KENDRIYA VIDYALAYA SANGATHAN LUCKNOW REGION

SECOND PREBOARD EXAM 2022-23

CLASS – XII

MATHEMATICS (041)

Time Allowed : 3 Hours | Maximum Marks : 80

General Instructions:

1 This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.

2 Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.

3 Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.

4 Section C has 6 Short Answer (SA)-type questions of 3 marks each.

5 Section D has 4 Long Answer (LA)-type questions of 5 marks each.

6 Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

SECTION A

(Multiple Choice Questions) Each question carries 1 mark

1 The corner points of the feasible region determined by the system of linear constraints are $(0, 10), (5, 5), (15, 15), (0, 20)$. Let $Z = px + qy$, where $p, q > 0$. Condition on p and q so that the maximum of Z occurs at both the points $(15, 15)$ and $(0, 20)$ is

(a) $p = q$ (b) $p = 2q$ (c) $q = 2p$ (d) $q = 3p$

2 If A is a square matrix such that $A^2 = A$, then the value of $7A - (I + A)^3$ is , where I is an identity matrix

(a) – I (b) I (c) A (d) A - I

3 If A_{ij} is the cofactor of the elements a_{ij} of the determinant $|2 \ -3 \ 5 \ 6 \ 0 \ 4 \ 1 \ 5 \ -7|$ then the value of $a_{32}.A_{32}$ is

(a) -110 (b) 0 (c) -22 (d) 110

4 If $f(x)$ is continuous at $x = 3$ and $f(x) = \{\frac{(x+3)^2-36}{x-3}, \ x \neq 3 \ k, \ x = 3$ then the value of k is

(a) 3 (b) 12 (c) 9 (d) 6

5 The solution set of the inequality $3x + 5y < 4$is

(a) an open half-plane not containing the origin.

(b) an open half-plane containing the origin.

(c) the whole XY-plane not containing the line $3x + 5y = 4$.
(d) a closed half plane containing the origin.

6 Given $\int e^x(tanx + 1)secxdx = e^x f(x) + c$, then $f(x)$ is
(a) $tanx$ (b) $secx$ (c) $secxtanx$ (d) none of these.

7 If m and n, respectively, are the order and the degree of the differential equation $\frac{d}{dx}\left[\left(\frac{dy}{dx}\right)\right]^4 = 0$, then $m + n =$

(a) 1 (b) 2 (c) 3 (d) 4

8 The general solution of the differential equation $ydx - xdy = 0$ is

(a) $y = cx$ (b) $xy = c$ (c)$x = cy^2$ (d) $y = cx^2$

9 If $f(x) = \int_0^x ysinydy$, then $f'(x)$ is

(a) $cosx + xsinx$ (b) $xsinx$ (c)$xcosx$ (d) $sinx + xcosx$

10 The scalar projection of the vector $3\hat{} - \jmath\hat{} - 2k\hat{}$ on the vector $\hat{} + 2\jmath\hat{} - 3k\hat{}$ is

(a) $\frac{7}{14}$ (b) $\frac{7}{\sqrt{14}}$ (c) 7 (d) $\frac{7}{2}$

11 Given a square matrix A of order 3×3, such that $|A| = 12$, then the value of $|A.adjA|$ is

(a) 1728 (b) 144 (c) 12 (d) none of these

12 If$|-\ 1\ 2\ 4\ 8\ | = |2\ x\ x\ -\ 4\ |, x =$?

(a) $\sqrt{2}$ (b) $\sqrt{4}$ (c)$2\sqrt{2}$ (d) $\pm2\sqrt{2}$

13 The value of $x + y$ from the equation $2(1\ 3\ 0\ x\ ) + (y\ 0\ 1\ 2\ ) = (5\ 6\ 1\ 8\ )$ is

(a) 3 (b) 6 (c) 9 (d) 5

14.If $\vec{a}$ and $\vec{b}$ are such that $|\vec{a}\cdot\vec{b}| = |\vec{a}\times\vec{b}|$, then the angle between $\vec{a}$ and $\vec{b}$ is

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$ (c)$\frac{\pi}{4}$ (d) $\frac{\pi}{2}$

15 The value of i. $(j \times k) + j.(k \times i) + k.(j \times i)$ is

(a) 1 (b) 3 (c) 0 (d) -1

16 If A and B are two independent events such that $P(A) = \frac{1}{2}, P(B) = p$ and $P(A \cup B) = \frac{3}{5}$, then the value of p is

(a) $\frac{1}{5}$ (b) $\frac{2}{5}$ (c)$\frac{3}{5}$ (d) $\frac{4}{5}$

17 In a triangle OAC, if B is the midpoint of side AC and $\vec{OA} = \vec{a}, \vec{OB} = \vec{b}$, then $\vec{OC}$ is

(a) $2\vec{b}$ (b) $2\vec{b} - \vec{a}$ (c)$\frac{\vec{a}+\vec{b}}{2}$(d) $\frac{2\vec{b}-\vec{a}}{2}$

18.If$x = t^2$ and $y = t^3$, then $\frac{dy}{dx}$ is equal to

(a) $\frac{2t}{3}$ (b) $2t$ (c)$3t$ (d) $\frac{3t}{2}$

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true but R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false but R is true.

19 Assertion (A) : f: $N \to N$ given by $f(x) = 5x$ is injective but not surjective

Reason (R) : If co-domain $\neq$ range , then the function is not surjective.

20 Assertion (A) : The direction-cosines of the line joining the points $(1, 0, 0)$ and $(0, 1, 1)$ is $\left(\frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$

Reason (R) : direction ratios = direction cosines.

SECTION B

This section comprises of very short answer type-questions (VSA) of 2 marks each

21.Find the principal value of $tan^{-1}(\sqrt{3}) - sec^{-1}(-2)$.

(OR)

Find the value of $sin^{-1}\left[sin\left(\frac{13\pi}{7}\right)\right]$

22.If $\vec{a} = 3i + 2j + 9k$and $\vec{b} = i + \mu j + 3k$, find μ, so that $\vec{a} + \vec{b}$ is perpendicular to $\vec{a} - \vec{b}$

23.Write the vector of magnitude 15 units in the direction of $i - 2j + 2k$.

24 Find $f'(x)$ if $f(x) = (sinx)^{sinx}$, for all $0 < x < \pi$

25 The total revenue in Rupees received from the sale of x units of a product is given by $R(x) = 13x^2 + 26x + 15$. Find the marginal revenue when $x = 7$.

SECTION C

(This section comprises of short answer type questions (SA) of 3 marks each)

26 Find $\int \frac{1}{\sqrt{3-2x-x^2}} dx$

27 Solve the following linear programming problem graphically. Minimize $Z = 3x + 9y$ subject to the constraints

$$x + 3y \leq 60, x + y \geq 10, x \leq y, x, y \geq 0$$

28.Evaluate: $\int \frac{x^2+1}{x^2-5x+6} dx$

29.Evaluate: $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{1+\sqrt{tanx}}$

30. Solve: $\left(xsin\frac{y}{x}\right)dy = \left(ysin\frac{y}{x} - x\right)dx$

(OR)

Solve: $ydx + \left(x - y^2\right)dy = 0$.

31.A couple has two children. Find the probability that both the children are girls, if it is known that the older child is a girl.

(OR)

A random variable X has the following probability distribution, determine k.

X	0	1	2	3	4	5	6	7
$P(X)$	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2 + k$

SECTION D

(This section comprises of long answer-type questions (LA) of 5 marks each)

32 Using matrix method, solve the following system of equations.

$$8x - 4y + z = 5$$

$10x + 6z = 4$

$8x + y + 6z = \frac{5}{2}$

(OR)

If $A = (1 \ -1 \ 0 \ 2 \ 3 \ 4 \ 0 \ 1 \ 2)$ and $B = (2 \ 2 \ -4 \ -4 \ 2 \ -4 \ 2 \ -1 \ 5)$ are two square matrices, find AB and hence solve the system of equations $x - y = 3, 2x + 3y + 4z = 17$ and $y + 2z = 7$

33. Find the shortest distance between the lines whose equations are

$$\vec{r}_1 = \hat{} + \jmath\hat{} + \lambda(2\hat{} - \jmath\hat{} + k\hat{}) \ and \ \vec{r}_2 = 2\hat{} + \jmath\hat{} - k\hat{} + \mu(3\hat{} - 5\jmath\hat{} + 2k\hat{})$$

(OR)

Find the vector equation of the line passing through the point $(1, 2, -4)$ and

perpendicular to the two lines $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$

34 Show that the relation S in the set $A = \{x \in Z: 0 \leq x \leq 12\}$ given by $S = \{(a, b): a, b \in Z, |a - b|$ is divisible by $4\}$ is an equivalence relation. Find the set of all elements related to 1 .

35 Using integration, find the area of the region in the first quadrant enclosed by the x-axis, the line $y = x$ and the circle $x^2 + y^2 = 32$.

SECTION E

(This section comprises of 3 case-study/passage-based questions of 4 marks each)

36.**Case study 1:**

Read the following information and answer the questions given below.

The temperature of a person during an intestinal illness is given by $f(x) = -0.1x^2 + 1.2x + 98.6, 0 \leq x \leq 12$, where $f(x)$ is the temperature in $^{\circ}F$ at x days.

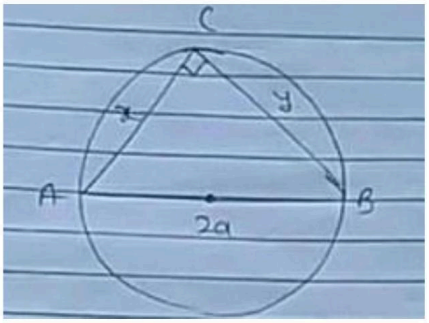
(i) Is the function differentiable in $(0, 12)$? Justify your answer. (1)

(ii) Find the critical point of the function in $(0, 12)$ (1)

(iii) Find the intervals in which the function $f(x)$ is strictly increasing or strictly decreasing. (2)

37.**Case study 2:**

In a circular field the authority wants to design a triangular field with the maximum possible area. The field is given by the following diagram with AB as a diameter of a circle and C is any point on the circle. Let $AC = x$ and $CB = y$



(a) Write y in terms of x

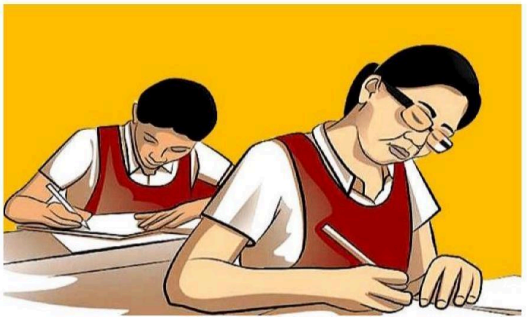
(b) Write the Area of triangle in terms of x

(c) Find the rate of change of area with respect to x.

(d) Find the value of $x$and y, when the area is maximum. $(4\times1 = 4)$

38 Case study 3:

In answering a question on a multiple-choice test for class XII, a student either knows the answer or guesses. Let $3/5$ be the probability that he knows the answer and $2/5$ be the probability that he guesses. Assume that a student who guesses at the answer will be correct with probability $1/3$. Let E 1, E2, E be the events that the student knows the answer,guesses the answer and answers correctly respectively.



Based on the above information, answer the following

1 What is the value of $P(E1)$? (1)

2 Find the value of $P(E|E1)$. (1)

3 What is the probability that the student knows the answer given that he answered it correctly? (2)