PIMs Thoughts

For me this book was an answer to a question I had fruitlessly scoured the internet for many times: “How do I learn math?”

The standard responses on Hacker News is Khan Academy, or Spivak’s Calculus… Which immediately struck me as suspicious, because they are VERY different resources. I tried the Khan Academy route. Truly an amazing site, with gifted instruction - but it wasn’t what I was looking for. Surely, there must be more to math than recognizing how to solve a class of problems?

Unlike many other software engineers math never came easily to me. When I was in kindergarten my family fixed my issues with basic counting by playing hours of Blackjack with me. In high school my math scores were all over the board, the only theme being an inability to understand why I should care about the subject, falling into the same ‘I’m not good at this’ ditch that so many people do.

What still strikes me as funny is that I got a 1 on my AP Calculus test (~F) and I got a 5 (~A+) on my AP English test. The determining factor was definitely that I loved re-reading poems until I found deeper understanding and new connections in the material - the exact thing I am now beginning to do with math. I often wonder if math was taught in a different way how different my life would be.

Luckily, I stumbled on A Programmer's Introduction to Mathematics. Its promise is the very thing that I desire: The ability to engage with any mathematics that I choose. Access to the magical part of the world, not just the tools that numerical libraries package up for me. New tools of thought, new abstractions, and what is so far the most difficult learning project of my life.

Meant for this to just be feedback, but I think the story is informative in a way that a list of likes / needs might not be. tldr; I love this book, earning mathematical maturity is hard.

Here is my experience with reading/working through it so far:

Thankful that it explains the ‘culture’ of math. So much math seems to start in the middle and these explanations shed light on some of the walls I’ve run into previously.

Understanding new things is an absolute rush. I knew this from programming, but in programming the computer tells you you are right. In math your brain tells you you are right. It’s a different thing.

I struggled with the Polynomial chapter to the extent that I put the book down and resumed with the chapter on Sets. I’m not sure why this happened. My best guess was perhaps it was the niggling feeling that we still weren’t getting to the bones of math (how can we define the polynomial function without first having a definition of a function?)... but this is just filling in reasons knowing where I am now. I still left this chapter with the tools I needed: How to read a definition and write down examples, and a bit more ability to parse mathematics. I don't think it really even dampened my excitement to temporarily wave the white flag on this chapter.

When I came back to the book I attacked Sets with abandon. Things felt better, I was starting to 'chunk' definitions for myself so I could more easily bring them to hand, my ability to slow down and understand concepts was better - but again something made me put the book down (but for a specific purpose this time).

It might be worth mentioning some things.

First: I have never encountered proofs before. Turns out many people see them in high school - I never did, and I took almost no math in college.

Second, I am (or at least consider myself) a learning machine. I've skilled up in vastly different industries. I've learned new programming languages and frameworks rapidly for work. I read a lot about "learning how to learn" and I utilize tools like Anki spaced-repetition software.

All this to say I have some discipline around learning, as well as some rules. And one of those rules is that when I detect a fundamental skill that I am missing I basically drop everything and acquire it. Proofs, it seemed to me (and the book told me!) are a fundamental building block, and I was starting to feel that I was delaying the inevitable by not getting them in my toolkit.

If ever one has underestimated the size of a task it was me making that choice. But, even with my new found (very basic) understanding of proofs I was able to come back to your book and, with plenty of sweat, not just get through but begin to really enjoy the chapter on Sets. I'm open to the idea that it might not even be knowledge about proofs, but the mathematical maturity I gained by chasing them that changed things.

So what did I do? First off I found a short but widely recommended book that teaches proofs: Keith Devlin's 'Introduction to Mathematical Thinking'. Something I learned about math books through this experience: number of pages has nothing to do with how long a book is, the number of questions is what really determines speed. I didn't get all of the way through this, but it got me over the True/False beginner stuff, a lot more practice with quantified notation, and I got to read and (try) to write some proofs.

Something I would like to write about someday is exactly how long to struggle with a problem before getting help. Devlin's book has no answer guide to its questions, and I was starting to feel like I might be tricking myself. Without (imo) enough preparation Devlin's book asks the reader to prove that the square root of three is irrational, and failing at this over a longish period of time I I felt like I had gotten what I was going to get from that resource and sought out some other beginner proof information. I then went through the 3rd chapter of 'How to Prove It', got a little from that, then found the 4th chapter of 'Book of Proof', which was my favorite of all.

What I really needed it seems was to see and write some proofs with simple concepts (proofs about odd and even worked well for this), understand the rules of the game, and build some more familiarity with what they look like. In PIMs you use the example of integers that are divisible by 57 being a subset of integers divisible by 3. At my current level of mathematical maturity I was surprised to see two equal statements used together in the way that proofs so often do.

The thing that polished this whole tour off was the most exciting: Elias Zakon's 'Basic Concepts of Mathematics'. There it was: the thing I barely knew I had been looking for, the bare metal of mathematics, the axioms of an ordered field. So exciting. I spent some time working through chapter 2, and, just like peeking at the C code that underlies Ruby, I was satisfied that it exists and ready to come back to PIMs.

This whole time, of course, my ability to read and think about math was increasing. This is one of the things that I don't know is completely describable: things come into focus faster now. It is analogous to the first time I saw a piece of JSON. In a literal sense I couldn't internally parse what I saw as keys and lists. With math the feeling is much more general. It's likely that your book gave me the tools I needed to try to learn about proofs, which in turn gave me the tools I needed to come back to your book. A process I have come to think of as "breaking your brain with mathematics" - changing the way you think in order to approach the subject.

This leads me to my biggest suggestion: Perhaps you should include an invitation to the reader to reach outside of the book while learning. I know that I was personally a little disappointed in myself that I had to bring in other resources, but my experience of proofs showed me that this is ridiculous. I'm glad your book doesn't go deeper into proofs: there are many resources for that. Your book it seems to me is more about guiding me through a foriegn city rather than making me a master of its internal infrastructure. Could just be a personal hang up though!

This ties to a second suggestion, which would be to not only recommend that the reader create their own examples (which you and Feynman are happily agreed upon) but to also suggest that if they cannot feel confident in their own examples that they go searching for ones they like. I was struggling with the product of two sets and Wikipedia immediately handed me the example of playing cards, and that stuck like glue. Sometimes my internal parser isn't strong enough to know if my example is valid, and a quick search can help me grok a subtlety I had missed.

I have to commend you on the many ways you point out that with math the struggle is the rule not the exception. I now feel that when I self-correct I have built a new muscle. When learning about 'Stable Marriages' I stared at the definition of stable bijections for a long time. I knew what it was trying to say, but to me it seemed to say the opposite. $rank\_m(w) < rank\_m(f(m))$. I consolidated so much knowledge about bijections, ranking functions, and functions themselves trying to figure out what I was doing wrong… and it turned out that I had forgotten that a lower rank is BETTER. I was reading “less than” as if it was “prefers less than”. The struggle was the part that solidified my understanding, the actual mistake was kind of silly.

I alluded to it earlier, but this might also be useful in your book: some guidance on how long to struggle with a question, exercise or concept. This is something I am writing about in my journal all the time: “What do you miss out on by looking up the answer?”. Is there a great reward for struggling through every problem with no help, or do I sometimes not have the tools I need to approach a problem.

PIMs is so enjoyable because it feels like I am learning something that is immediately applicable and impressive. I can (and now do!) apply sets in my work. The material in the book is still quite difficult for me (I’m still working through some of the questions in this chapter) but now that I have understood all of the concepts in the chapter I feel certain that the concepts in the book are all surmountable.

Thank you for creating such an excellent resource! My challenge to myself is to build up to a point where I can get through a chapter a week. There is so much mathematical information out there I still feel like I have just took the first step onto a vast playground.