# Economics Assignment: Rates and Margins

## Assignment Overview

Many fascinating economic questions involve rates and decisions at the margin. To study the health of an economy, we may look at the rate of capital accumulation, technological progress, or inflation growth. We are all likely interested in the rate of change of monetary value. Additionally, optimal decisions are made at the margin. Firms should consider the marginal profit from producing and selling one more unit of a good when deciding how much to produce. Consumers should consider the marginal benefit of consuming one more unit of a good when making their optimal decisions.

Whenever we are dealing with rates and margins, integration allows us to understand how those relate to totals. In this assignment, you will explore this in the context of 5 economics-related questions.

## 1. Monetary Conversion

Different areas of the world use different forms of currency, and the conversion factor between different types of currency is ever-fluctuating. Suppose that Stan lives in the United States and is planning a vacation to Europe. He looks at the conversion between U.S. Dollars and Euros over the previous year, and he determines that the following function accurately models the fluctuation in the value of 1 USD in Euros over time.

$$E(t) = \frac{1}{20}\left[sin(2t) + cos(t)\right] + \frac{17}{20}$$

where $E(t)$is the value of 1 USD in Euros at time $t$, where $t$ is in months.

1. *What was the average value (in Euros) of 1 US Dollar over the first 6 months of the year? (Note: $t = 0$ corresponds to the very beginning of year)*
2. *Stan is thinking of traveling to Europe in July and August. What was the average value of 1 US Dollar over those months in the previous year? (Note: This corresponds to the time from $t = 6$ to $t = 8$)*

3. *According to last year's data, what would be the worst month to travel? In other words, which month had the lowest average value of 1 US Dollar? (Hint: It may be useful to look at a graph first, then confirming your answer with calculations)*

## 2. Marginal Revenue and Total Revenue

The *total revenue* that a firm receives from producing $Q$ units of a good is defined by $TR(Q) = p * Q$, where $p$ is the price of the good. *Marginal revenue,* denoted $MR(Q)$, can be thought of as the amount of additional revenue the firm receives from selling the $Q$ th unit. In this problem, you will be given a marginal revenue function and asked to find the total revenue function under two different levels of competition that the firm faces.

1. *In a perfectly competitive market, a firm cannot influence the price of a good by changing its level of output. Therefore, its marginal revenue is constant and equal to the price $p$ because every unit produced is sold for $p$ dollars. Suppose that for a specific firm, $MR(Q) = 20$. If $TR(0) = 0$ (the firm earns no revenue if it produces nothing), what is the total revenue function of the firm?*
2. *If a firm is a monopoly (meaning it has no competitors), it can influence the price of the good by varying its level of output $Q$. Suppose that for a specific monopoly, $MR(Q) =- \frac{2}{7}Q + 100$. If $TR(0) = 0$, what is the total revenue function of the firm?*
3. *Explain the shape of the total revenue function you found in 2.2. Why would total revenue begin to decrease after a certain quantity?*

## 3. Capital Accumulation

In economics, the term "investment" refers to increasing a company or economy's capital. This could include purchasing a factory or land. The rate of investment within an economy can vary, and knowing the *total capital* or *total capital accumulation* over a period of time is important to determining the health of an economy.

Suppose the rate of investment in a particular economy is given by the function

$$I(t) = \frac{2ln(t+1)}{t+1}$$

where $I(t)$ is the rate of investment at time $t$. Let $K(t)$ denote the total amount of capital in the economy at time $t$, where $K'(t) = I(t)$.

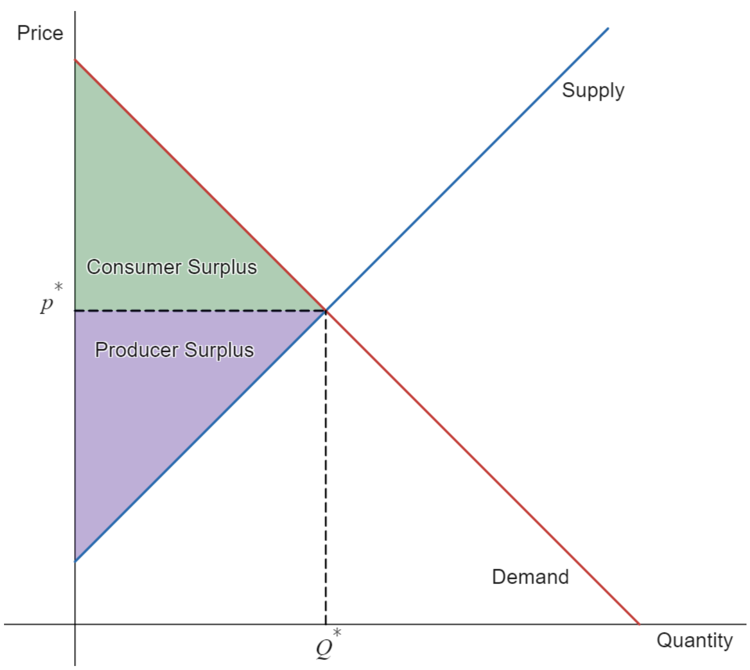
1. *If $K(0) = 500$, what is the total capital function $K(t)$?*
2. *What is the total amount of capital in the economy at time $t = 3$?*

3. *Using your answer from 3.1, find the capital accumulation between* $t = 1$ *and* $t = 5$ .

## 4. Consumer, Producer, and Total Surplus

Surplus is the benefit that results from trade. When a consumer purchases a good, the difference between what the consumer would be *willing* to pay and what the consumer *actually* paid is called consumer surplus. Similarly, when a producer sells a good, the difference between the revenue received and how much it cost to produce the good is called producer surplus.

The inverse demand curve can be thought of as the willingness-to-pay curve for consumers. The inverse supply curve can be thought of as the marginal cost curve for producers. Therefore, the area between the inverse demand curve and the horizontal line at the price of the good represents consumer surplus, and the area between the horizontal line at the price of the good and the inverse supply curve represents producer surplus. The two areas added together gives the total surplus from trade. The following is a graphical depiction of consumer and producer surplus when quantity $Q^*$ is traded at price $p^*$.




Suppose the inverse demand curve is given by

$$p = \quad - Q_D + 10$$

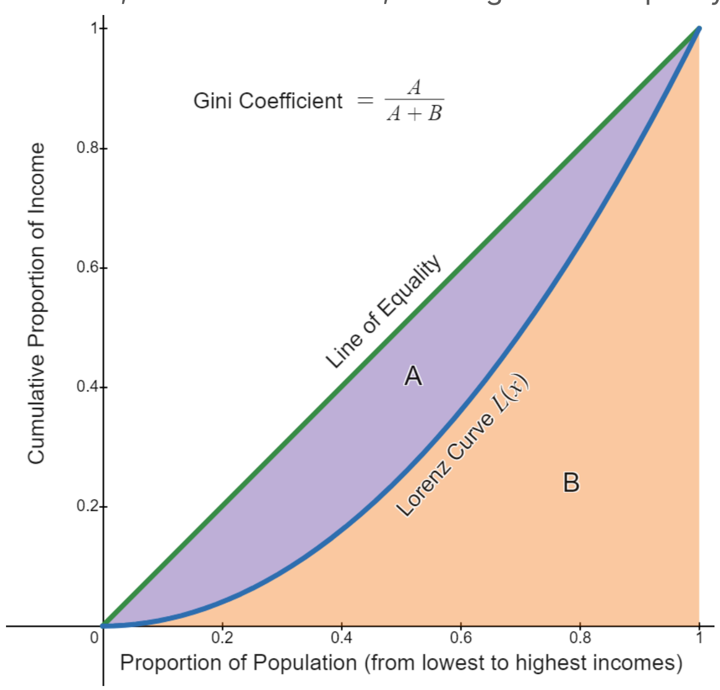
And the inverse supply curve is given by

$$p = \frac{1}{2} Q_S + 1$$

1. *Find an expression for $TS(Q)$, the total surplus when $Q$ units are traded. (Hint: What area on the graph are you trying to find?) At what quantity $Q^*$ is the total surplus maximized? What is the corresponding price $p^*$? Graph the curves and identify this point.*
2. *At the price and quantity found in 4.1, what is the consumer surplus? What is the producer surplus? (Hint: These should add together to make the total surplus). Are consumers or producers better off when that quantity of the good is traded?*

## 5. Measuring Income Inequality

Equality is an important concept in welfare economics. The Lorenz Curve and the Gini Coefficient help to measure the degree of income equality in an economy. As shown in the graph below, the Lorenz Curve (denoted $L(x)$) gives the cumulative proportion of an economy's income held by the lowest $x$ percent of income earners. Thus, if an economy has perfect income equality, the Lorenz Curve will be the same as the line of equality $y = x$. The larger the area $A$, on the other hand, the larger the inequality.



The Gini Coefficient quantifies the amount of inequality. As defined in the figure above, the Gini Coefficient can take on values from 0 (perfect equality) to 1 (perfect inequality). To be clear, $A$ is the area in purple between the line of equality and the Lorenz Curve, and $B$ is the area in orange between the Lorenz Curve and the x-axis.

1. *Suppose an economy has a Lorenz Curve* $L_1(x) = x^3$*. Calculate this economy's Gini Coefficient.*
2. *Suppose another economy has a Lorenz Curve* $L_2(x) = \frac{e^x - 1}{e - 1}$*. Calculate this economy's Gini Coefficient.*
3. *Based on your answers to the two previous questions, which economy (1 or 2) has the greatest amount of income inequality?*