's rotation angle. It helps in designing the cam profile to achieve the desired follower motion.*

🎬 **Explain the term 'pressure angle' in cam mechanisms.**

- The pressure angle is the angle between the direction of the follower's motion and the normal to the cam profile. It affects the force transmitted through the cam-follower interface and should be minimized to reduce friction and wear.

🎬 **What is a dwell period in cam motion?**

- The dwell period is the portion of the cam's rotation during which the follower remains stationary. During this time, the cam profile is designed to have no effect on the follower's position.

🎬 **Differentiate between a translating follower and an oscillating follower.**

- A translating follower moves in a straight line as the cam rotates, while an oscillating follower pivots around a fixed axis, causing angular motion.

🎬 **Define base circle in cam terminology.**

- The base circle is the smallest circle that can be drawn tangentially to the cam profile. It serves as the reference circle for defining other cam dimensions, such as the cam profile and follower displacement.

🎬 **What is the purpose of a cam profile?**

- The cam profile is the specific shape or contour of the cam's surface that dictates the motion of the follower. The profile is designed to achieve the desired movement of the follower during the cam's rotation.

🎬 **Explain the term 'rise' in cam mechanisms.**

- The rise in a cam mechanism refers to the portion of the cam's rotation where the follower moves away from the base circle, increasing its displacement. It is also known as the 'lift' or 'ascend' phase.

🎬 ***What is the function of a cam-follower system?***

- *The cam-follower system is used to control the motion of machine components. It provides precise control over the timing, displacement, and velocity of the follower, which is essential in applications like internal combustion engines.*

Describe the term 'fall' in cam motion.

- The fall in cam motion refers to the phase where the follower moves towards the base circle, decreasing its displacement. This phase is also known as the 'descent' phase.

What is a uniform velocity cam profile?

- A uniform velocity cam profile is designed so that the follower moves at a constant velocity during the rise and fall phases. This profile is characterized by linear displacement over time.

Explain the significance of the camshaft in a cam mechanism.

- The camshaft is the rotating shaft to which the cam is attached. It drives the cam and controls the timing and sequence of the follower's motion in various mechanisms, such as engines.

What is a cycloidal motion in cam design?

- Cycloidal motion refers to the cam profile designed based on a cycloid curve, which provides a smooth and continuous motion of the follower, minimizing acceleration and deceleration stresses.

UNIT II

1.Draw and Explain the types of cam in detail and explain cam nomenclature.

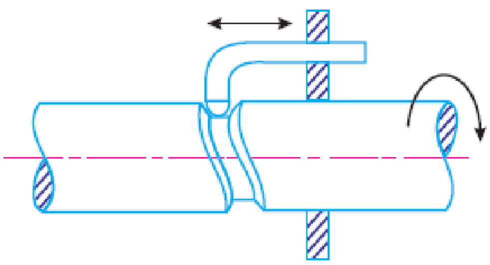
Classification of cams

a Radial or Disc cam

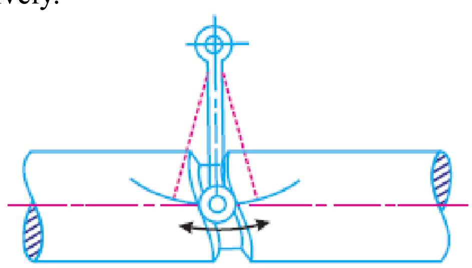
o In radial cams, the follower reciprocates or oscillates in a direction perpendicular to the cam axis. The cams as shown in Fig. 7.1 are all radial cams.

b Cylindrical cam

o In cylindrical cams, the follower reciprocates or oscillates in a direction parallel to the cam axis. The follower rides in a groove at its cylindrical surface. A cylindrical grooved cam with a reciprocating and an oscillating follower is shown in Fig. 7.2 (a) and (b)respectively.



(a) Cylindrical cam with reciprocating follower



(b) Cylindrical cam with oscillating follower

Terms used in radial cams

a Base circle

o It is the smallest circle that can be drawn to the camprofile.

b Trace point

o It is a reference point on the follower and is used to generate the pitch curve. In case of knife edge follower, the knife edge represents the trace point and the pitch curve corresponds to the cam profile. In a roller follower, the centre of the roller represents the tracepoint.

c Pressure angle

o It is the angle between the direction of the follower motion and a normal to the pitch curve. This angle is very important in designing a cam profile. If the pressure angle is too large, a reciprocating follower

Pitchpoint

o It is a point on the pitch curve having the maximum pressure angle.

e Pitchcircle

o It is a circle drawn from the centreof the cam through the pitch points.

f Pitchcurve

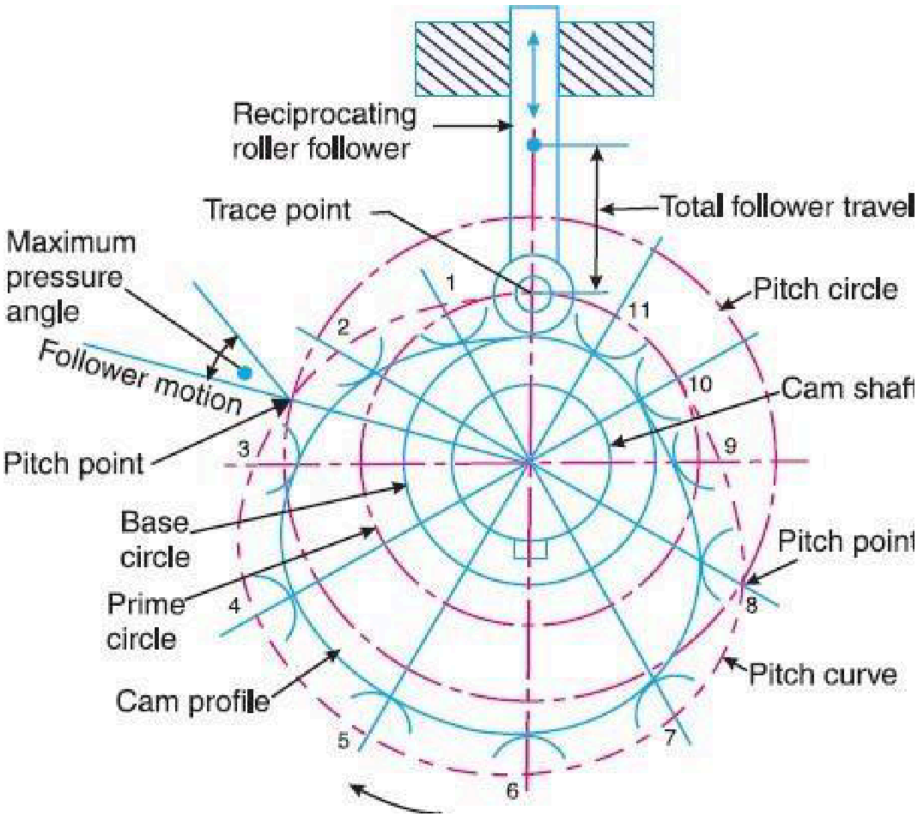
It is the curve generated by the trace point as the follower moves relative to the cam. For a knife edge follower, the pitch curve and the cam profile are same whereas for a roller follower, they are separated by the radius of the roller.

g Prime circle

o It is the smallest circle that can be drawn from the centre of the cam and tangent to the pitch curve. For a knife edge and a flat face follower, the prime circle and the base circle are identical. For a roller follower, the prime circle is larger than the base circle by the radius of the roller.

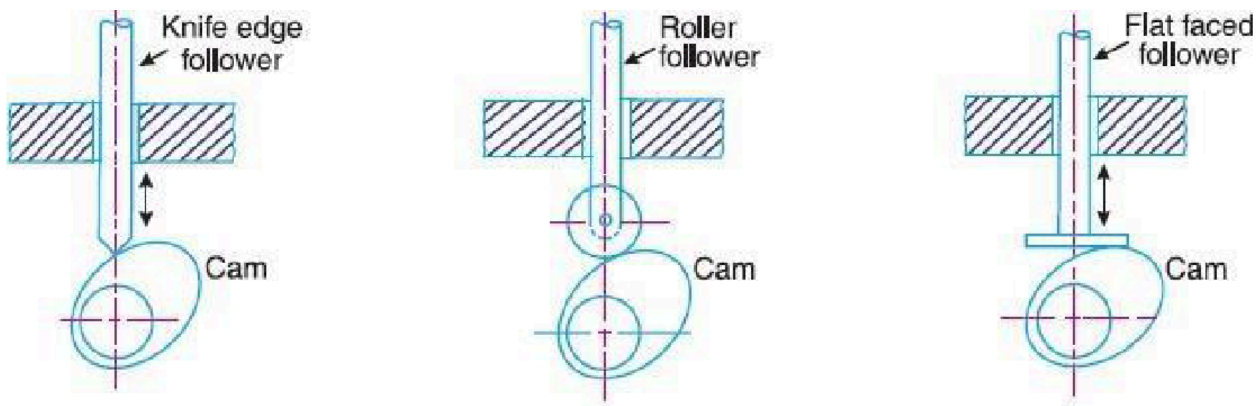
h Lift or Stroke

o It is the maximum travel of the follower from its lowest position to the top most position.

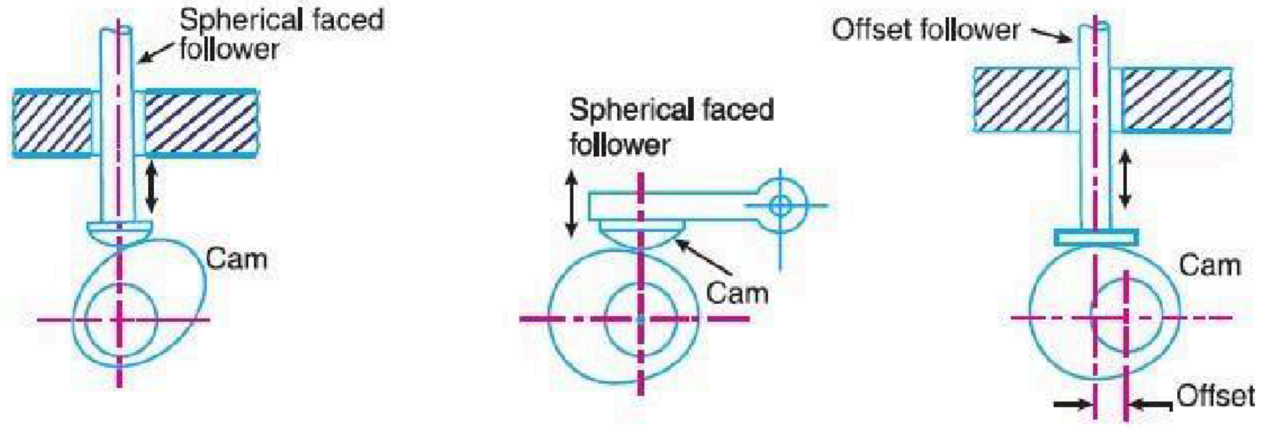


2. **Explain in detail about the types of followers.**

Classification of Followers :



(a) Cam with knife edge follower.   (b) Cam with roller follower.   (c) Cam with flat faced follower.



(d) Cam with spherical faced follower.   (e) Cam with spherical faced follower.   (f) Cam with offset follower.

According to surface in contact

A Knife edge follower

o When the contacting end of the follower has a sharp knife edge, it is called a knife edge follower, as shown in Fig. a

o The sliding motion takes place between the contacting surfaces (i.e. the knife edge and the cam surface). It is seldom used in practice because the small area of contacting surface results in excessive wear. In knife edge followers, a considerable side thrust exists between the follower and the guide.

b Roller follower

o When the contacting end of the follower is a roller, it is called a roller follower, as shown in Fig. (b). Since the rolling motion takes place between the contacting surfaces (i.e. the roller and thecam), therefore the rate of wear is greatly reduced.

o In roller followers also the side thrust exists between the follower and the guide. The roller followers are extensively used where more space is available such as in stationary gas and oil engines and aircraft engines.

c Flat faced or mush room follower

o When the contacting end of the follower is a perfectly flat face, it is called a flat-faced follower, as shown in Fig. 7.1 .It may be noted that the side thrust between the follower and the guide is much reduced in case of flat faced followers.

o The only side thrust is due to friction between the contact surfaces of the follower and the cam. The relative motion between these surfaces is largely of sliding nature but wear may be reduced by off-setting the axis of the follower, as shown in Fig. (f) so that when the cam rotates, the follower also rotates about its own axis.

o The flat faced followers are generally used where space is limited such as in cams which operate the valves of automobile engines.

d Spherical faced follower

o When the contacting end of the follower is of spherical shape, it is called a spherical faced follower, as shown in Fig. 7.1 (d). It may be noted that when a flat-faced follower is used in automobile engines, high surface stresses are produced. In order to minimize these stresses, the flat end of the follower is machined to a sphericalshape.

According to the motion of f ollower

a Reciprocating or TranslatingFollower

o When the follower reciprocates in guides as the cam rotates uniformly, it is known as reciprocating or translating follower. The followersas shown inFig. 7.1 (a) to (d) are all reciprocating or translating followers.

b Oscillating or RotatingFollower

o When the uniform rotary motion of the cam is converted into predetermined oscillatory motion of the follower,it is called oscillating or rotating follower. The follower, as shown in Fig 7.1 (e), is an oscillating or rotating

5. The corresponding points are marked as 1', 2'... 6' in the cam profile diagram also.
6. All the points 0 to 6' and 6' to 0 are connected using smooth curve.
7. The dwell portions can be connected by arcs taking O as centre.
8. The cam profile is drawn using dark line.

Example : 6.2

Draw the cam profile for the problem Example 6:1 if the follower is offset by 15 mm from the cam axis.

☞ ***Solution :***

✓ Knife - edge follower is offset by 15 *mm*.

✓ As the side of offset (LHS to cam axis or RHS to cam axis) is not given; the offset is taken at the RHS.

✓ [Since the displacement diagram is already available, no separate diagram is drawn for this problem]

✓ As the displacement diagram has been drawn with a scale of 1 : 1 (lift); the same is to be followed for the came profile also.

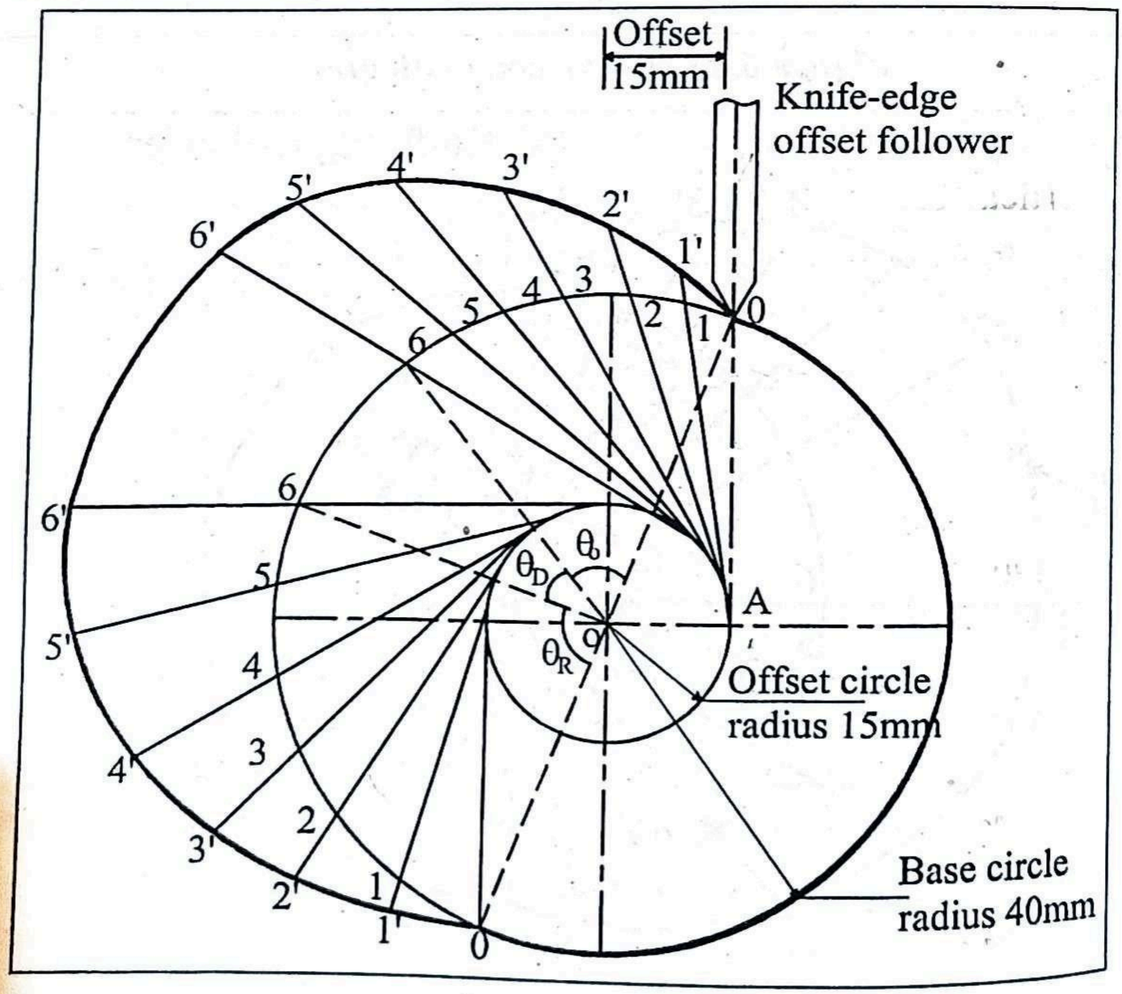


Figure 6.45

Cam profile - procedure :

1. The minimum cam circle is drawn (Radius 40 *mm*) taking any arbitary point O as the centre.
2. Taking O as centre, the offset circle is drawn with a radius of 15 *mm*.
3. As it is assumed that the offset of the follower is at the RHS; a vertical tangent is drawn from the right extreme point of the offset circle 'A' to meet the minimum circle at 0 (zero), the initial position of the follower.
4. The points 0 (zero) and circle centre O are joined. From the line 0O; angles θ_0, θ_D and θ_R are marked in anticlockwise direction.
5. The angles θ_0 and θ_R are divided into six equal parts (as in the displacement diagram) each and the points 0–6 and 6–0 are marked in the circumference of the base circle (minimum circle).

   No lines are to be drawn at this stage.
7. From these points ; tangents are drawn to the offset circle in the direction shown (initially the tangents will be terminating near the point A and the direction is maintained for all the points 1–6 and 6–0).
8. The vertical distances $1-1'$, $2-2'$, $3-3'$, $4-4'$, $5-5'$ and $6-6'$ are measured from the displacement diagram and marked in the extented tangents passing through the points 1–6 and 6–1.
9. The corresponding points are marked as $1'$, $2'$, ... $6'$ in the cam profile diagram.
10. All the points 0 to $6'$ and $6'$ to 0 are joined using a smooth curve.
11. The dwell portions can be connected by arcs taking O as the centre.
12. The cam profile is drawn using dark line.

Example : 6.3

Draw the profile of a cam rotating clockwise to raise a valve with uniform acceleration and retardation through 40 mm in 90°, keep it fully raised through 30° and to lower it with the same type of motion in 120°. The valve remains closed during the rest of the revolution. The minimum radius is 30 mm and the roller follower has a radius of 10 mm. The axis of the valve rod passes through the axis of the cam shaft.

☞ ***Solution :***

Given : $s = 40\ mm$; minimum cam radius = 30 *mm*; roller radius = 10 *mm*;

sequence : $\theta_0 = 90° \Rightarrow \theta_D = 30° \Rightarrow \theta_R = 120° \Rightarrow \theta_D = (360-240) = 120°$; uniform acceleration and retardation (parabolic) motion; inline roller follower.

Scale : For the lift in the displacement diagram and cam profile; 1 : 1 (1 *mm* = 1 *mm*)

Scale : For the cam angle; $2° = 1\ mm$.

The displacement diagram is drawn as discussed earlier.

θ_0 and θ_R are divided into 6 equal parts.

θ_D need not be divided.

Cam profile - procedure :

1. The minimum cam circle (base circle) with a radius of 30 *mm* and the prime circle with a radius of (radius of base circle + radius of cam) = 30+10 = 40 *mm* are drawn by taking a suitable arbitrary point O as the centre.
2. The initial position of the centre of the roller follower is fixed above the centre line of the cam and marked as 0 (zero).
3. The angles $\theta_0 = 90°$ followed by $\theta_D = 30°$ and $\theta_R = 120°$ are marked in the anticlokwise direction (cam rotates in the clockwise direction).

   θ_0 and θ_R are divided into six number of equal parts as in the displacement diagram.

   In the circumference of the prime circle, the divisions are numbered from 0–6 for θ_0 and 6–0 for θ_R.
4. The vertical distances 1–1', 2–2', 3–3', 4–4', 5–5' and 6–6' are measured from the displacement diagram and marked in the extended lines of O–1, O–2, O–3, O–4, O–5 and 0–6 respectively.
5. Taking these points (1', 2'... 6') as centres, circles of radius 10 mm are drawn representing the different roller follower positions.
6. Now, all the bottom of the circles are joined by a smooth curve. The curve will touch each circle at a single point only (ie, tangent to the circle).
7. The dwell portions can be connected by arcs taking O as centre.
8. The cam profile is drawn using dark line.

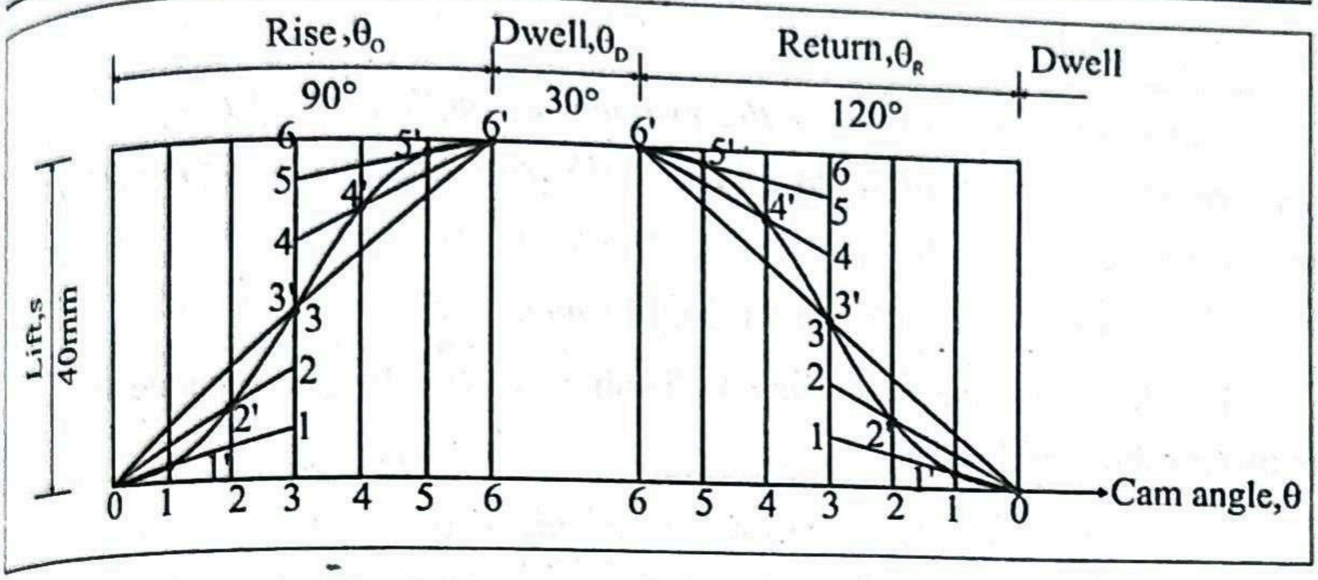


Figure 6.46 Displacement diagram

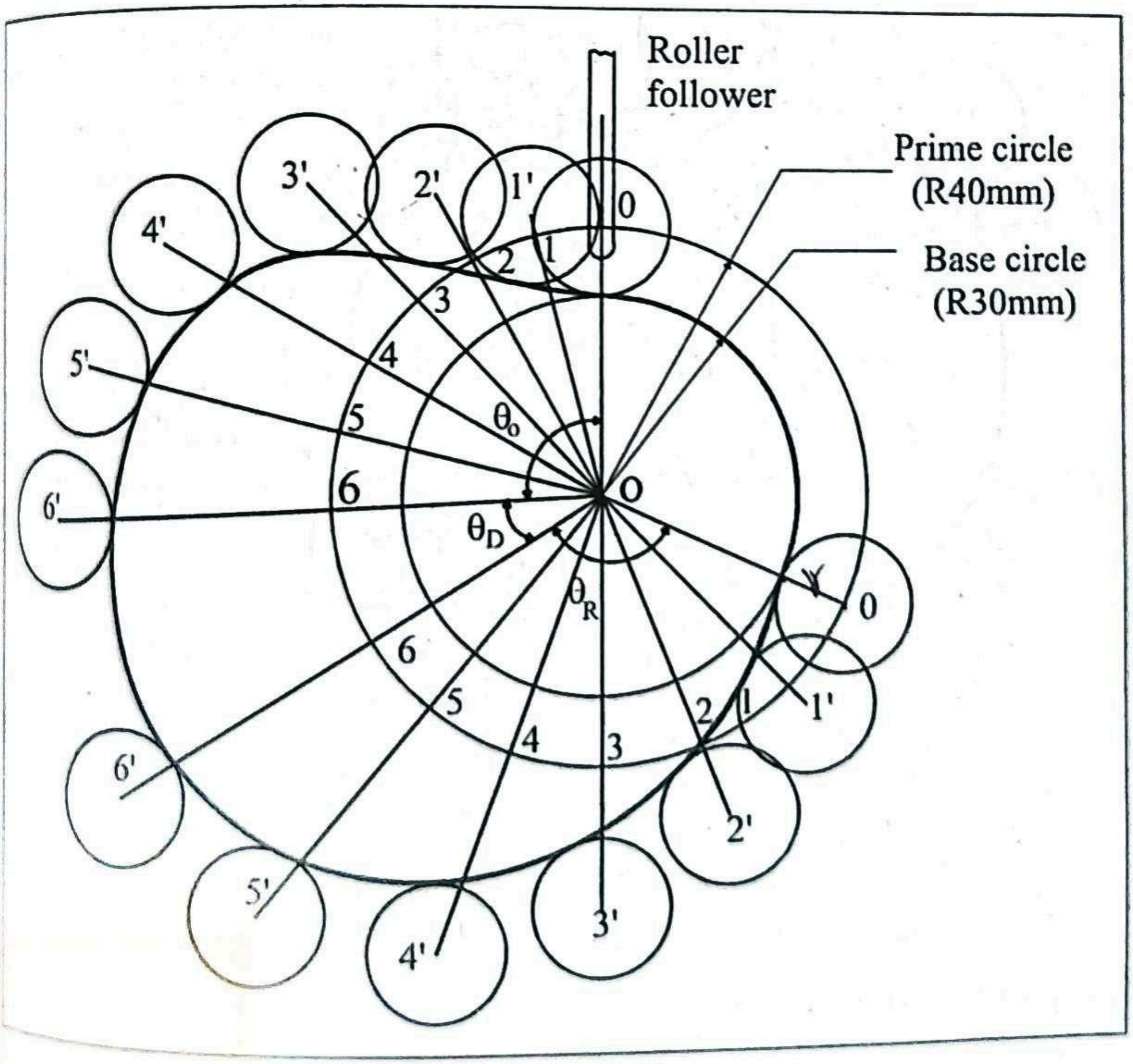


Figure 6.47 Cam profile

Example : 6.4

Draw the cam profile for the problem example 6.3 : if the centre of the roller follower is offset by 15 mm RHS from the axis of the cam shaft.

☞ ***Solution :***

Roller follower is offset at RHS by 15 *mm*.

[Since the displacement diagram is already available, no separate diagram is drawn for this problem]

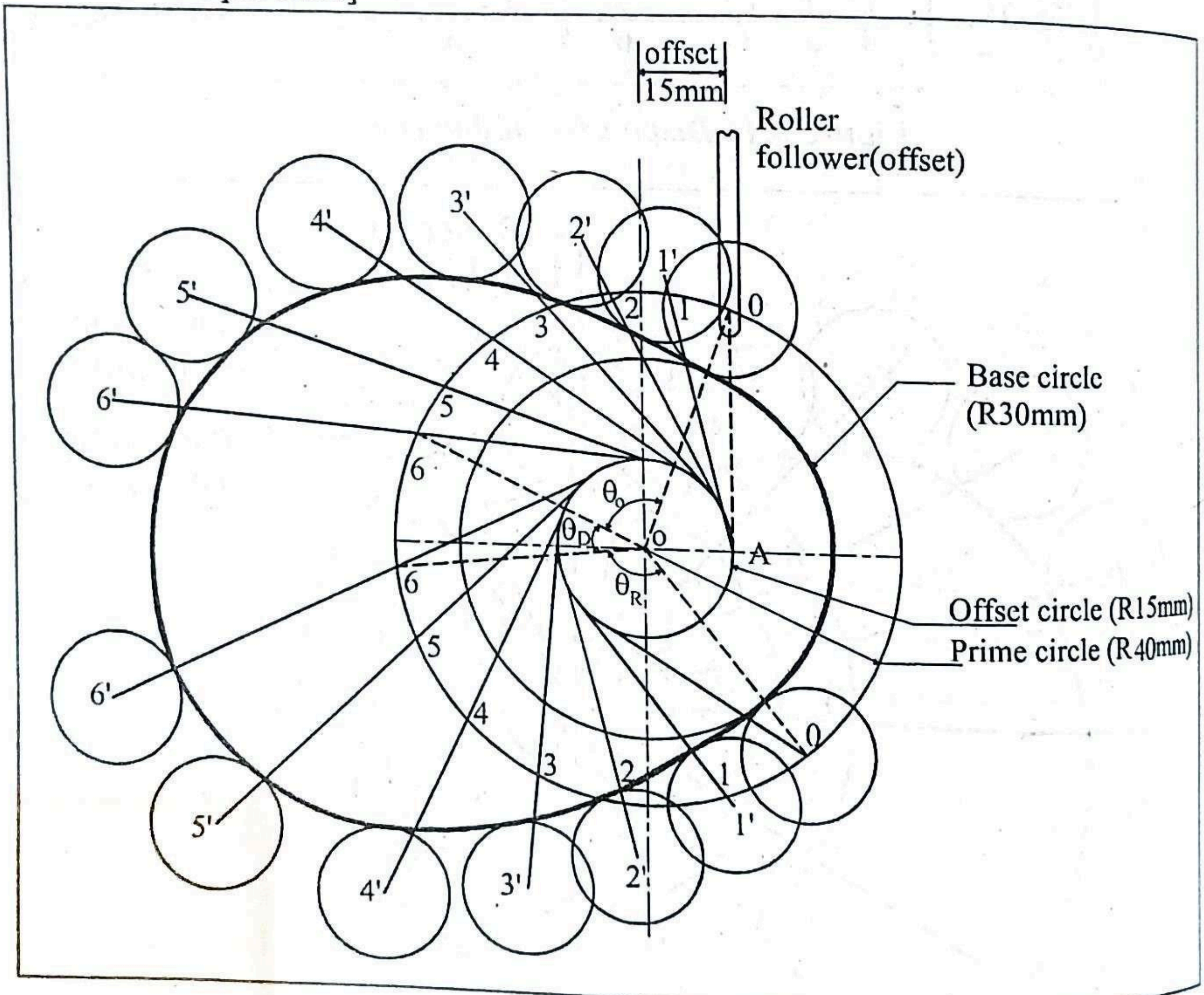


Figure 6.48

As the displacement diagram has been drawn (lift) with a scale of 1 : 1; the same is to be followed for the cam profile also.

Cam profile - procedure :

1. The minimum cam circle (R 30 *mm*), prime circle (R 40 *mm*) and offset circle (R 15 *mm*) are drawn taking an arbitrary point O as the centre.

2. As the follower offset is 15 *mm* at RHS, a vertical tangent is drawn from the right extreme point of the offset circle 'A' to meet the prime circle at 0 (zero), the initial position of the roller centre.
3. The points 0 (zero) and centre of the circle O are joined. From the line 0O; angles θ_0, θ_D and θ_R are marked in anticlockwise direction.
4. The angles θ_0 (90°) and θ_R (120°) are divided into six equal parts (as in the displacement diagram) each and the points 0–6 and 6–0 are marked in the circumference of the prime circle.

   No lines are to be drawn at this stage.
5. From these points; tangents are drawn to the offset circle, in the direction shown, initially the tangents will be terminating near the point A and the direction is maintained for all the points 1–6 and 6–0.
6. The vertical distances 1–1', 2–2', 3–3', 4–4', 5–5' and 6–6' are measured from the displacement diagram and marked in the extended tangents passing through the points 1–6 and 6–1.
7. The corresponding points are marked as 1', 2'... 6' in the cam profile diagram.
8. Taking these points as centres, circles of roller radius (R 10 *mm*) are drawn.
9. The bottom of all the circles (representing different positions of the roller follower) are smoothly connected by a curve.

   The curve will touch each circle at a single point only (ie, tangent to the circle).
10. The dwell portions can be connected by arcs taking O as centre.
11. The cam profile is drawn using dark line.

Example : 6.6

A cam operates a flat faced follower which moves with cycloidal motion during rise and fall. The minimum radius of cam = 30 mm; Lift of follower = 40 mm; angle of rise = 120°; angle of dwell = 60°; angle of descent = 90°. Draw the cam profile if the cam rotates in anticlockwise direction.

Given : minimum cam radius, $R_B = 30\ mm$; Lift, $s = 40\ mm$; cycloidal motion;

sequence : $\theta_0 = 120° \Rightarrow \theta_D = 60° \Rightarrow \theta_R = 90° \Rightarrow \theta_D$ = remaining; cam rotation - anticlockwise;

☞ ***Solution :***

Radius of circle : $2\pi R = 40\ mm,\ R = \dfrac{40}{2\pi} = 6.37\ mm$

Scale : For lift in the displacement diagram and cam profile; 1 : 1 (1 mm = 1 mm).

Scale : For the cam angle; $2° = 1\ mm$.

The displacement diagram is drawn as discussed earlier.

θ_0 and θ_R are divided into six equal parts.

θ_D need not be divided.

Cam rotates anticlockwise. ∴ The follower will be moved in clockwise direction, assuming that the cam is stationary.

Cam profile - procedure :

1. The minimum cam circle is drawn with radius 30 mm and any arbitrary point O as centre.
2. The initial position of the flat follower is fixed above the centre line of the cam as shown, and marked as 0 (zero).
3. As the cam rotates anticlockwise, $\theta_0 = 120°$ followed by $\theta_D = 60°$ and $\theta_R = 90°$ are marked in the clockwise direction.

   θ_0 and θ_R are divided into six equal parts and numbered from 0–6 for θ_0 and from 6–0 for θ_R.
4. The vertical distances 1–1', 2–2', 3–3', 4–4', 5–5' and 6–6' are measured from the displacement diagram and marked in the cam profile in the extended lines of O–1, O–2, O–3, O–4, O–5 and O–6 respectively.

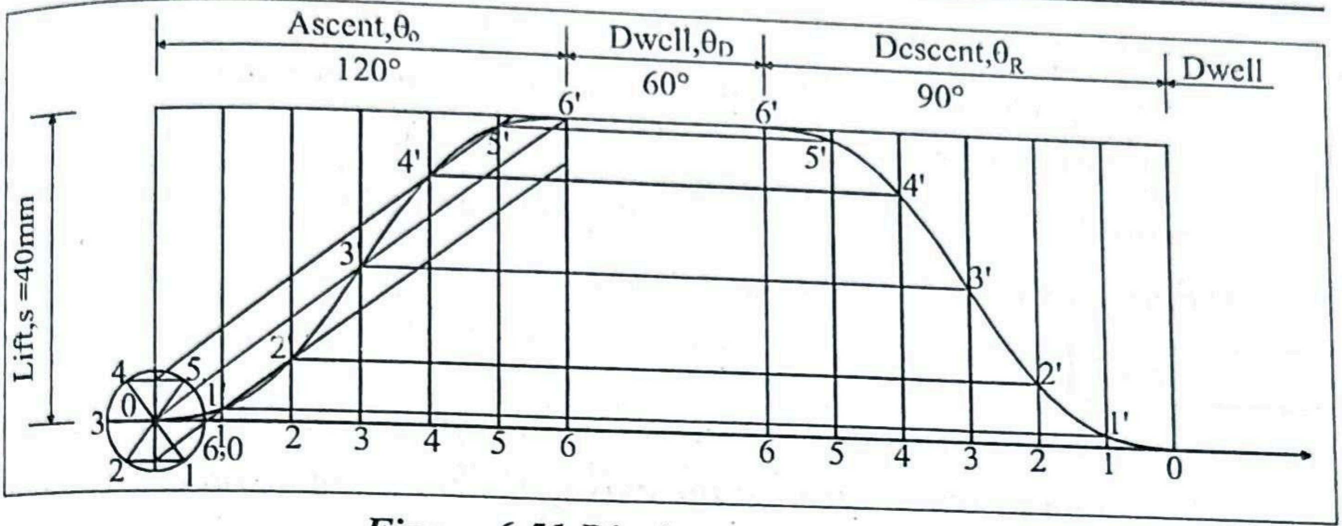


Figure 6.51 Displacement diagram

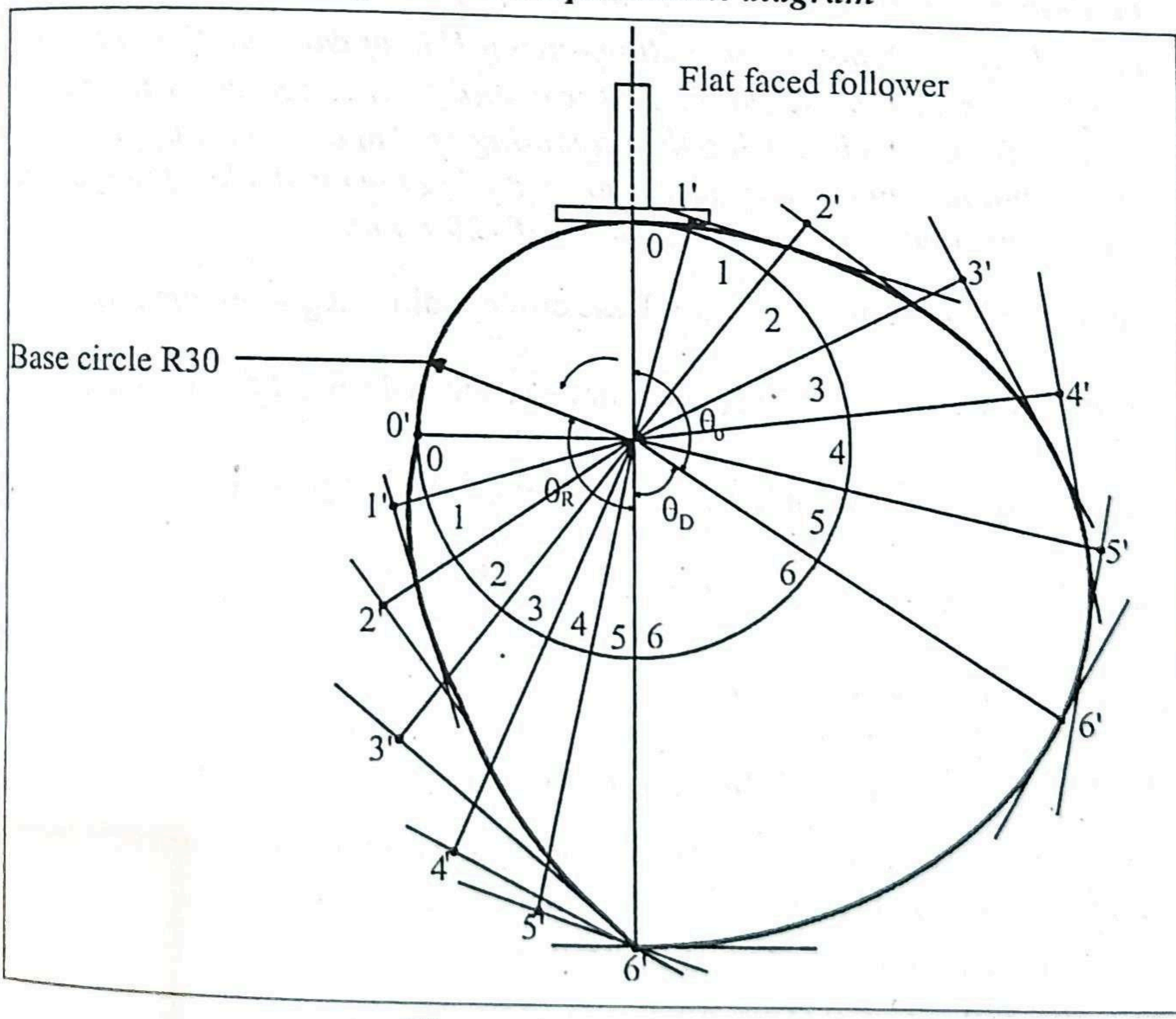


Figure 6.52 Cam profile

5. These points, 1'–6' and 6'–1' are the centres of the bottom face of the flat follower. As the width of the flat follower is not given; tangents are drawn through these points (extending both the sides liberally).

6. Now, the bottoms of the flat follower are connected by a smooth curve. Essentially, the curve will touch all the tangents only once and will not cross any of the tangents.
7. The dwell portion can be connected by an arc taking O as centre.
8. The cam profile is drawn using dark line.

Example : 6.9

Following data is related to a cam profile, in which the follower moves with SHM during the lift and returning it with uniform acceleration and deceleration, acceleration being half the deceleration.

Minimum radius of cam = 30 mm; Roller radius = 10 mm; Lift of follower = 45 mm; offset of follower axis = 12 mm; Angle of ascent = 70°; angle of descent = 120°; Angle of dwell between ascent and descent = 45°; speed of cam = 300 r.p.m; Draw the cam profile and determine maximum velocity and maximum acceleration during lift.

☞ ***Solution :***

Given : SHM during ascent; uniform acceleration and deceleration during descent; minimum cam radius, $R_B = 30\ mm$; roller radius, $r = 10\ mm$; Lift, $s = 45\ mm$; offset $= 12\ mm$; sequence : $\theta_0 = 70° \Rightarrow \theta_D = 45° \Rightarrow \theta_{R(a)} = \dfrac{\theta_{R(d)}}{2} \Rightarrow \theta_{R(d)}$; $\theta_R = 120°$;

$$N = 300 \text{ r.p.m} \Rightarrow \omega = \frac{2\pi(300)}{60} = 31.4159\ rad/s$$

Given : $\theta_{R(a)} = \dfrac{\theta_{R(d)}}{2}$ and $\theta_{R(a)} + \theta_{R(d)} = \theta_R = 120°$

$$\therefore \quad \frac{\theta_{R(d)}}{2} + \theta_{R(d)} = 120° \text{ and } \theta_{R(d)} = 80°;$$

$$\theta_{R(a)} = \frac{80}{2} = 40°.$$

Similarly; $s_a + s_d = 45\ mm$ and $s_a = \dfrac{s_d}{2}$

$$\therefore \quad \frac{s_d}{2} + s_d = 45\ mm \text{ and } s_d = 30\ mm;$$

$$s_a = \frac{30}{2} = 15\ mm$$

Displacement diagram :

✓ The outstroke portion of the follower motion with SHM is drawn as usual.

✓ The return stroke takes place with uniform acceleration and deceleration motion, but acceleration period is half of deceleration.

✓ ∴ In the displacement diagram;

The vertical lift of 45 mm is divided at the line no. 4 after two parts ($\because \theta_{R(a)} = 40°$, six parts represent 120° and 40° will be represented by 2 parts).

✓ Now, the displacement diagram can be completed as shown.

✓ Scale for the lift : 1 : 1

✓ Any suitable scale can be assigned for the cam angle θ.

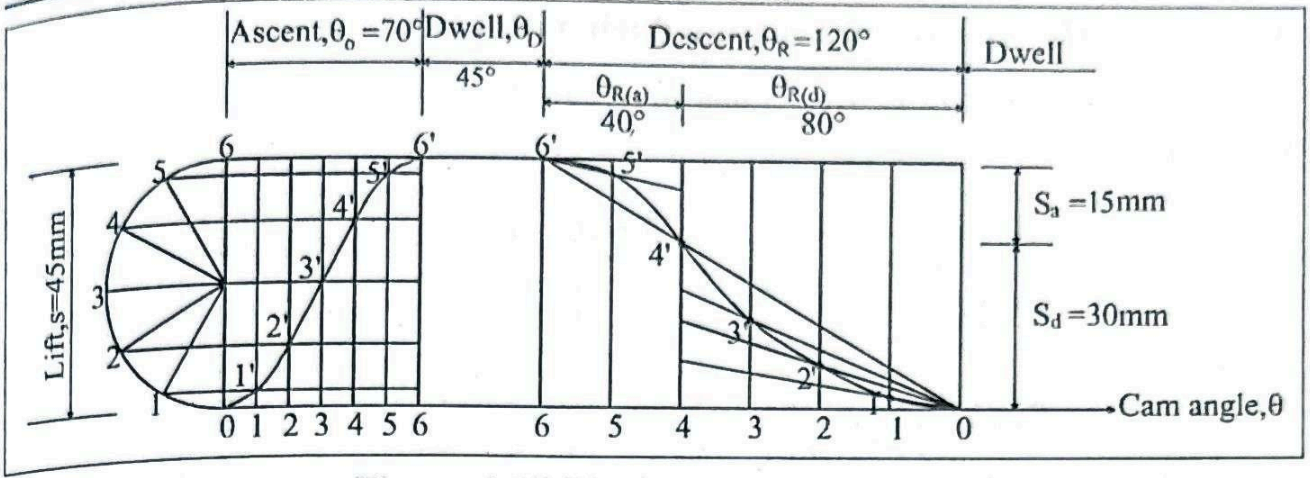


Figure 6.57 Displacement diagram

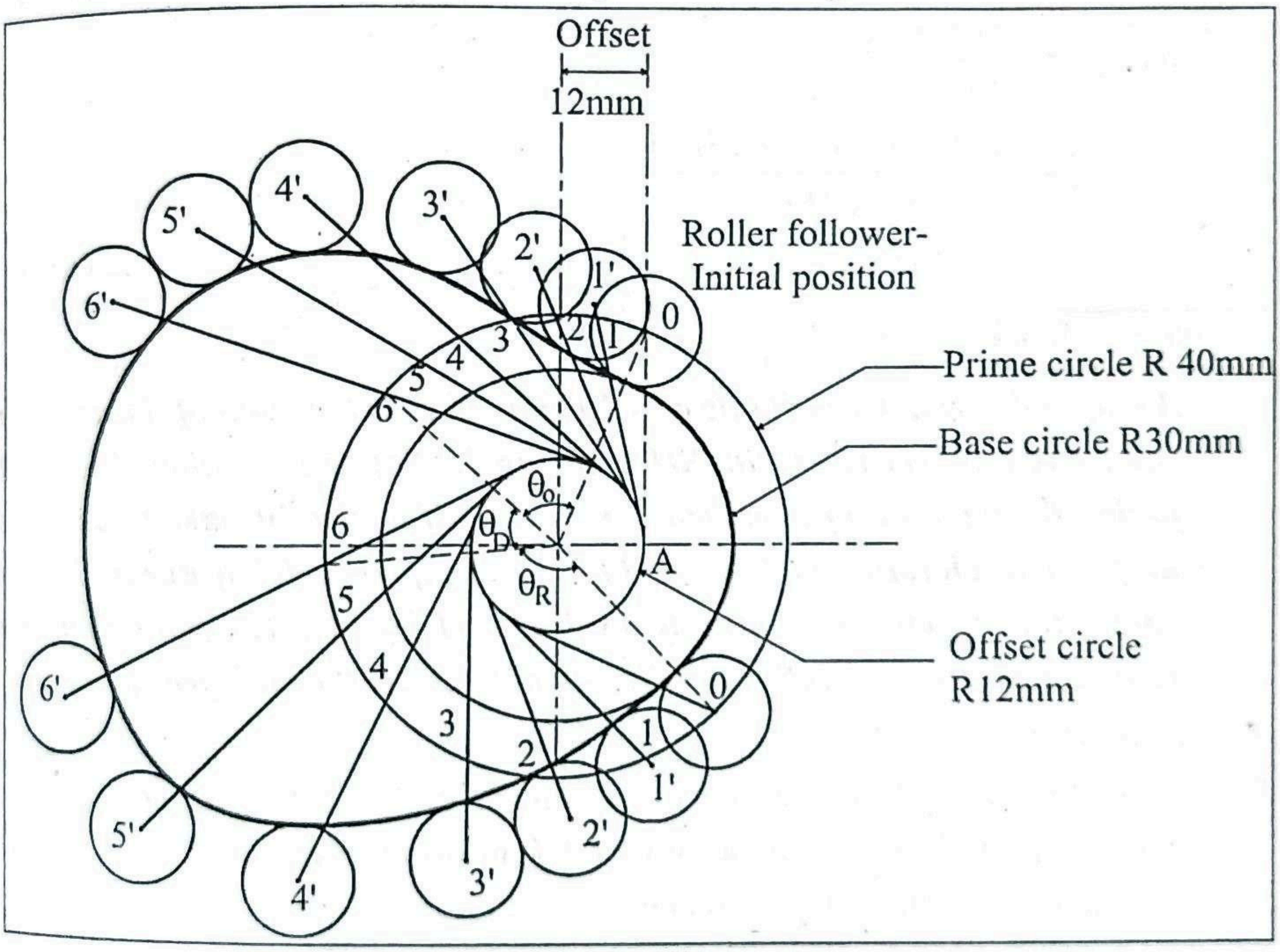


Figure 6.58 Cam profile

Cam profile :

✓ The cam profile is completed as discussed earlier, with a scale of 1 : 1.

✓ Care has to be taken while plotting the vertical distances 1 – 1' to 6 – 6'.

Outstroke (θ_0) portion : 1 – 1' to 6 – 6' are to be measured from the displacement diagram in the SHM region.

Return stroke (θ_R) portion : 6 – 6' to 1 – 1' are to be measured from the second half of the displacement diagram, the uniform acceleration and deceleration region.

Maximum velocity and acceleration during lift :

$\omega = 31.4159\ rad/s;\ s = 45\ mm = 45 \times 10^{-3}\ m$

For SHM;

$$\theta_0 = 70^\circ = \frac{\pi}{180} \times 70^\circ = 1.22173\ rad.$$

$$v_{0(max)} = \frac{\pi \omega s}{2\theta_0}$$

$$= \frac{\pi\ (31.4159)\ (45 \times 10^{-3})}{2 \times 1.22173}$$

$$= 1.8176\ m/s \qquad \text{... (Ans)}$$

$$a_{0(max)} = \frac{\pi^2 \omega^2 s}{2\theta_0^{\ 2}}$$

$$= \frac{\pi^2 (31.4159)^2\ (45 \times 10^{-3})}{2 \times (1.22173)^2}$$

$$= 146.8354\ m/s^2 \qquad \text{... (Ans)}$$

Example : 6.10

A knife-edge follower with its axis 10 mm offset to the right of the axis of a cam. The follower rises with SHM during 120° of cam rotation. The dwell period for next 30° of cam rotation. First half of the fall takes place with uniform acceleration and second half with uniform velocity during 120° of cam rotation. The remaining is dwell period for 90°. The minimum cam radius is 30 mm and follower lift is 40 mm. Draw the cam profile.

☞ ***Solution :***

Given : Knife-edge follower; 10 *mm* offset right side; $\theta_0 = 120^\circ$ (SHM); $\theta_D = 30^\circ$; $\theta_R = 120^\circ = 60^\circ$ (uniform acceleration) + 60° (uniform velocity); $\theta_D = 90^\circ$; s = 40 *mm*; minimum cam radius, $R_B = 30\ mm$.

Displacement diagram :

✓ Scale : For the lift; 1 : 1

For the cam angle; $2^\circ = 1\ mm$

✓ The displacement diagram is drawn as discussed.

✓ The rise (out stroke, θ_0) takes place with SHM.

✓ The fall (return stroke, θ_R) takes place initially with uniform acceleration for 60° and the remaining fall with uniform velocity for 60°.

Cam profile :

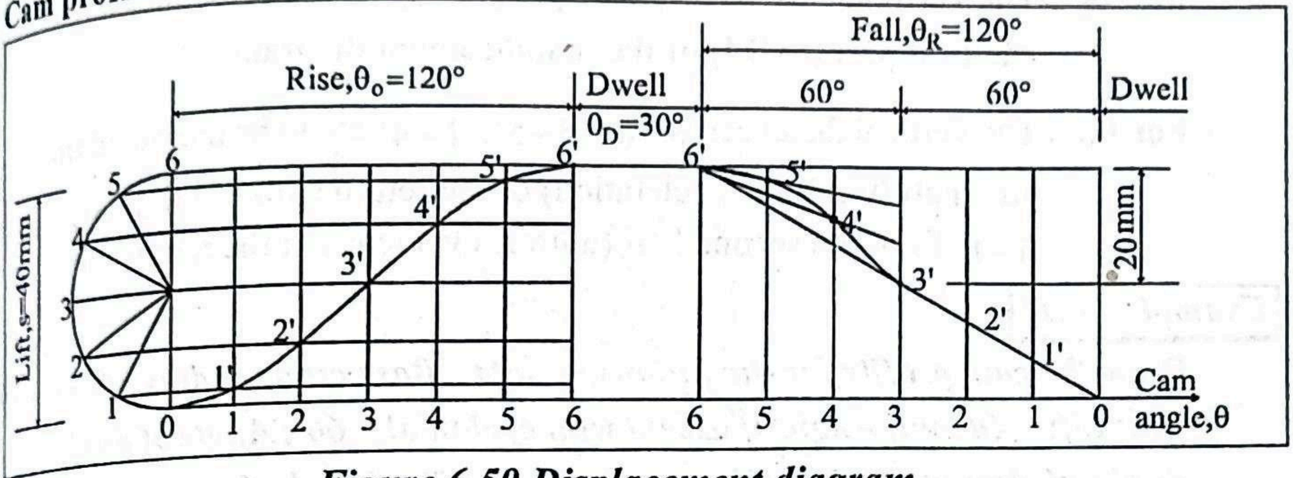


Figure 6.59 Displacement diagram

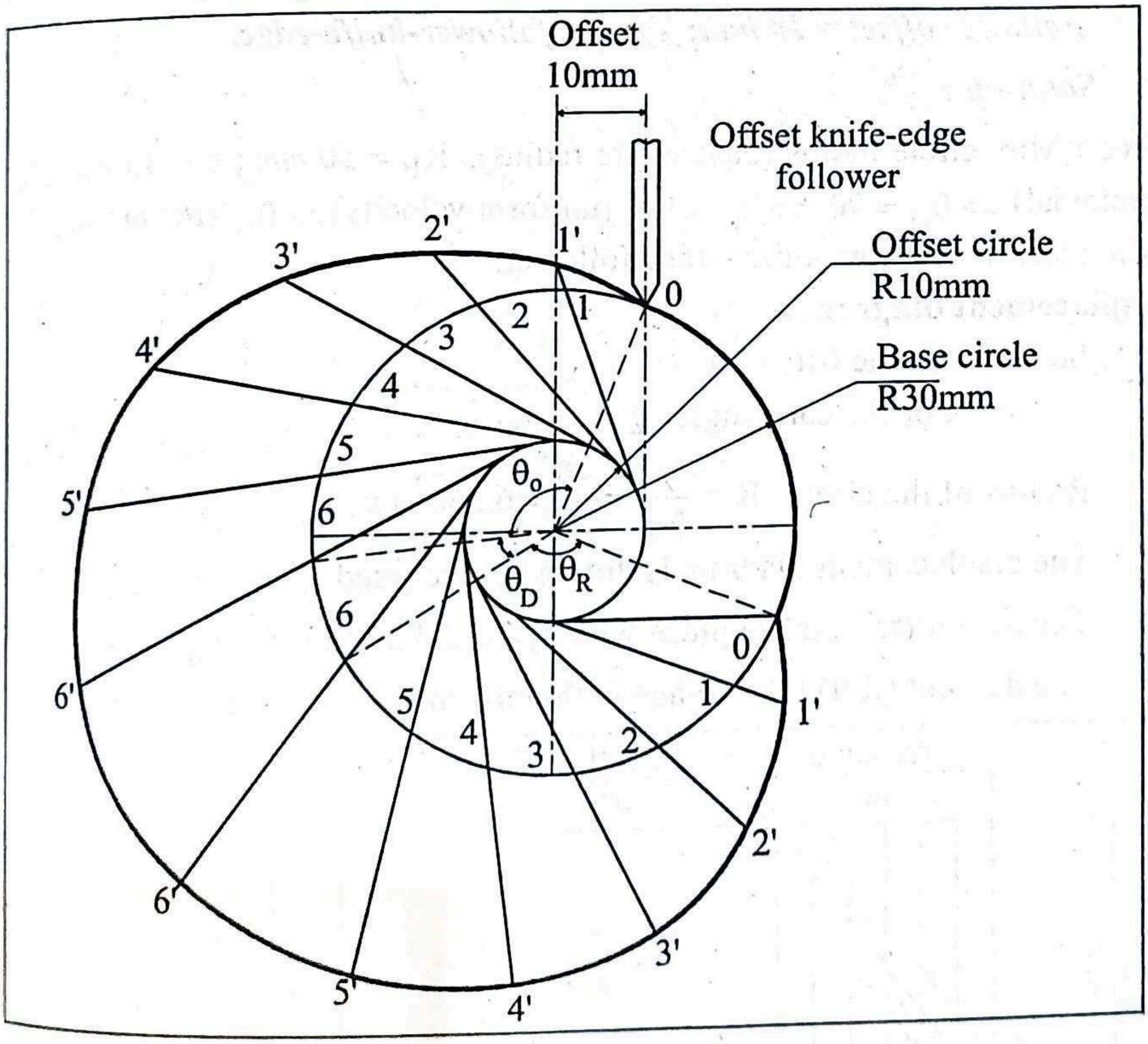


Figure 6.60 Cam profile

Scale : 1 : 1 since for the lift the scale is 1 : 1

Offset of the knife edge follower is 10 *mm* RHS (given).

The cam rotation is assumed to be in clockwise direction and hence the follower positions will be drawn in anticlockwise direction.

For θ_0 : The vertical distances $1-1'$, $2-2' \cdots 6-6'$ are to be measured from the first part (SHM) of the displacement diagram.

For θ_R : The vertical distances $6-6'$, $5-5'$, $4-4'$ are to be measured from the first half (uniform acceleration) of the return stroke and $3-3'$, $2-2'$, $1-1'$ from the second half (uniform velocity) of the return stroke.

Example : 6.11

Draw the cam profile for the following data : Base circle radius of cam = 50 mm; Lift = 40 mm; Angle of ascent with cycloidal = 60°; Angle of dwell = 90°; Angle of descent with uniform velocity = 90°; Speed of cam = 300 r.p.m. Follower offset = 10 mm; Type of follower-knife-edge.

☞ ***Solution :***

Give : Min. circle radius (base circle radius), $R_B = 50\ mm$; s = 40 *mm*; $\theta_0 = 60°$ (cycloidal) $\Rightarrow \theta_D = 90° \Rightarrow \theta_R = 90°$ (uniform velocity) $\Rightarrow \theta_D{}'$ (remaining); N = 300 *r.p.m.*; offset = 10 *mm*; knife - edge follower.

Displacement diagram :

Scale : for the lift; 1 : 1

For the cam angle; $2° = 1\ mm$

Radius of the circle, $R = \dfrac{s}{2\pi} = \dfrac{40}{2\pi} = 6.366\ mm$.

The displacement diagram is drawn as discussed.

The ascent (rise) takes place with cycloidal motion for $\theta_0 = 60°$.

The descent (fall) takes place with uniform velocity for $\theta_R = 90°$

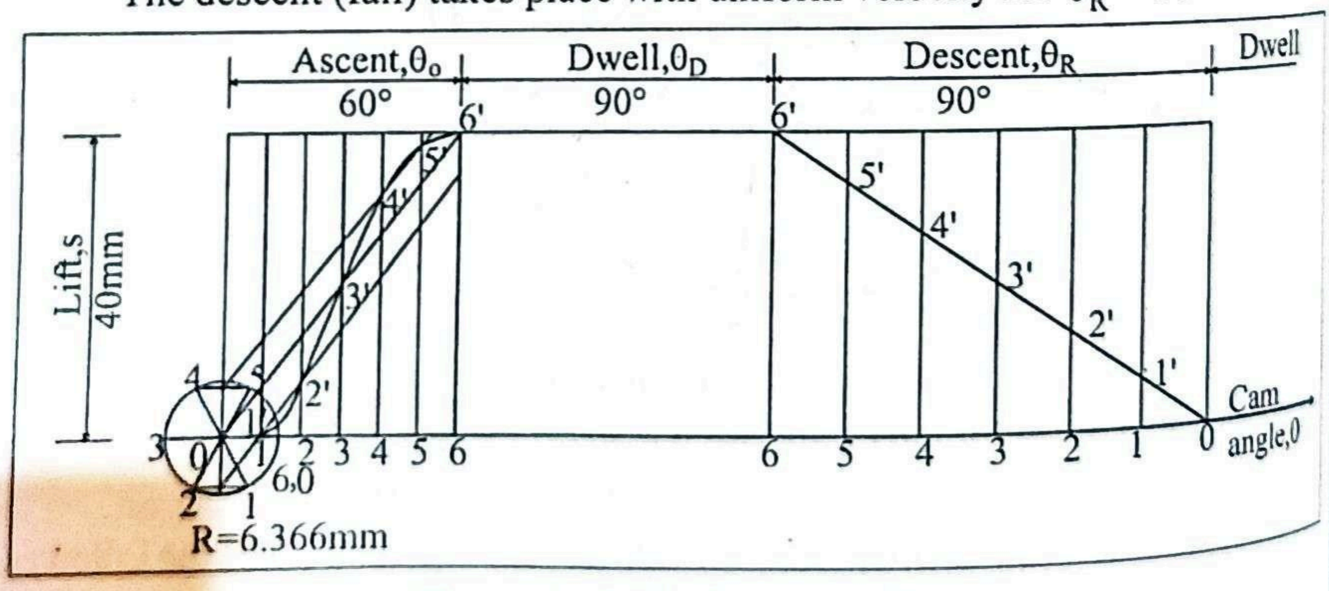


Figure 6.61 Displacement diagram

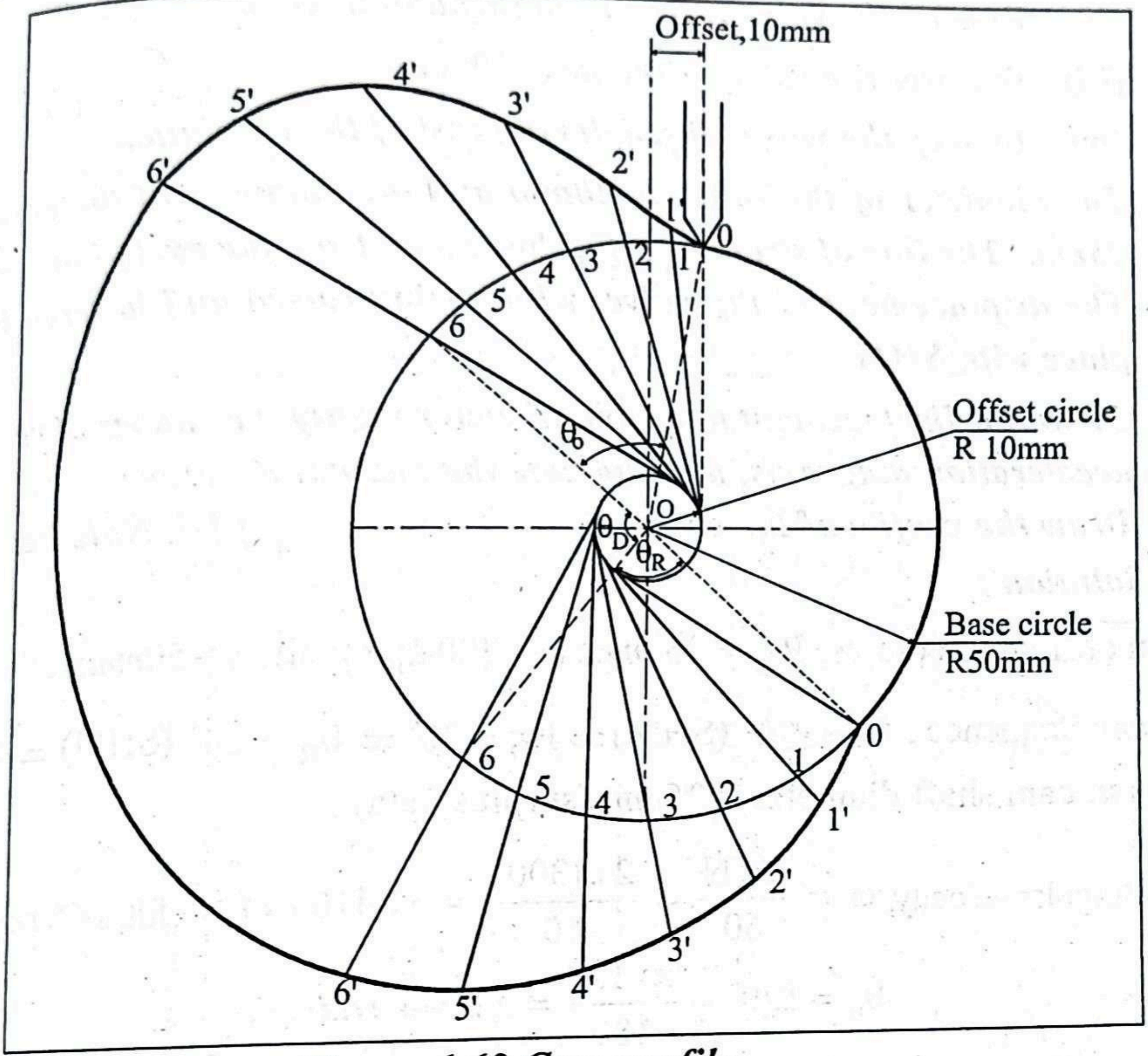


Figure 6.62 Cam profile

Cam profile :

Scale : 1 : 1 since for the lift the scale is 1 : 1.

Offset of the knife-edge follower = 10 *mm* (assumed in RHS).

Cam is assumed to be rotating in clockwise direction.

Hence, the follower positions are drawn in anticlockwise direction.

For θ_0 : Vertical distances $1-1'$, $2-2'$...$6-6'$ are to be measured from the first part (cycloidal) of the displacement diagram.

For θ_R : Vertical distances $6-6'$, $5-5'$...$1-1'$ are to be measured from the second part (uniform velocity) of the displacement diagram.

Example : 6.15

A roller follower cam with roller diameter 10 mm is rotating clockwise. The lift of the cam is 30 mm and the follower completes the lift with SHM during 120° of cam rotation. The dwell at lift is 30° of cam rotation. First half of the fall takes place with constant acceleration and retardation and the second half with constant velocity during 120° of the cam rotation. The rest is the dwell at fall. Draw the cam profile, taking the base radius as 25 mm.

☞ ***Solution :***

Given : roller diameter = 10*mm*; lift, s = 30*mm*; sequence : $\theta_0 = 120^\circ$ (SHM) $\rightarrow \theta_D = 30^\circ \rightarrow \theta_R = 120^\circ$ (parabolic-60°; constant velocity - 60°) $\rightarrow \theta_D$; minimum radius, $R_B = 25\ mm$.

Displacement diagram : (scale 1:1 for the lift; 2° = 1mm for the cam angle)

Outstroke (θ_0) : $\theta_0 = 120^\circ$ with SHM.

This portion of the diagram can be drawn as usual.

120° can be divided into 6 divisions of 20° and also the semi circle into six parts of 30° each.

Return stroke (θ_R): $\theta_R = 120^\circ$: 60° with parabolic followed by uniform velocity for the remaining 60°.

As usual we can not divide this 120° into 6 parts.

If we do so, 3 parts will be for parabolic and 3 parts will be for uniform velocity.

We know that, for constructing a parabolic curve we require even number of divisions like 4, 6, 8...

(For uniform velocity, the number of divisions is immaterial)

∴ $\theta_R = 120^\circ$ will be divided into 8 number of parts in this case.

Cam profile (scale 1:1) :

✓ Cam is rotating in clockwise direction (given).

✓ ∴ The follower positions will be drawn in anticlockwise direction.

✓ Outstroke, $\theta_0 = 120^\circ$ and divided into six equal parts as done in displacement diagram.

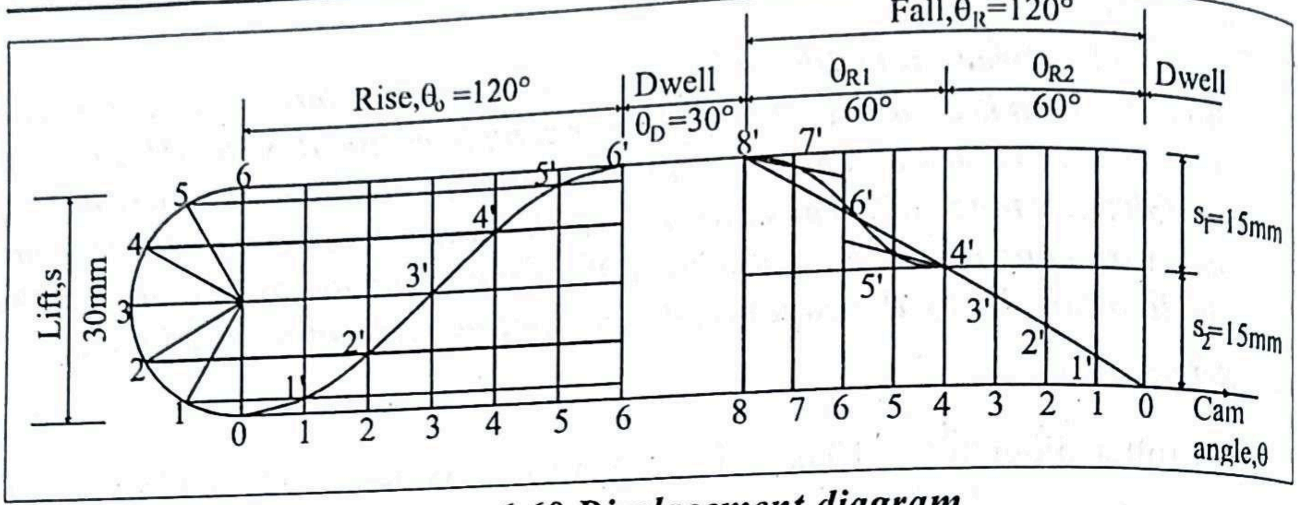


Figure 6.69 Displacement diagram

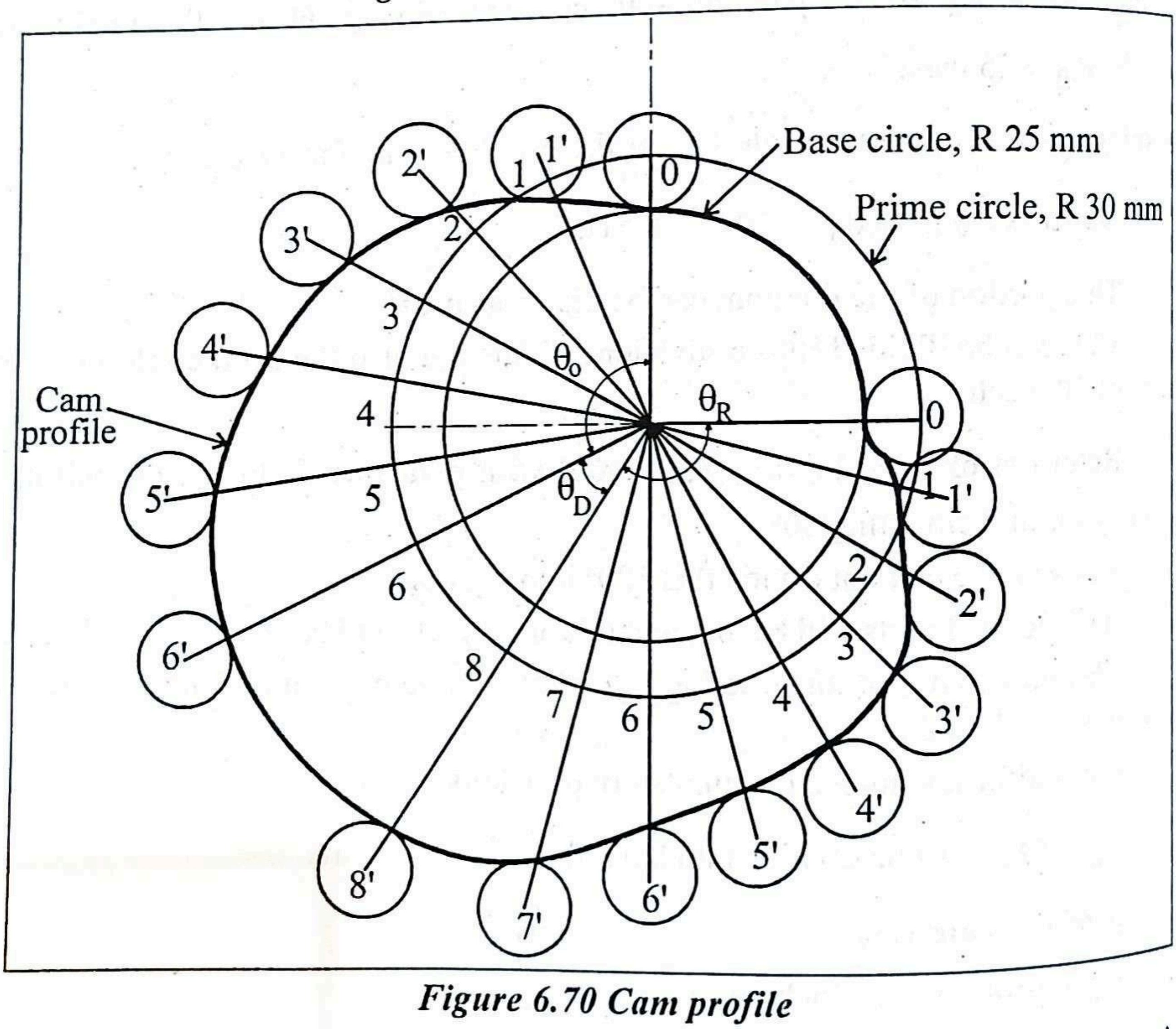


Figure 6.70 Cam profile

✓ Return stroke, θ_R =120° and divided into eight equal parts as done in displacement diagram.

✓ The respective vertical distances are measured from the displacement diagram and marked from the prime circle as shown.

✓ The cam profile can now be completed.

UNIT 3: FLYWHEELS AND BALANCING OF MASSES

1. **What is the primary function of a flywheel?**
   - A flywheel is used to store rotational energy and maintain a consistent speed in machines by smoothing out the fluctuations in the energy supply or load.
2. **How is the energy stored in a flywheel?**
   - The energy stored in a flywheel is given by the formula E=12Iω2E = \frac{1}{2} I \omega^2E=21Iω2, where III is the moment of inertia of the flywheel and ω\omegaω is its angular velocity.
3. **Define the term 'coefficient of fluctuation of speed'.**
   - The coefficient of fluctuation of speed is the ratio of the difference between maximum and minimum speeds to the mean speed of the flywheel. It indicates the flywheel's ability to regulate speed.
4. **What is the moment of inertia in the context of a flywheel?**
   - The moment of inertia of a flywheel is a measure of its resistance to changes in rotational speed. It depends on the mass distribution relative to the axis of rotation.
5. **Explain the term 'fluctuation of energy' in flywheels.**
   - Fluctuation of energy is the difference between the maximum and minimum kinetic energy of the flywheel during its rotation. It represents the energy that needs to be stored or released by the flywheel to smooth out speed variations.
6. **Why are flywheels used in engines?**
   - Flywheels are used in engines to minimize the fluctuations in the engine's speed during the power strokes, ensuring a smoother operation and consistent delivery of power.
7. **What factors influence the design of a flywheel?**
   - The design of a flywheel is influenced by factors such as the energy to be stored, the permissible fluctuation in speed, the mass distribution, and the material strength.
8. **What is the purpose of a rim-type flywheel?**
   - A rim-type flywheel concentrates most of its mass at the rim, which increases the moment of inertia and the energy storage capacity for a given weight.
9. **Differentiate between a solid disk and a rim-type flywheel.**
   - A solid disk flywheel has mass distributed evenly across its radius, while a rim-type flywheel has most of its mass concentrated at the outer edge, resulting in a higher moment of inertia for the same mass.
10. **What is the role of the flywheel in power generation?**
    - In power generation, flywheels are used to maintain consistent generator speed and output frequency, especially when there are fluctuations in the load or fuel supply.

Balancing of Masses

11. **What is balancing in mechanical systems?**
    - Balancing in mechanical systems refers to the process of ensuring that the rotating or reciprocating masses do not cause unwanted vibrations by equalizing the forces and moments acting on the system.
12. **Why is balancing important in rotating machinery?**

- Balancing is crucial in rotating machinery to prevent vibrations, reduce wear and tear on bearings and other components, and ensure smooth and efficient operation.

13. **Explain the difference between static and dynamic balancing.**
    - Static balancing ensures that the center of mass of a rotating object is on the axis of rotation, preventing it from tipping. Dynamic balancing ensures that the object does not produce unbalanced forces or moments when rotating at speed, addressing both static and inertia-related issues.
14. **What is the result of an unbalanced rotating mass?**
    - An unbalanced rotating mass can cause vibrations, noise, excessive wear on bearings, and even structural damage to the machinery due to the unbalanced centrifugal forces.
15. **Describe the concept of a balancing machine.**
    - A balancing machine is a device used to measure the unbalance in rotating parts. It detects the location and magnitude of unbalance so that corrective measures, like adding or removing material, can be applied.

1. Introduction to Static Force Analysis

Static force analysis involves determining the forces acting on the various components of a mechanism when it is in static equilibrium, i.e., when all the components are at rest or moving with constant velocity. In static equilibrium, the sum of all forces and the sum of all moments acting on each component must be zero.

2. Principles of Static Force Analysis

For a mechanism in static equilibrium, the following conditions must be satisfied:

$\sum \vec{F} = 0$ $\sum \vec{M} = 0$

where:

- $\sum \vec{F}$ is the vector sum of all external and internal forces,
- $\sum \vec{M}$ is the sum of all moments about any point.

These conditions are applied to each link or component of the mechanism to solve for the unknown forces and moments.

3. Four-Bar Linkage Mechanism

A four-bar linkage is a common planar mechanism consisting of four rigid links connected by revolute joints. The four links include:

- **Crank (Input link)**
- **Coupler**
- **Rocker (Output link)**
- **Fixed frame (Ground link)**

Let's perform static force analysis on a four-bar linkage in static equilibrium.

4. Assumptions

- The system is under static equilibrium (no acceleration).
- Neglect the effect of friction in the joints.
- The mechanism is driven by a force or torque applied on the input link (crank).
- The weight of the links is either negligible or incorporated into the external forces.

5. Static Force Analysis Methodology

a. Free Body Diagrams (FBDs)

To analyze the forces, we draw the free body diagram (FBD) of each link, isolating it from the rest of the mechanism. The FBD includes:

- All external forces (e.g., applied forces, reactions at joints).
- Internal forces (forces and moments transmitted from one link to another through the joints).

b. Equilibrium Equations

For each link in the mechanism, apply the equilibrium equations:

1. **Force Equilibrium:** $\sum F_x = 0$ and $\sum F_y = 0$
2. **Moment Equilibrium:** $\sum M = 0$

Let's consider the static force analysis of a four-bar linkage step by step.

7. Solving the Equations

The above equilibrium equations can be solved simultaneously to determine the unknown forces F_A, F_B, F_C, F_D, and the corresponding reaction forces at the joints. The complexity of the system may require the use of matrix methods or numerical techniques for solving the equations.

8. Example Problem

Consider a four-bar linkage where the crank is subjected to an input torque T. The goal is to find the reaction forces at the joints and the forces transmitted through the coupler and rocker.

Given:

- Crank length l_1, Coupler length l_2, Rocker length l_3
- Torque T
- Geometry of the linkage (angles θ, ϕ, ψ)

The forces are found by solving the equilibrium equations derived earlier.

Discuss the theory of fluctuation of energy and fluctuation of speed in a flywheel. Explain their significance in the design of flywheels used in mechanical systems. Derive the mathematical expressions for the fluctuation of energy and the coefficient of fluctuation of speed. Explain how these concepts are applied in the design of a flywheel for a mechanical system.

Answer:

1. Introduction to Flywheels:

A flywheel is a mechanical device specifically designed to store rotational energy. It is used in systems where the energy input to the system varies over time, and there is a need to smooth out these fluctuations. Common applications include engines, presses, and various other types of machinery.

The flywheel's primary function is to ensure a constant output speed despite fluctuations in the input energy. It achieves this by storing energy during periods of surplus and releasing it during periods of deficit.

2. Fluctuation of Energy:

Fluctuation of energy refers to the variation in the kinetic energy stored in a flywheel as it undergoes changes in its rotational speed. During each cycle of operation, the torque exerted on the flywheel varies due to varying forces (like combustion in an engine). This causes the rotational speed of the flywheel to change, leading to a fluctuation in the stored kinetic energy.

Kinetic Energy (E):

The kinetic energy stored in a flywheel rotating at angular velocity ω\omegaω is given by:
E=12Iω2E = \frac{1}{2} I \omega^2E=21Iω2

Where:

- III is the moment of inertia of the flywheel.
- ω\omegaω is the angular velocity.

Maximum and Minimum Energy:

- **Maximum Energy (E_max):** Stored when the flywheel rotates at its maximum speed ωmax\omega_{max}ωmax.
- **Minimum Energy (E_min):** Stored when the flywheel rotates at its minimum speed ωmin\omega_{min}ωmin.

Fluctuation of Energy (ΔE):

$$\Delta E = E_{max} - E_{min}$$

$$\Delta E = \frac{1}{2} I \omega_{max}^2 - \frac{1}{2} I \omega_{min}^2$$

$$\Delta E = \frac{1}{2} I (\omega_{max}^2 - \omega_{min}^2)$$

This fluctuation of energy is essential to be minimized in many mechanical systems to ensure steady operation.

3. Fluctuation of Speed:

The fluctuation of speed in a flywheel refers to the difference between its maximum and minimum speeds during its operation. This fluctuation is caused by the variation in the load or the input energy during a cycle.

Coefficient of Fluctuation of Speed (C_s):

The coefficient of fluctuation of speed is a dimensionless quantity that measures the relative change in speed during a cycle. It is defined as:

Cs=Nmax−NminNmeanC_s = \frac{N_{max} - N_{min}}{N_{mean}}Cs=NmeanNmax−Nmin

Where:

- NmaxN_{max}Nmax is the maximum speed of the flywheel.
- NminN_{min}Nmin is the minimum speed of the flywheel.
- NmeanN_{mean}Nmean is the mean speed of the flywheel.

This coefficient is crucial for determining the size and mass of the flywheel needed to keep the speed within desired limits.

4. Significance in Flywheel Design:

In designing a flywheel, both the fluctuation of energy and the fluctuation of speed are critical factors. The flywheel must be large enough to store sufficient energy to smooth out the fluctuations in the input torque but not so large that it becomes inefficient or cumbersome.

- **Minimizing Speed Fluctuations:** To minimize speed fluctuations, the flywheel must have a high moment of inertia, which typically requires more mass or a larger radius.
- **Energy Storage:** The flywheel should be able to store enough energy to cover the entire cycle's energy deficit without excessive speed variation.

5. Application in Mechanical Systems:

Flywheels are used in various mechanical systems where energy input is intermittent or varying, such as in:

- **Internal Combustion Engines:** To smooth out the pulsating torque from the engine cylinders.
- **Presses:** To provide the required energy during the pressing operation while maintaining a constant speed.
- **Power Generation:** To stabilize the rotational speed of turbines.

In each of these applications, the design of the flywheel involves calculating the required moment of inertia to ensure the fluctuations in speed and energy are within acceptable limits.

6. Example Application:

Consider a mechanical system where the input energy varies due to the operation of a piston in an internal combustion engine. The flywheel is required to keep the engine's speed within ±2% of the mean speed of 300 rpm.

- **Step 1:** Calculate the allowable speed variation:
  - Mean speed Nmean=300N_{mean} = 300Nmean=300 rpm.
  - Speed variation ΔN=2%\Delta N = 2\%ΔN=2% of 300 = 6 rpm.
  - Maximum speed Nmax=306N_{max} = 306Nmax=306 rpm.
  - Minimum speed Nmin=294N_{min} = 294Nmin=294 rpm.
- **Step 2:** Determine the required moment of inertia using the energy fluctuation equation:
  - Assume the engine develops a peak power output of 50 kW, and the flywheel stores sufficient energy during each cycle.

ΔE=12I(ωmax2−ωmin2)\Delta E = \frac{1}{2} I (\omega_{max}^2 - \omega_{min}^2)ΔE=21I(ωmax2−ωmin2)

Using the given data, the moment of inertia can be calculated to ensure the required energy is stored and the speed variation is kept within the desired limits.

Explain the theory of static and dynamic balancing in rotating machinery. Discuss the significance of each type of balancing and the consequences of imbalance in mechanical systems. Derive the conditions for static and dynamic balancing for a system of rotating masses. Provide examples of where static and dynamic balancing are critical in mechanical engineering applications.

Answer:

1. Introduction to Balancing:

Balancing in rotating machinery is crucial to ensure smooth operation and to minimize vibrations. An imbalance in a rotating system occurs when the mass distribution is uneven, causing centrifugal forces that can lead to excessive vibrations, noise, and even mechanical failure. Balancing can be classified into two types: **static balancing** and **dynamic balancing**.

- **Static Balancing** deals with the balance of forces in a single plane, ensuring that the center of mass of a rotating object is on its axis of rotation.
- **Dynamic Balancing** involves balancing in multiple planes and addresses not only the forces but also the moments about the axis of rotation.

2. Theory of Static Balancing:

Static Balancing is achieved when the mass distribution in a rotating object is such that the resultant centrifugal force is zero. This condition ensures that there is no tendency for the object to rotate due to imbalance, and the object remains stationary when placed in any orientation.

For static balancing:

- The sum of all centrifugal forces must be zero.

Let's consider a rotor with masses $m_1, m_2, \ldots, m_n$ attached at distances $r_1, r_2, \ldots, r_n$ from the axis of rotation. The centrifugal force for each mass is given by:

$$F_i = m_i \cdot r_i \cdot \omega^2$$

Where ω is the angular velocity.

For static balance, the vector sum of all these forces must be zero:

$$\sum_{i=1}^{n} F_i = \sum_{i=1}^{n} m_i \cdot r_i = 0$$

This implies that the center of mass of the system lies on the axis of rotation.

3. Theory of Dynamic Balancing:

Dynamic Balancing involves not only balancing the forces but also ensuring that the moments of these forces about the axis of rotation are zero. Dynamic balancing is necessary when the rotor is subjected to motion in multiple planes.

For dynamic balancing:

- The sum of the forces (as in static balancing) must be zero.
- The sum of the moments of these forces about any point on the axis must also be zero.

Consider a rotor with masses distributed in different planes. For dynamic balance, we must ensure:

$$\sum_{i=1}^{n} m_i \cdot r_i = 0$$

And also:

∑i=1nmi·ri·zi=0\sum_{i=1}^{n} m_i \cdot r_i \cdot z_i = 0∑i=1nmi·ri·zi=0

Where ziz_izi is the distance of the mass mim_imi from a reference plane.

4. Derivation of Conditions for Balancing:

Static Balancing Condition:

For a rotor to be statically balanced, the sum of the moments of all the masses about any axis perpendicular to the axis of rotation must be zero. Mathematically:

∑i=1nmi·ri=0\sum_{i=1}^{n} m_i \cdot r_i = 0∑i=1nmi·ri=0

This condition ensures that the center of mass lies on the axis of rotation, and there is no unbalanced force acting on the rotor.

Dynamic Balancing Condition:

In dynamic balancing, not only must the forces be balanced (as in static balancing), but the moments about any reference plane must also be zero. This ensures that there are no unbalanced moments acting on the rotor.

For dynamic balance:

1. **Force Balance:** ∑i=1nmi·ri=0\sum_{i=1}^{n} m_i \cdot r_i = 0∑i=1nmi·ri=0
2. **Moment Balance:** ∑i=1nmi·ri·zi=0\sum_{i=1}^{n} m_i \cdot r_i \cdot z_i = 0∑i=1nmi·ri·zi=0

Where ziz_izi represents the axial distance of each mass from the reference plane.

These conditions ensure that the rotor is dynamically balanced, meaning it will not experience unbalanced forces or moments during rotation.

5. Consequences of Imbalance:

If a rotor is not properly balanced, it can lead to several problems:

- **Vibration:** Unbalanced rotors produce vibrations, which can lead to noise, wear, and eventually failure of machine components.
- **Bearing Wear:** Excessive vibrations cause additional loads on the bearings, leading to premature wear and failure.
- **Structural Damage:** Prolonged operation with unbalanced rotors can cause damage to the machine structure or foundation.
- **Energy Loss:** Unbalanced rotors require more energy to maintain the same speed, leading to inefficiency.

6. Examples of Balancing in Mechanical Engineering:

- **Automobile Wheels:** Static and dynamic balancing of car wheels is essential to avoid vibrations at high speeds. Unbalanced wheels can cause uneven tire wear and poor vehicle handling.
- **Turbomachinery:** In turbines, compressors, and pumps, dynamic balancing is critical to prevent excessive vibrations that could lead to catastrophic failure.
- **Machine Tools:** In precision machinery like lathes and milling machines, balancing the rotating components ensures smooth operation and high-quality finishes.
- **Aircraft Propellers:** Proper balancing of propellers is essential to prevent vibrations that could damage the aircraft or reduce its efficiency.

Explain the theory of balancing rotating masses. Discuss the importance of balancing in rotating machinery and derive the conditions required for both static and dynamic balancing of a system with multiple rotating masses. Provide examples of applications where balancing rotating masses is critical in mechanical engineering.

Answer:

1. Introduction to Balancing of Rotating Masses:

Balancing of rotating masses is a fundamental aspect of mechanical engineering that ensures the smooth operation of rotating machinery. When a rotating system is not properly balanced, it can lead to vibrations, noise, excessive wear on bearings, and even catastrophic failure. Balancing involves arranging the masses in such a way that the resultant forces and moments acting on the rotor are minimized or eliminated.

Balancing can be categorized into two types:

- **Static Balancing:** Ensures that the center of mass of a rotating body is on the axis of rotation, preventing any net force acting on the rotor.
- **Dynamic Balancing:** Ensures that not only are the forces balanced (as in static balancing), but also the moments about the axis of rotation are zero, preventing any net couple acting on the rotor.

2. Static Balancing of Rotating Masses:

Static Balancing refers to the condition where the center of mass of a rotating object lies on its axis of rotation. If an object is statically balanced, it will not rotate due to gravity when placed on a knife-edge or any other pivot. In practical terms, this means that the net force acting on the rotor due to its mass distribution is zero.

Theory of Static Balancing:

Consider a rotor with masses m1,m2,…,mnm_1, m_2, \ldots, m_nm1,m2,…,mn attached at distances r1,r2,…,rnr_1, r_2, \ldots, r_nr1,r2,…,rn from the axis of rotation, and at angular positions θ1,θ2,…,θn\theta_1, \theta_2, \ldots, \theta_nθ1,θ2,…,θn. The centrifugal force for each mass is:

Fi=mi·ri·ω2F_i = m_i \cdot r_i \cdot \omega^2Fi=mi·ri·ω2

Where ω\omegaω is the angular velocity.

For the rotor to be statically balanced, the vector sum of all these forces must be zero:

$$\sum_{i=1}^{n} F_i = 0$$

In terms of components, this can be expressed as:

$$\sum_{i=1}^{n} m_i \cdot r_i \cdot \cos \theta_i = 0 \quad \text{and} \quad \sum_{i=1}^{n} m_i \cdot r_i \cdot \sin \theta_i = 0$$

This ensures that there is no net force acting on the rotor in any direction, meaning the center of mass lies on the axis of rotation.

3. Dynamic Balancing of Rotating Masses:

Dynamic Balancing is a more comprehensive form of balancing that ensures the rotor is balanced not only in terms of forces but also in terms of moments. In dynamic balancing, we consider the balance of forces in multiple planes along the axis of rotation.

Theory of Dynamic Balancing:

Dynamic balancing involves ensuring that:

1. The vector sum of all the forces acting on the rotor is zero (same as static balancing).
2. The vector sum of all the moments about any plane perpendicular to the axis of rotation is also zero.

Consider a system with multiple masses $m_1, m_2, \ldots, m_n$ located at different distances from the axis of rotation and in different planes. Let the distances from a reference plane be $z_1, z_2, \ldots, z_n$.

For dynamic balancing, the following conditions must be met:

1. **Force Balance:** $\sum_{i=1}^{n} m_i \cdot r_i = 0$
2. **Moment Balance:** $\sum_{i=1}^{n} m_i \cdot r_i \cdot z_i = 0$

Where r_i is the radial distance of the mass m_i from the axis of rotation, and z_i is the axial distance from the reference plane.

These conditions ensure that there is no net force or couple acting on the rotor, thus preventing both translational and rotational vibrations.

4. Importance of Balancing:

Balancing is critical in rotating machinery for several reasons:

- **Reduced Vibrations:** Properly balanced rotors experience minimal vibrations, leading to smoother operation and reduced wear and tear on machine components.

- **Extended Bearing Life:** Imbalance in rotating parts causes additional forces on the bearings, leading to premature failure. Balancing reduces these forces and extends the life of the bearings.
- **Increased Efficiency:** Machines with balanced rotors operate more efficiently as less energy is lost to vibrations.
- **Safety:** Unbalanced rotors can lead to catastrophic failures, especially in high-speed machinery. Balancing improves the safety of these machines.

5. Applications of Balancing Rotating Masses:

- **Automobile Engines:** In internal combustion engines, the crankshaft and other rotating components must be balanced to reduce vibrations and ensure smooth engine operation.
- **Turbines:** In turbines used in power plants, the blades and shafts must be dynamically balanced to prevent destructive vibrations that could lead to failure.
- **Rotating Machinery:** Fans, blowers, and other rotating machinery in HVAC systems require balancing to operate quietly and efficiently.
- **Aerospace Applications:** In aircraft engines and propellers, balancing is crucial for ensuring the safety and reliability of the aircraft.

Explain the theory of balancing reciprocating masses in a single-cylinder engine. Discuss the challenges associated with balancing reciprocating masses and the methods used to mitigate the imbalance. Provide the conditions necessary for achieving partial balancing and explain why complete balancing is not possible in a single-cylinder engine.

Answer:

1. Introduction to Balancing of Reciprocating Masses:

In a single-cylinder engine, the reciprocating motion of the piston generates unbalanced forces due to the inertia of the moving masses (such as the piston, connecting rod, and other related components). These unbalanced forces lead to vibrations, which can affect the engine's performance, cause discomfort to users, and result in increased wear and tear on the engine components.

Balancing reciprocating masses in a single-cylinder engine is a complex task because the motion of the piston is not uniform; it involves continuous acceleration and deceleration. The goal is to minimize the unbalanced forces and reduce vibrations, but achieving complete balancing is impossible due to the nature of the reciprocating motion.

2. Theory of Reciprocating Masses:

When a piston moves inside a cylinder, it undergoes reciprocating motion, which is a combination of linear displacement and rotational motion (due to the crankshaft). This motion creates inertial forces that vary in magnitude and direction throughout the engine cycle.

The main forces acting on the piston include:

- **Primary Forces:** These are due to the mass of the piston and connecting rod and are proportional to the angular velocity of the crankshaft. They act along the line of the piston’s motion.
- **Secondary Forces:** These arise due to the non-uniform motion of the connecting rod, causing slight variations in the piston’s motion that are not directly in line with the crankshaft rotation.

The total force on the piston can be expressed as:

F=mr·ω2·r·(cosθ+λ·cos2θ)F = m_r \cdot \omega^2 \cdot r \cdot (\cos\theta + \lambda \cdot \cos 2\theta)F=mr·ω2·r·(cosθ+λ·cos2θ)

Where:

- FFF is the net force on the piston.
- mrm_rmr is the reciprocating mass (including the piston and a portion of the connecting rod).
- ω\omegaω is the angular velocity of the crankshaft.
- rrr is the crank radius.
- θ\thetaθ is the crank angle.
- λ\lambdaλ is the ratio of the length of the connecting rod to the crank radius (l/rl/rl/r).

The first term mr·ω2·r·cosθm_r \cdot \omega^2 \cdot r \cdot \cos\thetamr·ω2·r·cosθ represents the **primary force**, and the second term mr·ω2·r·λ·cos2θm_r \cdot \omega^2 \cdot r \cdot \lambda \cdot \cos 2\thetamr·ω2·r·λ·cos2θ represents the **secondary force**.

3. Challenges in Balancing Reciprocating Masses:

Balancing the reciprocating masses is challenging due to the following reasons:

1. **Unidirectional Forces:** The primary and secondary forces act in a single direction (along the axis of piston movement), making it difficult to counterbalance them using rotating masses.
2. **Non-Uniform Motion:** The connecting rod's motion introduces secondary forces that vary in magnitude and direction, adding complexity to the balancing process.
3. **Varying Inertia:** As the crankshaft rotates, the inertia of the reciprocating masses changes, resulting in fluctuating forces that are difficult to completely balance.

4. Methods of Partial Balancing:

Since complete balancing is impossible, engineers aim for **partial balancing** to reduce the most significant unbalanced forces. The common methods include:

- **Balancing Primary Forces:** A counterweight is added to the crankshaft opposite the crankpin. This counterweight is designed to balance the primary forces by generating an opposing centrifugal force.
- **Compromising on Secondary Forces:** Due to the complexity of secondary forces, they are often left unbalanced or partially balanced. This approach reduces the overall vibrations but does not eliminate them entirely.

Condition for Partial Balancing:

For partial balancing of primary forces, the counterweight mcm_cmc is designed such that:

mc·rc=mr·rm_c \cdot r_c = m_r \cdot rmc·rc=mr·r

Where:

- mcm_cmc is the mass of the counterweight.
- rcr_crc is the distance of the counterweight from the axis of rotation.

This condition ensures that the primary reciprocating forces are balanced, reducing the unbalanced forces that cause engine vibrations.

5. Why Complete Balancing is Impossible:

Complete balancing in a single-cylinder engine is impossible due to the nature of reciprocating motion:

1. **Non-Circular Motion:** Unlike rotating masses, which can be completely balanced by counterweights, the reciprocating motion is linear and cannot be fully counteracted by rotating counterweights.
2. **Secondary Forces:** The secondary forces, which are due to the non-linear motion of the connecting rod, cannot be fully balanced without introducing additional complexity and components.
3. **Harmonic Content:** The force generated by the reciprocating mass has multiple harmonics (primary and secondary), and it is not possible to balance all harmonics simultaneously with a single rotating mass.

6. Applications and Significance:

- **Automobile Engines:** Partial balancing is used in single-cylinder engines in motorcycles, small cars, and other vehicles to reduce vibrations and improve ride comfort.
- **Small Machinery:** Single-cylinder engines are commonly used in small machinery, such as lawn mowers and generators, where partial balancing helps in reducing operational noise and wear.
- **Stationary Engines:** In stationary applications, such as pumps and compressors, partial balancing improves efficiency and longevity by reducing the strain on bearings and other components.

6. Example Problem:

Example: Consider a rotor with three masses m1=5 kgm_1 = 5 \, \text{kg}m1=5kg, m2=3 kgm_2 = 3 \, \text{kg}m2=3kg, and m3=4 kgm_3 = 4 \, \text{kg}m3=4kg located at radii r1=0.2 mr_1 = 0.2 \, \text{m}r1=0.2m, r2=0.15 mr_2 = 0.15 \, \text{m}r2=0.15m, and r3=0.25 mr_3 = 0.25 \, \text{m}r3=0.25m, and angular positions θ1=0∘\theta_1 =

0^\circθ1=0∘, θ2=120∘\theta_2 = 120^\circθ2=120∘, and θ3=240∘\theta_3 = 240^\circθ3=240∘. Determine whether the system is statically balanced.

Four masses m_1, m_2, m_3 and m_4 are 200 kg, 300 kg, 240 kg and 260 kg respectively. The corresponding radii of rotation are 0.2 m, 0.15 m, 0.25 m and 0.3 m respectively and the angles between successive masses are 45°, 75° and 135°. Find the position and magnitude of the balance mass required, if its radius of rotation is 0.2 m.

$m_1 = 200$ kg	$r_1 = 0.2$ m	$\theta_1 = 0°$
$m_2 = 300$ kg	$r_2 = 0.15$ m	$\theta_2 = 45°$
$m_3 = 240$ kg	$r_3 = 0.25$ m	$\theta_3 = 45° + 75° = 120°$
$m_4 = 260$ kg	$r_4 = 0.3$ m	$\theta_4 = 120° + 135° = 255°$
$m_1r_1 = 200 \times 0.2 = 40$	$r_C = 0.2$ m	
$m_2r_2 = 300 \times 0.15 = 45$		
$m_3r_3 = 240 \times 0.25 = 60$		
$m_4r_4 = 260 \times 0.3 = 78$		

$\Sigma mr + m_cr_c = 0$

$40\cos 0° + 45\cos 45° + 60\cos 120° + 78\cos 255° + m_cr_c\cos\theta_c = 0$ and

$40\sin 0° + 45\sin 45° + 60\sin 120° + 78\sin 255° + m_cr_c\sin\theta_c = 0$

Squaring, adding and then solving,

$$m_c r_c = \sqrt{\begin{matrix}(40\cos 0° + 45\cos 45° + 60\cos 120° + 78\cos 255°)^2 + \\ (40\sin 0° + 45\sin 45° + 60\sin 120° + 78\sin 255°)^2\end{matrix}}$$

$$m_c \times 0.2 = \sqrt{(21.6)^2 + (8.5)^2}$$

$$= 23.2 \text{ kg.mm}$$

$$m_c = 116 \text{ kg}$$

$$\tan\theta_c = \frac{-\Sigma mr\sin\theta}{-\Sigma mr\cos\theta} = \frac{-8.5}{-21.6} = 0.3935$$

$$\theta_c = 21°28'$$

θ_c lies in the third quadrant (numerator is negative and denominator is negative).

$$\theta_c = 180 + 21°28'$$

$$\theta_c = 201°28'$$

Graphical Method:

- For graphical method draw the vector diagram with the above values, to some suitable scale, as shown in Fig. 1.4. The closing side of the polygon *ae* represents the resultant force. By measurement, we find that *ae* = 23 kg-m.

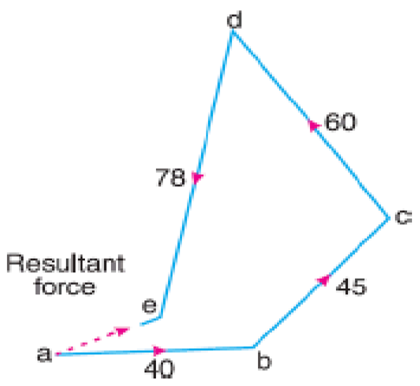


Fig. 1.4 Vector Diagram

- The balancing force is equal to the resultant force.Since the balancing force is proportional to *m.r*, therefore

$$m \times 0.2 = \text{vector } ea = 23 \text{ kg-m or } m_c = 23/0.2$$

$$m_c = 115 \text{ kg.}$$

- By measurement we also find that the angle of inclination of the balancing mass (*m*) from the horizontal or positive X-axis,

$$\theta_c = 201°.$$

A shaft carries four masses A, B, C and D of magnitude 200 kg, 300 kg, 400 kg and 200 kg respectively and revolving at radii 80 mm, 70 mm, 60 mm and 80 mm in planes measured from A at 300 mm, 400 mm and 700 mm. The angles between the cranks measured anticlockwise are A to B 45°, B to C 70° and C to D 120°. The balancing masses are to be placed in planes X and Y. The distance between the planes A and X is 100 mm, between X and Y is 400 mm and between Y and D is 200 mm. If the balancing masses revolve at a radius of 100 mm, find their magnitudes and angular positions.

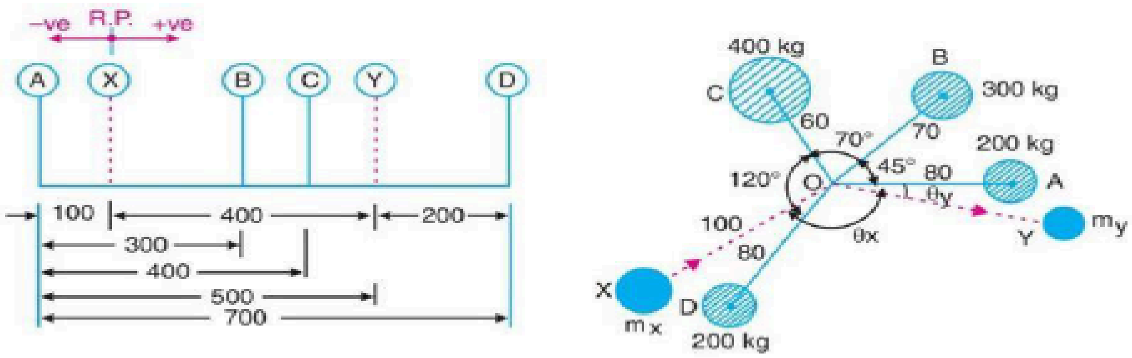
m_A = 200 kg	r_A = 80 mm	$\theta_A = 0°$	l_A = -100 mm
m_B = 300 kg	r_B= 70 mm	$\theta_B = 45°$	l_B = 200 mm
m_C = 400 kg	r_C = 60 mm	$\theta_C = 45° + 70° = 115°$	l_C = 300 mm
m_D = 200 kg	r_D = 80 mm	$\theta_D = 115° + 120° = 235°$	l_D = 600 mm
	$r_X = r_Y$ = 100 mm		l_Y = 400 mm

Let m_X = Balancing mass placed in plane *X*, and

m_Y = Balancing mass placed in plane *Y*.

The position of planes and angular position of the masses (assuming the mass A as horizontal) are shown in Fig. 1.5 (*a*) and (*b*) respectively.

Assume the plane *X* as the reference plane (*R.P.*). The distances of the planes to the right of plane *X* are taken as + ve while the distances of the planes to the left of plane *X* are taken as –ve.



(a) Position of planes. *(b) Angular position of masses.*

Fig. 1.6

$m_A r_A l_A = 200 \times 0.08 \times (-0.1) = -1.6$ kg.m²	$m_A r_A = 200 \times 0.08 = 16$ kg.m
$m_B r_B l_B = 300 \times 0.07 \times 0.2 = 4.2$ kg.m²	$m_B r_B = 300 \times 0.07 = 21$ kg.m
$m_C r_C l_C = 400 \times 0.06 \times 0.3 = 7.2$ kg.m²	$m_C r_C = 400 \times 0.06 = 24$ kg.m
$m_D r_D l_D = 200 \times 0.08 \times 0.6 = 9.6$ kg.m²	$m_D r_D = 200 \times 0.08 = 16$ kg.m

Analytical Method:

For unbalanced couple

$\Sigma mrl + m_Y r_Y l_Y = 0$

$$m_y r_y l_y = \sqrt{(\Sigma mrl\cos\theta)^2 + (\Sigma mrl\sin\theta)^2}$$

$$m_y r_y l_y = \sqrt{\begin{aligned}&(-1.6\cos 0° + 4.2\cos 45° + 7.2\cos 115° + 9.6\cos 235°)^2 + \\ &(-1.6\sin 0° + 4.2\sin 45° + 7.2\sin 115° + 9.6\sin 235°)^2\end{aligned}}$$

$$m_y r_y l_y = \sqrt{(-7.179)^2 + (1.63)^2}$$

$$m_y \times 0.1 \times 0.4 = 7.36$$

m_Y = 184 kg.

$$\tan\theta_y = \frac{-\Sigma mrl\sin\theta}{-\Sigma mrl\cos\theta} = \frac{-1.63}{-(-7.179)} = -0.227$$

$$\theta_y = -12°47'$$

θ_Y lies in the fourth quadrant (numerator is negative and denominator is positive).

$\theta_Y = 360 - 12°47'$

$\theta_Y = 347°12'$

For unbalanced centrifugal force

$\Sigma mr + m_x r_X + m_Y r_Y = 0$

For unbalanced centrifugal force

$\Sigma mr + m_x r_X + m_Y r_Y = 0$

$$m_x r_x = \sqrt{(\Sigma mr\cos\theta + m_y r_y\cos\theta_y)^2 + (\Sigma mr\sin\theta + m_y r_y\sin\theta_y)^2}$$

$$m_x r_x = \sqrt{\begin{aligned}&(16\cos 0° + 21\cos 45° + 24\cos 115° + 16\cos 235° + 18.4\cos 347°12')^2 + \\ &(16\sin 0° + 21\sin 45° + 24\sin 115° + 16\sin 235° + 18.4\sin 347°12')^2\end{aligned}}$$

$$m_x r_x = \sqrt{(29.47)^2 + (19.42)^2}$$

$$m_x \times 0.1 = 35.29$$

m_x = 353 kg.

$$\tan\theta_x = \frac{-\Sigma mr\sin\theta}{-\Sigma mr\cos\theta} = \frac{-19.42}{-29.47} = 0.6589$$

$\theta_X = 33°22'$

θ_X lies in the third quadrant (numerator is negative and denominator is negative).

$\theta_X = 180 + 33°22'$

$\theta_X = 213°22'$

Graphical Method:

The balancing masses and their angular positions may be determined graphically as discussed below :

UNIT 4: FREE AND FORCED VIBRATIONS

1. **What are free vibrations?**
   - Free vibrations occur when a system oscillates naturally without external forces after being displaced from its equilibrium position. The motion continues at the system's natural frequency.
2. **Define natural frequency.**
   - *o*
   - *o*
3. **What is damping in the context of vibrations?**
   - Damping refers to the resistance to motion that gradually reduces the amplitude of vibrations over time, eventually bringing the system to rest. It is caused by factors like friction and material properties.
4. **Explain the difference between undamped and damped free vibrations.**
   - In undamped free vibrations, the system oscillates indefinitely without a decrease in amplitude. In damped free vibrations, the amplitude gradually decreases due to energy loss, and the system eventually stops vibrating.
5. **What is meant by critical damping?**
   - Critical damping is the minimum amount of damping required to prevent oscillations. In a critically damped system, the system returns to its equilibrium position without oscillating.
6. **Define the term 'overdamping'.**
   - Overdamping occurs when the damping in a system is greater than the critical damping. The system returns to equilibrium more slowly without oscillating.
7. **What is underdamping?**
   - Underdamping occurs when the damping is less than critical damping. The system oscillates with gradually decreasing amplitude until it eventually stops.
8. **Explain the concept of a damped natural frequency.**
   - Damped natural frequency is the frequency at which a damped system oscillates. It is slightly lower than the undamped natural frequency due to the presence of damping.
9. **What are the factors affecting the natural frequency of a system?**
   - The natural frequency of a system depends on its mass and stiffness. Specifically, it is inversely proportional to the square root of mass and directly proportional to the square root of stiffness.
10. **What is a simple harmonic oscillator?**
    - A simple harmonic oscillator is a system that experiences simple harmonic motion, where the restoring force is directly proportional to the displacement from equilibrium and acts in the opposite direction.

Forced Vibrations

11. **What are forced vibrations?**
    - Forced vibrations occur when a system is subjected to a continuous external force or excitation. The system vibrates at the frequency of the external force rather than its natural frequency.
12. **Define resonance in the context of forced vibrations.**
    - Resonance occurs when the frequency of the external force matches the natural frequency of the system, leading to a significant increase in the amplitude of vibrations.

13. **What is the phase difference in forced vibrations?**
    - The phase difference is the angular difference between the displacement of the vibrating system and the external forcing function. It varies with the frequency of the external force and the damping in the system.
14. **Explain the concept of transmissibility in forced vibrations.**
    - Transmissibility is the ratio of the amplitude of the forced vibrations of a system to the amplitude of the external force. It indicates how much of the external vibration is transmitted to the system.
15. **How does damping affect the resonance amplitude?**
    - Damping reduces the resonance amplitude in forced vibrations. The higher the damping, the lower the amplitude at resonance, preventing excessive oscillations and potential damage.

(11) Types of Vibratory motion:

The following types of vibratory motion are important from the subject point of view :

1. ***Free or natural vibrations.*** When no external force acts on the body, after giving it an initial displacement, then the body is said to be under ***free or natural vibrations.*** The frequency of the free vibrations is called ***free or natural frequency.***

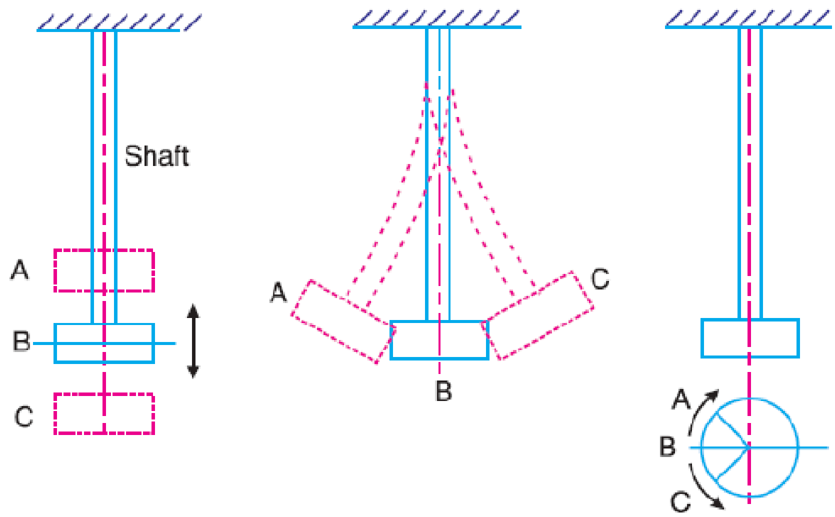
2. ***Forced vibrations.*** When the body vibrates under the influence of external force, then the body is said to be under ***forced vibrations.*** The external force applied to the body is a periodic disturbing force created by unbalance. The vibrations have the same frequency as the applied force.

Note : When the frequency of the external force is same as that of the natural vibrations, resonance takes place.

3. ***Damped vibrations.*** When there is a reduction in amplitude over every cycle of vibration, the motion is said to be ***damped vibration.*** This is due to the fact that a certain amount of energy possessed by the vibrating system is always dissipated in overcoming frictional resistances to the motion.

(12) Types of Vibration:

(a)Longitudinal vibration (b)Transverse Vibration (c)Torsional Vibration.



B = Mean position ; A and C = Extreme positions.

(a) Longitudinal vibrations. (b) Transverse vibrations. (c) Torsional vibrations.

(13) Longitudinal Vibration:

When the particles of the shaft or disc moves parallel to the axis of the shaft, then the vibrations known as longitudinal vibrations.

(14) Free undamped longitudinal vibrations;

When a body is allowed to vibrate on its own, after giving it an initial displacement, then the ensuring vibrations are known as free or natural vibrations. When the vibrations take place parallel to the axis of constraint and no damping is provided, then it is called free undamped longitudinal vibrations.

(15) Natural frequency of free undamped longitudinal vibration:

(15.a) Equilibrium method or Newton's method:

Consider a constraint (*i.e.* spring) of negligible mass in an unstrained position, .

Let s = Stiffness of the constraint. It is the force required to produce unit displacement in the direction of vibration. It is usually expressed in N/m.

m = Mass of the body suspended from the constraint in kg,

W = Weight of the body in newtons = $m.g$,

(2) Basic elements of vibration system:

- Mass or Inertia
- Springiness or Restoring element
- Dissipative element (often called damper)
- External excitation

(3) Causes of vibration:

Unbalance: This is basically in reference to the rotating bodies. The uneven distribution of mass in a rotating body contributes to the unbalance. A good example of unbalance related vibration would be the "vibrating alert" in our mobile phones. Here a small amount of unbalanced weight is rotated by a motor causing the vibration which makes the mobile phone to vibrate. You would have experienced the same sort of vibration occurring in your front loaded washing machines that tend to vibrate during the "spinning" mode.

Misalignment: This is an other major cause of vibration particularly in machines that are driven by motors or any other prime movers.

Bent Shaft: A rotating shaft that is bent also produces the the vibrating effect since it losses it rotation capability about its center.

Gears in the machine: The gears in the machine always tend to produce vibration, mainly due to their meshing. Though this may be controlled to some extent, any problem in the gearbox tends to get enhanced with ease.

(4) Effects of vibration:

(a)Bad Effects:

The presence of vibration in any mechanical system produces unwanted noise, high stresses, poor reliability, wear and premature failure of parts. Vibrations are a great source of human discomfort in the form of physical and mental strains.

(b)Good Effects:

A vibration does useful work in musical instruments, vibrating screens, shakers, relive pain in physiotherapy.

(5) Methods of reduction of vibration:

- -unbalance is its main cause, so balancing of parts is necessary.
- -using shock absorbers.
- -using dynamic vibration absorbers.
- -providing the screens (if noise is to be reduced)

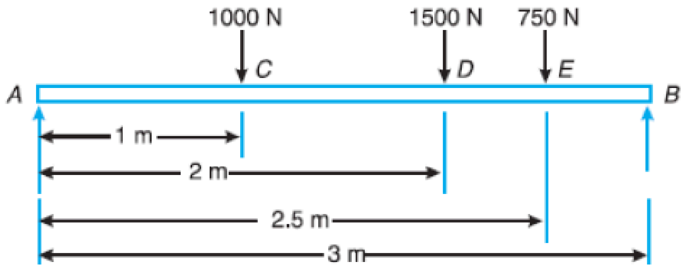
(6) Types of vibratory motion:

- Free Vibration
- Forced Vibration

A shaft 50 mm diameter and 3 metres long is simply supported at the ends and carries three loads of 1000 N, 1500 N and 750 N at 1 m, 2 m and 2.5 m from the left support. The Young's modulus for shaft material is 200 GN/m2. Find the frequency of transverse vibration.

Solution. Given : d = 50 mm = 0.05 m ; l = 3 m, W_1 = 1000 N ; W_2 = 1500 N ; W_3 = 750 N; E = 200 GN/m^2 = 200 × 10^9 N/m^2

The shaft carrying the loads is shown in Fig. 23.13



We know that moment of inertia of the shaft,

$$I = \frac{\pi}{64} \times d^4 = \frac{\pi}{64}(0.05)^4 = 0.307 \times 10^{-6}\,\text{m}^4$$

and the static deflection due to a point load W,

$$\delta = \frac{Wa^2b^2}{3\,EIl}$$

∴ Static deflection due to a load of 1000 N,

$$\delta_1 = \frac{1000 \times 1^2 \times 2^2}{3 \times 200 \times 10^9 \times 0.307 \times 10^{-6} \times 3} = 7.24 \times 10^{-3} \text{ m}$$

. . . (Here a = 1 m, and b = 2 m)

Similarly, static deflection due to a load of 1500 N,

$$\delta_2 = \frac{1500 \times 2^2 \times 1^2}{3 \times 200 \times 10^9 \times 0.307 \times 10^{-6} \times 3} = 10.86 \times 10^{-3} \text{ m}$$

. . . (Here a = 2 m, and b = 1 m)

and static deflection due to a load of 750 N,

$$\delta_3 = \frac{750 (2.5)^2 (0.5)^2}{3 \times 200 \times 10^9 \times 0.307 \times 10^{-6} \times 3} = 2.12 \times 10^{-3} \text{ m}$$

. . . (Here a = 2.5 m. and b = 0.5 m)

We know that frequency of transverse vibration,

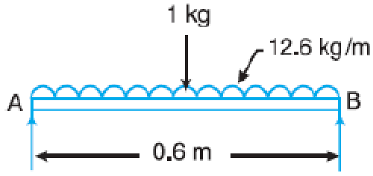
$$f_n = \frac{0.4985}{\sqrt{\delta_1 + \delta_2 + \delta_3}} = \frac{0.4985}{\sqrt{7.24 \times 10^{-3} + 10.86 \times 10^{-3} + 2.12 \times 10^{-3}}}$$

$$= \frac{0.4985}{0.1422} = 3.5 \text{ Hz}$$ **Ans.**

Calculate the whirling speed of a shaft 20 mm diameter and 0.6 m long carrying a mass of 1 kg at its mid-point. The density of the shaft material is 40 Mg/m³, and Young's modulus is 200 GN/m². Assume the shaft to be freely supported.

Solution. Given : d = 20 mm = 0.02 m ; l = 0.6 m ; m_1 = 1 kg ; ρ = 40 Mg/m^3 = 40×10^6 g/m^3 = 40×10^3 kg/m^3 ; E = 200 GN/m^2 = 200×10^9 N/m^2

We know that moment of inertia of the shaft,



$$I = \frac{\pi}{64} \times d^4 = \frac{\pi}{64}(0.02)^4 \text{ m}^4$$

$$= 7.855 \times 10^{-9} \text{ m}^4$$

Since the density of shaft material is 40×10^3 kg/m^3, efore mass of the shaft per metre length,

$$m_S = \text{Area} \times \text{length} \times \text{density} = \frac{\pi}{4}(0.02)^2 \times 1 \times 40 \times 10^3 = 12.6 \text{ kg/m}$$

We know that static deflection due to 1 kg of mass at the centre,

$$\delta = \frac{Wl^3}{48\,EI} = \frac{1 \times 9.81 (0.6)^3}{48 \times 200 \times 10^9 \times 7.855 \times 10^{-9}} = 28 \times 10^{-6} \text{ m}$$

and static deflection due to mass of the shaft,

$$\delta_S = \frac{5wl^4}{384EI} = \frac{5 \times 12.6 \times 9.81 (0.6)^4}{384 \times 200 \times 10^9 \times 7.855 \times 10^{-9}} = 0.133 \times 10^{-3} \text{ m}$$

∴ Frequency of transverse vibration,

$$f_n = \frac{0.4985}{\sqrt{\delta + \frac{\delta_S}{1.27}}} + \frac{0.4985}{\sqrt{28 \times 10^{-6} + \frac{0.133 \times 10^{-3}}{1.27}}}$$

$$= \frac{0.4985}{11.52 \times 10^{-3}} = 43.3 \text{ Hz}$$

Let N_c = Whirling speed of a shaft.

We know that whirling speed of a shaft in r.p.s. is equal to the frequency of transverse ibration in Hz , therefore

$$N_c = 43.3 \text{ r.p.s.} = 43.3 \times 60 = 2598 \text{ r.p.m.}$$ **Ans.**

. *A shaft 1.5 m long, supported in flexible bearings at the ends carries two wheels each of 50 kg mass. One wheel is situated at the centre of the shaft and the other at a distance of 375 mm from the centre towards left. The shaft is hollow of external diameter 75 mm and internal diameter 40 mm. The density of the shaft material is 7700 kg/m³ and its modulus of elasticity is 200 GN/m². Find the lowest whirling speed of the shaft, taking into account the mass of the shaft.*

Solution. l = 1.5 m ; $m_1 = m_2$ = 50 kg ; d_1 = 75 mm = 0.075 m ; d_2 = 40 mm = 0.04 m ; ρ = 7700 kg/m³ ; E = 200 GN/m² = 200 × 10^9 N/m²

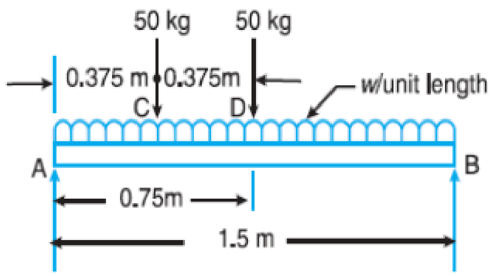


Fig. 23.16

moment of inertia of the shaft,

$$I = \frac{\pi}{64}\left[(d_1)^4 - (d_2)^4\right] = \frac{\pi}{64}\left[(0.075)^4 - (0.04)^4\right] = 1.4 \times 10^{-6}\,\text{m}^4$$

Since the density of shaft material is 7700 kg/m³, therefore mass of the shaft per metre length,

$$m_S = \text{Area} \times \text{length} \times \text{density} = \frac{\pi}{4}\left[(0.075)^2 - (0.04)^2\right]1 \times 7700 = 24.34 \text{ kg/m}$$

static deflection due to a load W

$$= \frac{Wa^2b^2}{3EIl} = \frac{m.ga^2b^2}{3EIl}$$

∴ Static deflection due to a mass of 50 kg at C,

$$\delta_1 = \frac{m_1 ga^2b^2}{3EIl} = \frac{50 \times 9.81(0.375)^2(1.125)^2}{3 \times 200 \times 10^9 \times 1.4 \times 10^{-6} \times 1.5} = 70 \times 10^{-6} \text{ m}$$

. . . (Here a = 0.375 m, and b = 1.125 m)

Similarly, static deflection due to a mass of 50 kg at D

$$\delta_2 = \frac{m_1 g a^2 b^2}{3EIl} = \frac{50 \times 9.81(0.75)^2 (0.75)^2}{3 \times 200 \times 10^9 \times 1.4 \times 10^{-6} \times 1.5} = 123 \times 10^{-6} \text{ m}$$

. . . (Here $a = b = 0.75$ m)

static deflection due to uniformly distributed load or mass of the shaft,

$$\delta_S = \frac{5}{384} \times \frac{wl^4}{EI} = \frac{5}{384} \times \frac{24.34 \times 9.81(1.5)^4}{200 \times 10^9 \times 1.4 \times 10^{-6}} = 56 \times 10^{-6} \text{ m}$$

. . . (Substituting, $w = m_S \times g$)

We know that frequency of transverse vibration,

$$f_n = \frac{0.4985}{\sqrt{\delta_1 + \delta_2 + \frac{\delta_S}{1.27}}} = \frac{0.4985}{\sqrt{70 \times 10^{-6} + 123 \times 10^{-6} + \frac{56 \times 10^{-6}}{1.27}}} \text{ Hz} = 32.4 \text{ Hz}$$

Since the whirling speed of shaft (N_c) in r.p.s. is equal to the frequency of transverse vibration in Hz, therefore

N_c = 32.4 r.p.s. = 32.4 × 60 = 1944 r.p.m. **Ans.**

A stepped shaft of 0.05 m in diameter for the first 0.6 m length, 0.08 m diameter for the next 1.8 m and 0.03 m diameter for the remaining 0.25 m length. While the 0.05 m diameter end is fixed, the 0.03 m diameter end of the shaft carries a rotor of mass moment of inertia 14.7 kg-m^2. If the modulus of elasticity of the shaft material is 0.83 x 10^{11} N/m^2, find the natural frequency of torsional oscillations, neglecting the inertia effect of the shaft.

Given Data:

- Diameters:
  - $d_1 = 0.05\,\mathrm{m}$
  - $d_2 = 0.08\,\mathrm{m}$
  - $d_3 = 0.03\,\mathrm{m}$
- Lengths:
  - $L_1 = 0.6\,\mathrm{m}$
  - $L_2 = 1.8\,\mathrm{m}$
  - $L_3 = 0.25\,\mathrm{m}$
- Mass Moment of Inertia: $I = 14.7\,\mathrm{kg\text{-}m}^2$
- Modulus of Elasticity (Shear Modulus): $G = 0.83 \times 10^{11}\,\mathrm{N/m}^2$

Step 1: Calculate Polar Moment of Inertia

The polar moment of inertia J for a circular sl of diameter d is given by:

$$J = \frac{\pi d^4}{32}$$

For the three sections:

$$J_1 = \frac{\pi (0.05)^4}{32} \, \text{m}^4$$

$$J_2 = \frac{\pi (0.08)^4}{32} \, \text{m}^4$$

$$J_3 = \frac{\pi (0.03)^4}{32} \, \text{m}^4$$

Step 2: Calculate Torsional Stiffness

The torsional stiffness K of a shaft section is given by:

$$K = \frac{GJ}{L}$$

For the three sections:

$$K_1 = \frac{GJ_1}{L_1}, \quad K_2 = \frac{GJ_2}{L_2}, \quad K_3 = \frac{GJ_3}{L_3}$$

Step 3: Equivalent Torsional Stiffness

The equivalent torsional stiffness for the stepped shaft can be found by considering the stiffness of each section:

$$\frac{1}{K_{\text{eq}}} = \frac{1}{K_1} + \frac{1}{K_2} + \frac{1}{K_3}$$

Step 4: Natural Frequency

The natural frequency f is given by:

$$f = \frac{1}{2\pi} \sqrt{\frac{K_{\text{eq}}}{I}}$$

Calculation:

Let's proceed with the calculations based on these steps.

I'll first calculate the polar moments of inertia J_1, J_2, J_3, the torsional stiffnesses K_1, K_2, K_3, the equivalent torsional stiffness K_{eq}, and finally the natural frequency f.

Here are the calculated values:

- **Polar Moment of Inertia:**
  - $J_1 = 6.14 \times 10^{-7}\,\mathrm{m}^4$
  - $J_2 = 4.02 \times 10^{-6}\,\mathrm{m}^4$
  - $J_3 = 7.95 \times 10^{-8}\,\mathrm{m}^4$
- **Torsional Stiffness:**
  - $K_1 = 84,880.27\,\text{N-m/rad}$
  - $K_2 = 185,423.78\,\text{N-m/rad}$
  - $K_3 = 26,401.16\,\text{N-m/rad}$
- **Equivalent Torsional Stiffness:** $K_{eq} = 18,164.82\,\text{N-m/rad}$
- **Natural Frequency:** $f = 5.59\,\mathrm{Hz}$

So, the natural frequency of torsional oscillations is approximately **5.59 Hz.**

2) A stepped shaft of 0.05 m in diameter for the first 0.6 m length, 0.08 m diameter for the next 1.8 m and 0.03 m diameter for the remaining 0.25 m length. While the 0.05 m diameter end is fixed, the 0.03 m diameter end of the shaft carries a rotor of mass moment of inertia 14.7 kg-m^2. If the modulus of elasticity of the shaft material is 0.83 x 10^{11} N/m^2, find the natural frequency of torsional oscillations, neglecting the inertia effect of the shaft.

Given Data:

- Mass, $m = 10\,\mathrm{kg}$
- **Natural Rubber:**
  - Stiffness, $k_1 = 3000\,\mathrm{N/m}$
  - Damping coefficient, $c_1 = 100\,\mathrm{N}$\cdotps$\mathrm{/m}$
- **Felt:**
  - Stiffness, $k_2 = 12000\,\mathrm{N/m}$
  - Damping coefficient, $c_2 = 330\,\mathrm{N}$\cdotps$\mathrm{/m}$

Step 1: Equivalent Stiffness

For two springs in series, the equivalent stiffness k_{eq} is given by:

$$\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2}$$

Step 2: Undamped Natural Frequency

The undamped natural frequency ω_n is given by:

$$\omega_n = \sqrt{\frac{k_{eq}}{m}}$$

and the undamped natural frequency in Hz:

$$f_n = \frac{\omega_n}{2\pi}$$

Step 3: Equivalent Damping Coefficient

For damping coefficients in series, the equivalent damping coefficient c_{eq} is given by:

$$c_{eq} = c_1 + c_2$$

Step 4: Damped Natural Frequency

The damped natural frequency ω_d is given by:

$$\omega_d = \omega_n\sqrt{1 - \left(\frac{c_{\text{eq}}}{2m\omega_n}\right)^2}$$

and the damped natural frequency in Hz:

$$f_d = \frac{\omega_d}{2\pi}$$

Calculation:

Let's proceed with these calculations.

It seems like the calculation for the damped natural frequency resulted in a math error, which typically happens when the damping is so high that the system becomes over-damped (i.e., the expression inside the square root becomes ne ↓ ve). This occurs when the damping ratio ζ is greater than 1.

Let's handle this by checking the damping ratio first. If the system is over-damped, we'll recognize it and calculate accordingly. [>-]

Here are the results:

- **Equivalent Stiffness:** $k_{\text{eq}} = 2400\,\text{N/m}$
- **Undamped Natural Frequency:** $f_n = 2.47\,\text{Hz}$
- **Equivalent Damping Coefficient:** $c_{\text{eq}} = 430\,\text{N}\backslash\text{cdotps/m}$
- **Damping Status:** The system is **overdamped.**

Since the system is overdamped, it does not exhibit oscillatory behavior, so there is no damped natural frequency (i.e., $f_d = 0$). [>-]

UNIT 5: MECHANISMS FOR CONTROL

Mechanisms for Control

1. **What is a governor in mechanical systems?**
   - A governor is a device used to regulate the speed of an engine or machine by adjusting the fuel supply or other control inputs in response to changes in load.
2. **Explain the working principle of a centrifugal governor.**

    - A centrifugal governor operates by using rotating flyweights that move outward due to centrifugal force as the speed increases. This movement adjusts the fuel supply or throttle to maintain a constant speed.
3. **What is the purpose of a flyball governor?**
    - A flyball governor, also known as a Watt governor, controls the speed of an engine by adjusting the fuel input. It uses spinning balls that move outward as the engine speed increases, regulating the fuel valve to maintain steady operation.
4. **Define the term 'sensitivity' in the context of governors.**
    - Sensitivity refers to the ability of a governor to respond to small changes in speed. A highly sensitive governor quickly adjusts the control mechanism in response to slight variations in engine speed.
5. **What is hunting in a governor?**
    - Hunting occurs when a governor over-corrects for speed changes, causing the engine to oscillate between high and low speeds instead of stabilizing at the desired speed.
6. **Differentiate between isochronous and non-isochronous governors.**
    - An isochronous governor maintains a constant speed regardless of load changes, while a non-isochronous governor allows slight variations in speed depending on the load.
7. **What is a servomechanism?**
    - A servomechanism is an automatic device that uses feedback to control the position, speed, or torque of a mechanical system. It is commonly used in applications requiring precise control.
8. **Explain the principle of feedback control in servomechanisms.**
    - Feedback control involves measuring the output of a system and comparing it to the desired setpoint. The difference (error) is used to adjust the input to bring the output closer to the desired value.
9. **What is the role of a proportional-integral-derivative (PID) controller?**
    - A PID controller is used in control systems to continuously calculate the error between a desired setpoint and the actual output, and then apply corrective actions to minimize the error over time.
10. **Define the term 'dead zone' in control systems.**
    - The dead zone is a range of input values in which a control system does not respond. It represents the insensitivity of the system to small changes in input.
11. **What is the function of a tachogenerator in control mechanisms?**
    - A tachogenerator is a device that converts rotational speed into an electrical signal, typically used in feedback control systems to monitor and adjust the speed of motors.
12. **Explain the concept of damping in control systems.**
    - Damping in control systems refers to the reduction of oscillations or vibrations in response to a disturbance. Proper damping helps achieve a stable and smooth response without excessive overshoot.
13. **What is the purpose of a cam-follower mechanism in control systems?**
    - A cam-follower mechanism is used to convert rotary motion into linear motion or vice versa. It is commonly used in control systems to achieve precise movement patterns, such as in automated machinery.
14. **Differentiate between open-loop and closed-loop control systems.**

- An open-loop control system operates without feedback, meaning it does not adjust based on the output. A closed-loop system uses feedback to continuously monitor and adjust the output to match the desired input.

15. **What is the role of a limiter in a control mechanism?**
    - A limiter restricts the range of output in a control system, preventing the system from exceeding predefined limits, which can protect the system from damage or malfunction.

The arms of a Porter governor are each 250 mm long and pivoted on the governor axis. The mass of each ball is 5 kg and the mass of the central sleeve is 30 kg. The radius of rotation of the balls is 150 mm when the sleeve begins to rise and reaches a value of 200 mm for maximum speed. Determine the speed range of the governor. If the friction at the sleeve is equivalent of 20 N of load at the sleeve, determine how the speed range is modified.

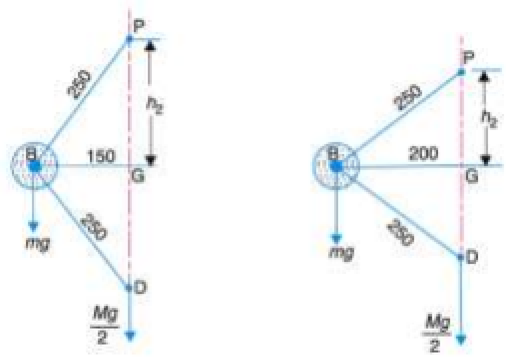
1) The arms of a Porter governor are each 250 mm long and pivoted on the governor axis. The mass of each ball is 5 kg and the mass of the central sleeve is 30 kg. The radius of rotation of the balls is 150 mm when the sleeve begins to rise and reaches a value of 200 mm for maximum speed. Determine the speed range of the governor. If the friction at the sleeve is equivalent of 20 N of load at the sleeve, determine how the speed range is modified.

Given : $BP = BD = 250$ mm ; $m = 5$ kg ; $M = 30$ kg ; $r1 = 150$ mm ; $r2 = 200$ mm

First of all, let us find the minimum and maximum speed of the governor. The minimum and maximum position of the governor is shown in Fig. (a) and (b) respectively.

Let N1 = Minimum speed when r1 = BG = 150 mm, and

N2 = Maximum speed when r2 = BG = 200 mm.



(a) Minimum position. (b) Maximum position.

Speed range of the governor

From Fig. (*a*), we find that height of the governor,

$$h_1 = PG = \sqrt{(PB)^2 - (BG)^2} = \sqrt{(250)^2 - (150)^2} = 200 \text{ mm} = 0.2 \text{ m}$$

$$(N_1)^2 = \frac{m + M}{m} \times \frac{895}{h_1} = \frac{5 + 30}{5} \times \frac{895}{0.2} = 31\,325$$

$$N_1 = 177 \text{ r.p.m.}$$

Height of the governor,

$$h_2 = PG = \sqrt{(PB)^2 - (BG)^2} = \sqrt{(250)^2 - (200)^2} = 150 \text{ mm} = 0.15 \text{ m}$$

$$(N_2)^2 = \frac{m + M}{m} \times \frac{895}{h_2} = \frac{5 + 30}{5} \times \frac{895}{0.15} = 41\,767$$

$$N_2 = 204.4 \text{ r.p.m.}$$

speed range of the governor= $N2 - N1 = 204.4 - 177 = 27.4$ r.p.m.

Speed range when friction at the sleeve is equivalent of 20N of load (i.e. when F = 20 N)

We know that when the sleeve moves downwards, the friction force (F) acts upwards and the minimum speed is given by

$$(N_1)^2 = \frac{m.g + (M.g - F)}{m.g} \times \frac{895}{h_1}$$

$$= \frac{5 \times 9.81 + (30 \times 9.81 - 20)}{5 \times 9.81} \times \frac{895}{0.2} = 29500$$

$$\therefore \quad N_1 = 172 \text{ r.p.m.}$$

We also know that when the sleeve moves upwards, the frictional force (F) acts downwards and the maximum speed is given by

$$(N_2)^2 = \frac{m.g + (M.g + F)}{m.g} \times \frac{895}{h_2}$$

$$= \frac{5 \times 9.81 + (30 \times 9.81 + 20)}{5 \times 9.81} \times \frac{895}{0.15} = 44\,200$$

$$\therefore \quad N_2 = 210 \text{ r.p.m.}$$

Speed range of the governor = $N2 - N1 = 210 - 172 = 38$ r.p.m.

The arms of a Porter governor are 300 mm long. The upper arms are pivoted on the axis of rotation. The lower arms are attached to a sleeve at a distance of 40 mm from the axis of rotation. The mass of the load on the sleeve is 70 kg and the mass of each ball is 10 kg. Determine the equilibrium speed when the radius of rotation of the balls is 200 mm. If the friction is equivalent to a load of 20 N at the sleeve, what will be the range of speed for this position ?

2)The arms of a Porter governor are 300 mm long. The upper arms are pivoted on the axis of rotation. The lower arms are attached to a sleeve at a distance of 40 mm from the axis of rotation. The mass of the load on the sleeve is 70 kg and the mass of each ball is 10 kg. Determine the equilibrium speed when the radius of rotation of the balls is 200 mm. If the

friction is equivalent to a load of 20 N at the sleeve, what will be the range of speed for this position ?

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Solution. Given : BP = BD = 300 mm ; DH = 40 mm ; M = 70 kg ; m = 10 kg ; r = BG = 200 mm

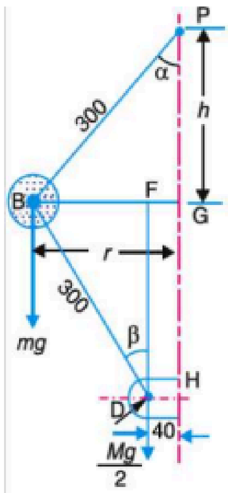
Equilibrium speed when the radius of rotation r = BG = 200 mm

Let N = Equilibrium speed.

The equilibrium position of the governor is shown in Fig. From the figure, we find that height of the governor,

$$h = PG = \sqrt{(BP)^2 - (BG)^2} = \sqrt{(300)^2 - (200)^2} = 224 \text{ mm}$$

$$= 0.224\text{m}$$



All dimensions in mm.

BF = BG – FG = 200 – 40 = 160 . . . (FG = DH)

and $$DF = \sqrt{(DB)^2 - (BF)^2} = \sqrt{(300)^2 - (160)^2} = 254 \text{ mm}$$

$$\therefore \tan\alpha = BG/PG = 200 / 224 = 0.893$$

and $$\tan\beta = BF/DF = 160 / 254 = 0.63$$

$$\therefore \quad q = \frac{\tan\beta}{\tan\alpha} = \frac{0.63}{0.893} = 0.705$$

We know that

$$N_2 = \frac{m + \frac{M}{2}(1+q)}{m} \times \frac{895}{h}$$

$$= \frac{10 + \frac{70}{2}(1+0.705)}{10} \times \frac{895}{0.224} = 27\,840$$

$\therefore \quad N_2 = 167$ r.p.m. **Ans.**

Range of speed when friction is equivalent to load of 20 N at the sleeve (i.e. when F = 20 N)

Let N1 = Minimum equilibrium speed, and
N2 = Maximum equilibrium speed.

We know that when the sleeve moves downwards, the frictional force (F) acts upwards and the minimum equilibrium speed is given by

$$(N_1)^2 = \frac{m.g + \left(\frac{M.g - F}{2}\right)(1+q)}{m.g} \times \frac{895}{h}$$

$$= \frac{10 \times 9.81 + \left(\frac{70 \times 9.81 - 20}{2}\right)(1+0.705)}{10 \times 9.81} \times \frac{895}{0.224} = 27\,144$$

N1 = 164.8 r.p.m.

We also know that when the sleeve moves upwards, the frictional force (F) acts downwards and the maximum equilibrium speed is given by

$$(N_2)^2 = \frac{m.g + \left(\frac{M.g + F}{2}\right)(1+q)}{m.g} \times \frac{895}{h}$$

$$= \frac{10 \times 9.81 + \left(\frac{70 \times 9.81 + 20}{2}\right)(1+0.705)}{10 \times 9.81} \times \frac{895}{0.224} = 28\,533$$

$\therefore \quad N_2 = 169$ r.p.m.

We know that range of speed = $N_2 - N_1$ = – 164.8 = 4.2 r.p.m.

A Proell governor has equal arms of length 300 mm. The upper and lower ends of the arms are pivoted on the axis of the governor. The extension arms of the lower links are each 80 mm long and parallel to the axis when the radii of rotation of the balls are 150 mm and 200 mm. The mass of each ball is 10 kg and the mass of the central load is 100 kg. Determine the range of speed of the governor.

4) A Proell governor has equal arms of length 300 mm. The upper and lower ends of the arms are pivoted on the axis of the governor. The extension arms of the lower links are each 80 mm long and parallel to the axis when the radii of rotation of the balls are 150 mm and 200 mm. The mass of each ball is 10 kg and the mass of the central load is 100 kg. Determine the range of speed of the governor.

Given :

PF = DF = 300 mm ; BF = 80 mm ; m = 10 kg ; M = 100 kg ;r_1 = 150 mm; r_2 = 200 mm

First of all, let us find the minimum and maximum speed of the governor. The minimum and maximum position of the governor is shown in Fig

Let N_1 = Minimum speed when radius of rotation, r_1 = FG = 150 mm ; and
N_2 = Maximum speed when radius of rotation , r_2 = FG = 200 mm.

From Fig. (a), we find that height of the governor,

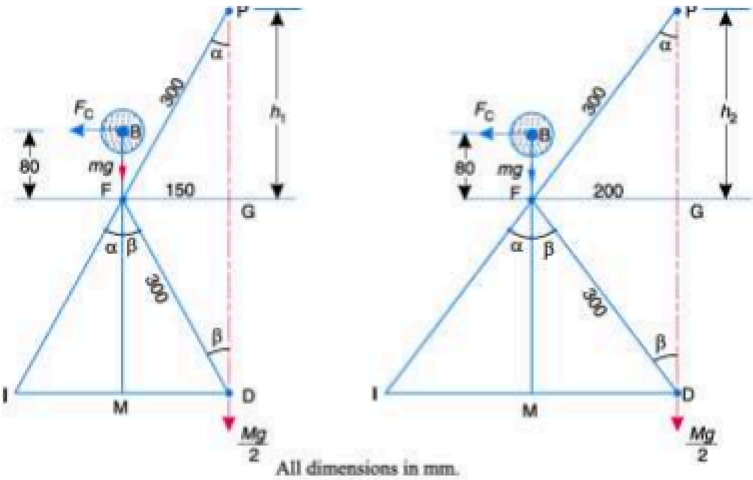
$$h_1 = PG = \sqrt{(PF)^2 - (FG)^2} = \sqrt{(300)^2 - (150)^2} = 260 \text{ mm} = 0.26 \text{ m}$$

$$FM = GD = PG = 260 \text{ mm} = 0.26 \text{ m}$$
$$BM = BF + FM = 80 + 260 = 340 \text{ mm} = 0.34 \text{ m}$$

$$(N_1)^2 = \frac{FM}{BM}\left(\frac{m+M}{m}\right)\frac{895}{h_1} \qquad \ldots (\because \alpha = \beta \text{ or } q = 1)$$

$$= \frac{0.26}{0.34}\left(\frac{10+100}{10}\right)\frac{895}{0.26} = 28\,956 \quad \text{or} \quad N_1 = 170 \text{ r.p.m.}$$

Now from Fig. *b*, we find that height of the governor,



All dimensions in mm.

(*a*) Minimum position. (*a*) Maximum position.

$$h_2 = PG = \sqrt{(PF)^2 - (FG)^2} = \sqrt{(300)^2 - (200)^2} = 224 \text{ mm} = 0.224 \text{ m}$$

and $$FM = GD = PG = 224 \text{ mm} = 0.224 \text{ m}$$

$\therefore$ $$BM = BF + FM = 80 + 224 = 304 \text{ mm} = 0.304 \text{ m}$$

We know that $(N_2)^2 = \frac{FM}{BM}\left(\frac{m + M}{m}\right)\frac{895}{h_2}$ ... ($\because$ $\alpha = \beta$ or $q = 1$)

$$= \frac{0.224}{0.304}\left(\frac{10 + 100}{10}\right)\frac{895}{0.224} = 32\ 385 \quad \text{or} \quad N_2 = 180 \text{ r.p.m.}$$

We know that range of speed

$$= N_2 - N_1 = 180 - 170 = 10 \text{ r.p.m.}$$ **Ans.**

Note : The example may also be solved as discussed below :

From Fig. 18.13 (*a*), we find that

$$\sin\alpha = \sin\beta = 150 / 300 = 0.5 \quad \text{or} \quad \alpha = \beta = 30°$$

and $$MD = FG = 150 \text{ mm} = 0.15 \text{ m}$$

$$FM = FD\cos\beta = 300\cos 30° = 260 \text{ mm} = 0.26 \text{ m}$$

$$IM = FM\tan\alpha = 0.26\tan 30° = 0.15 \text{ m}$$

$$BM = BF + FM = 80 + 260 = 340 \text{ mm} = 0.34 \text{ m}$$

$$ID = IM + MD = 0.15 + 0.15 = 0.3 \text{ m}$$

We know that centrifugal force,

$$F_C = m\,(\omega_1)^2\, r_1 = 10\left(\frac{2\pi N_1}{60}\right)^2 0.15 = 0.0165\,(N_1)^2$$

Now taking moments about point *I*,

$$F_C \times BM = m.g \times IM + \frac{M.g}{2} \times ID$$

or $$0.0165\,(N_1)^2\, 0.34 = 10 \times 9.81 \times 0.15 + \frac{100 \times 9.81}{2} \times 0.3$$

$$0.0056\,(N_1)^2 = 14.715 + 147.15 = 161.865$$

$\therefore$ $$(N_1)^2 = \frac{161.865}{0.0056} = 28\ 904 \quad \text{or} \quad N_1 = 170 \text{ r.p.m.}$$

Similarly N_2 may be calculated.

A Hartnell governor having a central sleeve spring and two right-angled bell crank levers moves between 290 r.p.m. and 310 r.p.m. for a sleeve lift of 15 mm. The sleeve arms and the ball arms are 80 mm and 120 mm respectively. The levers are pivoted at 120 mm from the governor axis and mass of each ball is 2.5 kg. The ball arms are parallel to the governor axis at the lowest equilibrium speed. Determine : 1. loads on the spring at the lowest and the highest equilibrium speeds, and 2. stiffness of the spring.

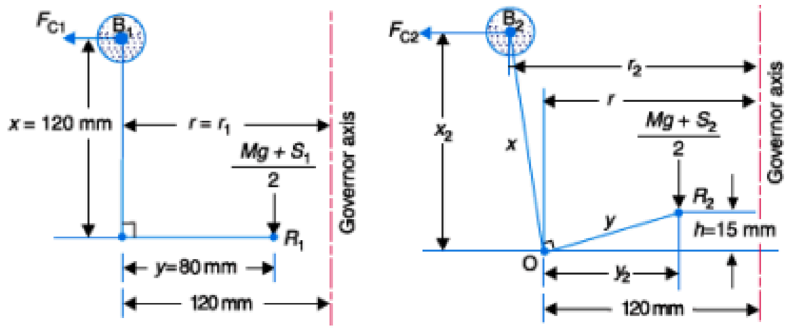
A Hartnell governor having a central sleeve spring and two right-angled bell crank levers moves between 290 r.p.m. and 310 r.p.m. for a sleeve lift of 15 mm. The sleeve arms and the ball arms are 80 mm and 120 mm respectively. The levers are pivoted at 120 mm from the governor axis and mass of each ball is 2.5 kg. The ball arms are parallel to the governor axis at the lowest equilibrium speed. Determine : 1. loads on the spring at the lowest and the highest equilibrium speeds, and 2. stiffness of the spring.

Solution. Given : N1 = 290 r.p.m. or ω_1 = 2 _ × 290/60 = 30.4 rad/s ; N2 = 310 r.p.m. or ω_2 = 2 _ × 310/60 = 32.5 rad/s ; h = 15 mm = 0.015 m ; y = 80 mm = 0.08 m ; x = 120 mm = 0.12 m ; r = 120 mm = 0.12 m ; m = 2.5 kg

1. Loads on the spring at the lowest and highest equilibrium speeds

Let S = Spring load at lowest equilibrium speed, and
S2 = Spring load at highest equilibrium speed.

Since the ball arms are parallel to governor axis at the lowest equilibrium speed (i.e. at N1 = 290 r.p.m.), as shown in Fig. (a), therefore r = r1 = 120 mm = 0.12 m



(a) Lowest position.

(b) Highest position.

We know that centrifugal force at the minimum speed,
$FC1 = m\,(\omega_1)^2\,r1 = 2.5\,(30.4)^2\,0.12 = 277$ N

Now let us find the radius of rotation at the highest equilibrium speed, *i.e.* at $N2 = 310$ r.p.m. The position of ball arm and sleeve arm at the highest equilibrium speed is shown in Fig. (*b*).

Let $r2$ = Radius of rotation at $N2 = 310$ r.p.m.

$$h = (r_2 - r_1)\,\frac{y}{x}$$

$$r_2 = r_1 + h\left(\frac{x}{y}\right) = 0.12 + 0.015\left(\frac{0.12}{0.08}\right) = 0.1425 \text{ m}$$

Centrifugal force at the maximum speed,

$$FC_2 = m\,(\omega_2)^2\,r_2 = 2.5 \times (32.5)2 \times 0.1425 = 376 \text{ N}$$

Neglecting the obliquity effect of arms and the moment due to the weight of the balls, we have for lowest position,

$$M.g + S_1 = 2F_{C1} \times \frac{x}{y} = 2 \times 277 \times \frac{0.12}{0.08} = 831 \text{ N}$$

$$\therefore \quad S_2 = 831 \text{ N} \qquad (\because M = 0)$$

and for highest position,

$$M.g + S_2 = 2F_{C2} \times \frac{x}{y} = 2 \times 376 \times \frac{0.12}{0.08} = 1128 \text{ N}$$

$$\therefore \quad S_1 = 1128 \text{ N} \qquad (\because M = 0)$$

2. *Stiffness of the spring*

We know that stiffness of the spring,

$$s = \frac{S_2 - S_1}{h} = \frac{1128 - 831}{15} = 19.8 \text{ N/mm}$$

A Porter governor has equal arms each 250 mm long and pivoted on the axis of rotation. Each ball has a mass of 5 kg and the mass of the central load on the sleeve is 25 kg. The radius of rotation of the ball is 150 mm when the governor begins to lift and 200 mm when the governor is at maximum speed. Find the range of speed, sleeve lift, governor effort and power of the governor in the following cases :

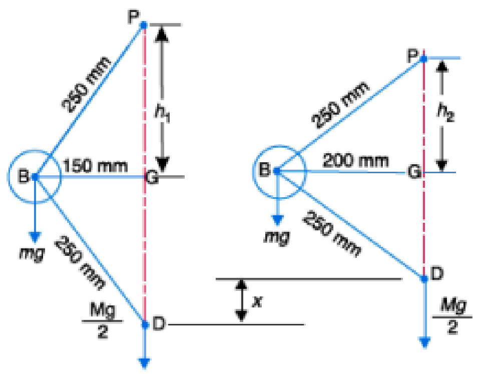
1. When the friction at the sleeve is neglected, and
2. When the friction at the sleeve is equivalent to 10 N.

Given : $BP = BD = 250$ mm ; $m = 5$ kg ; $M = 25$ kg ; $r1 = 150$ mm ; $r2 = 200$ mm ; $F = 10$ N

1. When the friction at the sleeve is neglected

First of all, let us find the minimum and maximum speed of rotation. The minimum and maximum position of the governor is shown in Fig. 18.34 (*a*) and (*b*) respectively.

Let $N1$ = Minimum speed, and
N2 = Maximum speed.



(*a*) Minimum position. (*b*) Maximum position.

From Fig a

$$h_1 = PG = \sqrt{(BP)^2 - (BG)^2} = \sqrt{(250)^2 - (150)^2} = 200 \text{ mm} = 0.2 \text{ m}$$

From Fig. (*b*),

$$h_2 = PG = \sqrt{(BP)^2 - (BG)^2} = \sqrt{(250)^2 - (200)^2} = 150 \text{ mm} = 0.15 \text{ m}$$

We know that $(N_1)^2 = \frac{m + M}{m} \times \frac{895}{h_1} = \frac{5 + 25}{5} \times \frac{895}{0.2} = 26\,850$

$\therefore$ $N_1 = 164$ r.p.m.

$$(N_2)^2 = \frac{m+M}{m} \times \frac{895}{h_2} = \frac{5+25}{5} \times \frac{895}{0.15} = 35\,800$$

$\because$ $N_2 = 189$ r.p.m.

Range of speed

We know that range of speed = N2 – N1 = 189 – 164 = 25 r.p.m.

Sleeve lift

We know that sleeve lift, x = 2 ($h_1 - h_2$) = 2 (200 – 150) = 100 mm = 0.1 m

Governor effort

Let c = Percentage increase in speed.

We know that increase in speed or range of speed,

c.N1 = $N_2 - N_1$ = 25 r.p.m.

c = $25/N_1$ = 25/164 = 0.152

We know that governor effort
P = c (m + M) g = 0.152 (5 + 25) 9.81 = 44.7 N

Power of the governor
We know that power of the governor = P.x = 44.7 × 0.1 = 4.47 N-m

2. When the friction at the sleeve is taken into account

$$(N_1)^2 = \frac{m.g + (M.g - F)}{m.g} \times \frac{895}{h_1}$$
$$= \frac{5 \times 9.81 + (25 \times 9.81 - 10)}{5 \times 9.81} \times \frac{895}{0.2} = 25\,938$$
$$N_1 = 161 \text{ r.p.m.}$$
$$(N_2)^2 = \frac{m.g + (M.g + F)}{m.g} \times \frac{895}{h_2}$$
$$= \frac{5 \times 9.81 + (25 \times 9.81 + 10)}{5 \times 9.81} \times \frac{895}{0.15} = 37\,016$$
$$N_2 = 192.4 \text{ r.p.m.}$$

Range of speed
We know that range of speed = N2 – N1 = 192.4 – 161 = 31.4 r.p.m.

Sleeve lift
The sleeve lift (x) will be same as calculated above.
Sleeve lift, x = 100 mm = 0.1 m

Governor effort
Let c = Percentage increase in speed.
We know that increase in speed or range of speed,
c.N1 = N2 – N1 = 31.4 r.p.m.
c = 31.4/N1 = 31.4/161 = 0.195

We know that governor effort,
P = c (m.g + M.g + F) = 0.195 (5 × 9.81 + 25 × 9.81 + 10) N= 57.4 N

Power of the governor
We know that power of the governor= P.x = 57.4 × 0.1 = 5.74 N-m

The turbine rotor of a ship has a mass of 3500 kg. It has a radius of gyration of 0.45 m and a speed of 3000 r.p.m. clockwise when looking from stern. Determine the gyroscopic couple and its effect upon the ship: 1. when the ship is steering to the left on a curve of 100 m radius at a speed of 36 km/h. 2. when the ship is pitching in a simple harmonic motion, the bow falling with its maximum velocity. The period of pitching is 40 seconds and the total angular displacement between the two extreme positions of pitching is 12 degrees.

5. The turbine rotor of a ship has a mass of 3500 kg. It has a radius of gyration of 0.45 m and a speed of 3000 r.p.m. clockwise when looking from stern. Determine the gyroscopic couple and its effect upon the ship:

1. when the ship is steering to the left on a curve of 100 m radius at a speed of 36 km/h.
2. when the ship is pitching in a simple harmonic motion, the bow falling with its maximum velocity. The period of pitching is 40 seconds and the total angular displacement between the two extreme positions of pitching is 12 degrees.

Solution

Given : m = 3500 kg ; k = 0.45 m; N = 3000 r.p.m. or $\omega = 2\pi \times 3000/60 = 314.2$ rad/s

1. When the ship is steering to the left Given: R =100 m ; v = km/h = 10 m/s

We know that mass moment of inertia of the rotor,

$$I = m.k2 = 3500\ (0.45)2 = 708.75 \text{ kg-m2}$$

and angular velocity of precession,

$$\omega_p = v/R = 10/100 = 0.1 \text{ rad/s}$$

∴ Gyroscopic couple,

$$C = I.\omega.\ \omega_p = 708.75 \times 314.2 \times 0.1 = 22\ 270 \text{ N-m}$$
$$= 22.27 \text{ kN-m Ans.}$$

When the rotor rotates clockwise when looking from the stern and the ship takes a left turn, the effect of the reactive gyroscopic couple is to raise the bow and lower the stern.

2. When the ship is pitching with the bow falling

Given: tp = 40 s

Since the total angular displacement between the two extreme positions of pitching is 12°

(i.e. $2\varphi = 12°$), therefore amplitude of swing,

$$\varphi = 12 / 2 = 6° = 6 \times \pi/180 = 0.105 \text{ rad}$$

and angular velocity of the simple harmonic motion,

$$\omega 1 = 2\pi / tp = 2\pi / 40 = 0.157 \text{ rad/s}$$

We know that maximum angular velocity of precession,

$$\omega_p = \varphi.\omega 1 = 0.105 \times 0.157 = 0.0165 \text{ rad/s}$$

∴ Gyroscopic couple,

$$C = I.\omega.\ \omega_p = 708.75 \times 314.2 \times 0.0165 = 3675 \text{ N-m}$$
$$= 3.675 \text{ kN-m Ans.}$$

When the bow is falling (i.e. when the pitching is downward), the effect of the reactive gyroscopic couple is to move the ship towards port side.

1. Introduction to Gyroscopic Stabilization

Gyroscopic stabilization refers to the phenomenon where a rotating body, due to its angular momentum, resists changes to its axis of rotation. This resistance to external forces that try to change the direction of the axis is utilized in various mechanical systems to provide stability. The principle is widely applied in navigation and stabilization systems for vehicles like ships, airplanes, and even bicycles.

2. Gyroscopic Principles

When a rigid body spins about an axis, it possesses angular momentum (L), which is given by:

$$L = I \cdot \omega$$

where:

- I is the moment of inertia of the body about the axis of rotation,
- ω is the angular velocity.

If an external torque (T) is applied perpendicular to the axis of rotation, it induces a change in angular momentum, causing the axis of rotation to precess (change direction) rather than the spinning object tipping over.

The rate of precession Ω is related to the applied torque by:

$$T = \frac{dL}{dt} = I \cdot \omega \cdot \Omega$$

This equation implies that the faster the object spins, the more stable it is against external disturbances.

3. Applications of Gyroscopic Stabilization

a. Marine Vessels

In ships, gyroscopes are used to stabilize the vessel against rolling in rough seas. The gyroscope's angular momentum helps counteract the tilting effect of waves. Stabilizing gyroscopes are mounted on gimbals that allow them to freely rotate in any direction. When the ship rolls, the gyroscope's precession produces a counteracting torque that helps level the ship.

b. Aircraft

Gyroscopic principles are crucial in aircraft for maintaining orientation and stability. For instance, in an airplane, the spinning rotors of the engines or the gyroscopic instruments help maintain the aircraft's heading and altitude by resisting changes in orientation. This resistance provides stability against pitch, roll, and yaw movements.

c. Bicycles and Motorcycles

The wheels of a bicycle or motorcycle act as gyroscopes. As the wheel spins, it stabilizes the bike, making it easier to maintain balance. The faster the wheels spin, the more stable the bike becomes.

5. Example: Gyroscopic Stabilization in Ships

Consider a ship equipped with a gyroscopic stabilizer. Suppose the stabilizer has a moment of inertia III and spins with angular velocity ω\omegaω. When the ship rolls due to a wave, a torque TTT is applied perpendicular to the spin axis of the gyroscope. The gyroscope generates a precession motion that counteracts the rolling motion, stabilizing the ship.

The centrifugal governors are based on the balancing of centrifugal force on the rotating balls by an equal and opposite radial force, known as the ***controlling force****.It consists of two balls of equal mass, which are attached to the arms as shown in Fig. 1. These balls are known as ***governor balls* or *fly balls*.** The balls revolve with a spindle, which is driven by the engine through bevel gears. The upper ends of the arms are pivoted to the spindle, so that the balls may rise up or fall down as they revolve about the vertical axis. The arms are connected by the links to a sleeve, which is keyed to the spindle. This sleeve revolves with the spindle; but can slide up and down. The balls and the sleeve rises when the spindle speed increases, and falls when the speed decreases. In order to limit the travel of the sleeve in upward and downward directions, two stops *S*, *S* are provided on the spindle. The sleeve is connected by a bell crank lever to a throttle valve. The supply of the working fluid decreases when the sleeve rises and increases when it falls. When the load on the engine increases, the engine and the governor speed decreases. This results in the decrease of centrifugal force on the balls. Hence the balls move inwards and the sleeve moves downwards. The downward movement of the sleeve operates a throttle valve at the other end of the bell crank lever to increase the supply of working fluid and thus the engine speed is increased. In this case, the extra power output is provided to balance the increased load. When the load on the engine decreases, the engine and the governor speed increases, which results in the increase of centrifugal force on the balls. Thus the balls move outwards and the sleeve rises upwards. This upward movement of the sleeve reduces the supply of the working

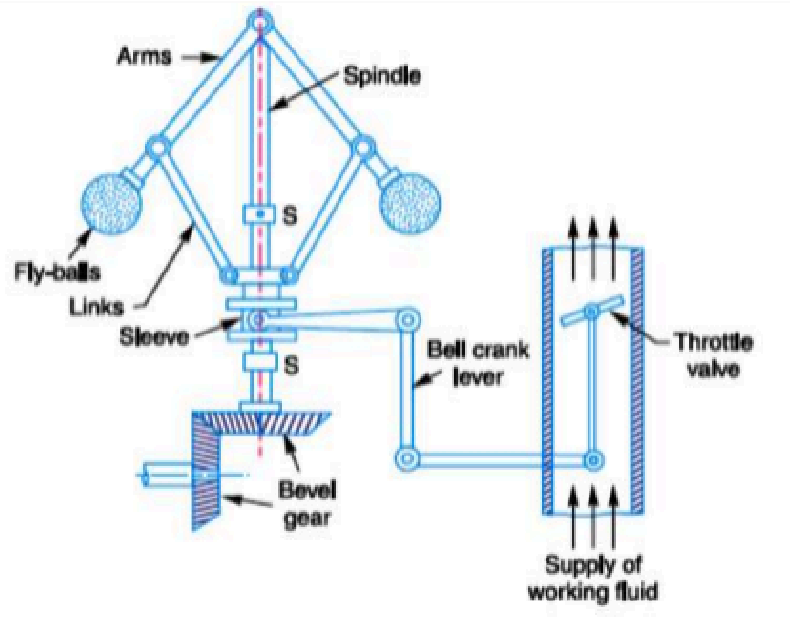


Fig. 1. Centrifugal governor

Terms Used in Governors

The following terms used in governors are important from the subject point of view ;

1. *Height of a governor*. It is the vertical distance from the centre of the ball to a point where the axes of the arms (or arms produced) intersect on the spindle axis. It is usually denoted by *h*.

2. *Equilibrium speed*. It is the speed at which the governor balls, arms etc., are in complete equilibrium and the sleeve does not tend to move upwards or downwards.

3. *Mean equilibrium speed*. It is the speed at the mean position of the balls or the sleeve.

4. *Maximum and minimum equilibrium speeds*. The speeds at the maximum and minimum radius of rotation of the balls, without tending to move either way are known as maximum and minimum equilibrium speeds respectively.

Note : There can be many equilibrium speeds between the mean and the maximum and the mean and the minimum equilibrium speeds.

5. *Sleeve lift*. It is the vertical distance which the sleeve travels due to change in equilibrium speed.