

Potential energy and potential

$$G = 6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$$

$$\text{mass of the Earth} = 6.0 \times 10^{24} \text{ kg}$$

$$\text{radius of the Earth} = 6.4 \times 10^6 \text{ m}$$

$$\text{gravitational field strength close to the surface of the Earth is } 9.8 \text{ N kg}^{-1}$$

- 1) What is the gravitational potential energy of a 60 kg student on the surface of the Earth? What then, is the minimum energy that would be required to get this student completely out of the Earth's gravitational field?
- 2) What is the potential you experience at the surface of the Earth? How would the answer be different if your friend who weighs more than you worked it out?
- 3) What is the potential at a height of 36 000 km from the Earth's surface? This is the height of a geostationary orbit. What is the potential difference between the surface of the Earth, and geostationary orbit height?
- 4) So, what minimum energy is required to launch a 150 kg satellite into geostationary orbit? Why is the actual value a lot more?
- 5) What is the potential difference between the top and bottom of an office block on the surface of the Earth that has a height of 50 m? (Hint – this close to the surface of the Earth, the field is roughly uniform, so you can use simple equations for change in GPE or change in potential).

Data required:

$$G = 6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$$

$$\text{mass of the Earth} = 6.0 \times 10^{24} \text{ kg}$$

$$\text{radius of the Earth} = 6.4 \times 10^6 \text{ m}$$

$$\text{gravitational field strength close to the surface of the Earth is } 9.8 \text{ N kg}^{-1}$$

- 1) What is the gravitational potential energy of a 60 kg student on the surface of the Earth? What then, is the minimum energy that would be required to get this student completely out of the Earth's gravitational field?

$$\text{GPE} = -GMm/r = (-6.67 \times 10^{-11} \times 6.0 \times 10^{24} \times 60) / 6.4 \times 10^6 = -3.8 \times 10^9 \text{ J}$$

To get the student away from the Earth's field completely will require taking him to infinity where his GPE will be zero. Thus we need to give him at least $3.8 \times 10^9 \text{ J}$ of energy somehow.

- 2) What is the potential you experience at the surface of the Earth? How would the answer be different if your friend who weighs more than you worked it out?

$$V = -GM/r = (-6.67 \times 10^{-11} \times 6.0 \times 10^{24}) / 6.4 \times 10^6 = -6.3 \times 10^7 \text{ J kg}^{-1} \text{ or } -63 \text{ MJ kg}^{-1}$$

The answer is the same for everyone – potential is a property of the field and all objects at the same point in the field are subject to the same potential, even though their potential energies may be different.

- 3) What is the potential at a height of 36 000 km from the Earth's surface? This is the height of a geostationary orbit. What is the potential difference between the surface of the Earth, and geostationary orbit height?

$$V = -GM/r = (-6.67 \times 10^{-11} \times 6.0 \times 10^{24}) / (6.4 \times 10^6 + 3.6 \times 10^7) = -9.4 \times 10^6 \text{ J kg}^{-1} \text{ or } -9.4 \text{ MJ kg}^{-1}$$

The potential difference is then $-9.4 - (-63) = 53 \text{ MJ kg}^{-1}$ (or $5.3 \times 10^7 \text{ J kg}^{-1}$).

Note that this potential difference is positive; indicating that to get to geostationary height, a large amount of energy would be required. There is no problem with potential differences being positive, but absolute values of potential themselves must always be negative.

- 4) So, what minimum energy is required to launch a 150 kg satellite into geostationary orbit? Why is the actual value a lot more?

Well, we now know that the potential difference between the Earth's surface and the geostationary orbit height is $+5.3 \times 10^7 \text{ J kg}^{-1}$. In other words, for each kg of mass lifted from Earth to geostationary orbit, $5.3 \times 10^7 \text{ J}$ of energy is required. Therefore, for a satellite of mass 150 kg, energy required = $150 \times 5.3 \times 10^7 = 8.0 \times 10^9 \text{ J}$ (or 8.0 GJ).

Actually, much more energy is required because there is drag with the Earth's atmosphere to overcome on the way up, and also the rocket itself (and its fuel) have mass that needs to be lifted into orbit too! The answer we've arrived at is only the energy required to lift the satellite against the gravitational pull of the Earth.

- 5) What is the potential difference between the top and bottom of an office block on the surface of the Earth which has a height of 50 m? (Hint – this close to the surface of the Earth, and the field is roughly uniform, so you can use simple equations for change in GPE or change in potential). What minimum energy do you use in walking up to the top of the building?

For a uniform field, change in potential = $gh = 9.8 \times 50 = 490 \text{ J kg}^{-1}$

For a 60 kg student, the energy required to climb to the top of the building is then $60 \times 490 = 2.94 \times 10^4 = 2.9 \times 10^4 \text{ J}$ or 29 kJ. For information, that's about 7 kcal where a kcal is what is referred to as a 'calorie' on diet adverts. A chocolate bar is typically 200 calories or more! Students can rest assured that they use up more energy than this in climbing to the top of the building because their bodies are not 100 % efficient, and they warm up.